Robust Decentralized Controller Design for Power Systems Using Matrix Inequalities Approaches

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***Abstract*-- This paper presents robust decentralized power system stabilizer (PSS) design approaches for power system that can be expressed as minimizing a linear objective function under linear matrix inequality (LMI) in tandem with bilinear matrix inequality (BMI) constraints. In particular, the paper addresses two approaches with their practical implications for large power systems. These approaches are: i) based on the concept of interconnection method for designing robust decentralized dynamic output feedback controllers that guarantee robust connective stability of the overall system, and ii) based on parameter continuation method involving matrix inequalities for designing reduced-order decentralized H_∞ dynamic output feedback controllers. Furthermore, the paper proposes algorithms to solve such optimization problems using sequential linear matrix inequality programming and general parameterized two-stage matrix inequalities optimization methods. It is also shown that the approaches presented in this paper can be used for designing realistic robust PSSs, notably so-called reduced-order robust PSSs design for power systems.**

***Index Terms*--Decentralized control, interconnected systems, nonlinear systems, optimization methods, robustness.**

I. Introduction

THE electric power systems such as those in Europe and North America have recently experienced unprecedented changes due to the emergence of deregulation in the sector and the development of competitive electricity market for generations and energy services. These changes have caused a noticeable uncertainty in the load flows, and have pushed the networks further to their operational limits. Besides, the integration of offshore wind generation plants into the existing network is also expected to have a significant impact on the load flow of the system as well as the dynamic behaviour of the network. On the other hand, the transmission grids have seen very little expansion due to environmental restrictions. As a result, available transmission and generation facilities are highly utilized with large amounts of power interchanges taking place through tie-lines and geographical regions. It is also expected that this trend will continue in the future and result in more stringent operational requirements to maintain reliable services and adequate system dynamic performances. Critical controls like excitation systems, power system stabilizers, static VAR compensators will play increasingly key roles in maintaining adequate system dynamic performance. Moreover, proper design of these control systems that takes into account the continual changes in the structure of the network is imperative to ensure/guarantee robustness over wide operating conditions in the system.

In order to address the problems described above, robust decentralized control mechanisms have been proposed which guarantee accurate prediction of system responses and system robustness to disturbances under various operation conditions. However, decentralized control of large power systems presents new challenges. Available analysis methods do not allow controlling and/or modelling of complex interactions among subsystems, particularly when large power systems are driven to the point of nonlinearity. This paper presents the various facets of this research area including methodological, structural and computational issues pertaining to the formulation of robust decentralised controller design problems.

In the recent past, designing decentralized controller structure for interconnected large power systems that conforms to each subsystem was one of the predominant subjects [1]–[4], and has also been intensively studied with most attention of guaranteeing the connective stability of the whole system despite the interconnection uncertainties among the different subsystems. A large number of results concerning robust decentralized stabilization of interconnected power systems based on the interconnection modelling approach - an approach which explicitly takes into accounts the interactions among the different subsystems - have been reported (to cite a few; [5]–[11]). In the work of [8], interesting decentralized turbine/governor controller scheme for power system has been presented. The salient feature of this approach is the use LMI optimization technique [12] to address the problem of robust stability in the presence of interconnection uncertainties among the subsystems. However, the local state feedback controllers designed through the approach need the corresponding state information of the subsystems and which may be either impossible or simply impractical to obtain measurements of the state information for all individual subsystems. An

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extension to robust static exciter output feedback controller scheme for power system, based on the approach outlined in [13], was presented in [9]. However, the additional information structure constraints that are used for decomposing the controller strategy might also lead to non-true decentralized controller schemes for large power system. On the other hand, efforts have also been made to extend the application of robust control techniques to large power systems, such as L_∞ - optimization [14], [15], H_∞ - optimization [16], [17], structured singular value (SSV or μ) technique [18], [19]. Specially, the work presented in [19] relies on the sequential μ - synthesis technique where the design procedure is carried-out successively for each local input-output pairs in the system. Though the individual controllers are sequentially designed to guarantee the robust stability and performance of the whole system, the reliability of the decentralized controller depends on the order in which the design procedures of the individual controllers are carried-out. Moreover, a failure in the lower loop might affect the stability and/or the performance of the whole system. It is also clear from the nature of the problem that the order of the controller increases for each sequentially designed local controller. Another attempt is also made to apply a linear parameter varying (LPV) technique for designing decentralized power system stabilizers for a large power system [20]. The resulting controllers designed through this technique, however, are typically high order - at least as high as the system since the technique relies on H_∞ - optimization; and besides the problem formulation attempts to solve an infinite-dimensional LMI type optimization where the latter problem is computationally very demanding. Moreover, the approach did not consider the complete power system model with all interconnections in the design formulation.

This paper mainly focuses on developing robust decentralized control techniques for power systems, with special emphasis on problems that can be expressed as minimizing a linear objective function under LMI in tandem with BMI constraints where the latter types of constraints render a computational challenge in designing decentralized controllers. Therefore, this paper proposes alternative computational schemes that solve the robust decentralized controllers design problem for power systems using: i) sequential linear matrix inequality programming, and, ii) generalized parameter continuation method involving matrix inequalities. Moreover, these algorithms are computationally efficient and can be conveniently implemented with the available Semidefinite Optimization (SDO) solvers such as MATLAB *LMI Toolbox* [21]. The local convergence properties of these algorithms for designing (sub)-optimal robust decentralized controllers have shown the effectiveness of the approaches for designing decentralized power system stabilizers (PSSs).

The rest of the paper is organized in five sections. The next two sections, i.e., Section II and III, are written to provide a self-contained treatment of the proposed approaches used for designing robust decentralized controllers for power systems. In Section II, the robust decentralized dynamic output feedback controller design problem for an interconnected power system is formulated in terms of minimizing a linear objective function under linear matrix inequality (LMI) constraints coupled through bilinear matrix equation. A sequential LMI programming algorithm which is used to solve (sub)-optimally such design problem is also presented in this section. Section III presents robust decentralized H_∞ controller design for power system based on a generalized parameter continuation method involving matrix inequalities. Moreover, Section IV provides design examples for these approaches using real data for generators, excites and governors from industry. Simulation results for power system stabilizer together with performance indices are also given for both approaches. Finally, summary of the approaches is given in Section VI.

II. Robust Decentralized Dynamic Output Feedback Controller Design for Power Systems: An LMI Approach

A. Mathematical Model for Large - Scale Systems with Interconnection Terms

Consider a large-scale interconnected system $\mathbf{S}$ composed of N subsystems $\mathbf{S}_i$, $i=1,2,\ldots,N$ described by the following equations:

$$\begin{aligned}\dot{\mathbf{x}}_i(t) &= \mathbf{A}_i\mathbf{x}_i(t)+\mathbf{B}_i\mathbf{u}_i(t)+\sum_{j=1}^{N}\left(\mathbf{M}_{ij}+\Delta\mathbf{M}_{ij}(t)\right)\mathbf{x}_j(t)\\ \mathbf{y}_i(t) &= \mathbf{C}_i\mathbf{x}_i(t)\end{aligned} \tag{1}$$

where $\mathbf{x}_i(t)\in\Re^{n_i}$ is the state vector, $\mathbf{u}_i(t)\in\Re^{m_i}$ is the control variable, $\mathbf{y}_i(t)\in\Re^{q_i}$ is the output variable of the subsystem $\mathbf{S}_i$. The matrices $\mathbf{A}_i$, $\mathbf{B}_i$, $\mathbf{C}_i$ and $\mathbf{M}_{ij}$ are constant matrices of appropriate dimensions conformable to each $\mathbf{S}_i$. Furthermore, the matrix $\mathbf{M}_{ij}$ represents the interconnections and/or interactions among the subsystems. The term $\Delta\mathbf{M}_{ij}(\cdot)$ is intentionally introduced to take into account the effect of any deviation from the given operating condition due to nonlinearities and structural changes in the system.

The interconnections and uncertainties terms in (1), which are used to characterize the interactions among the subsystems and the effects of nonlinearities within each subsystem of a power system, can be rewritten in the following form

$$\mathbf{h}_i(t,\mathbf{x})=\sum_{j=1}^{N}\left(\mathbf{M}_{ij}+\Delta\mathbf{M}_{ij}(t)\right)\mathbf{x}_j(t) \tag{2}$$

Furthermore, assume that the following quadratic constraints hold:

$$\mathbf{h}_i(t,\mathbf{x})^{\mathrm{T}}\mathbf{h}_i(t,\mathbf{x}) \preceq \xi_i^2\mathbf{x}^T(t)\mathbf{H}_i^T\mathbf{H}_i\mathbf{x}(t) \tag{3}$$

where $\xi_i>0$ are parameters related to interconnection uncertainties in the system and $\mathbf{H}_i$ are matrices that reflect the nature of interconnections among subsystems. Moreover, assume that the pairs $(\mathbf{A}_i,\mathbf{B}_i)$ and $(\mathbf{A}_i,\mathbf{C}_i)$ are stabilizable and detectable, respectively.

With the assumption of no "overlapping" among $\mathbf{x}_i(t)$, the state variable $\mathbf{x}(t) \in \Re^n$ of the overall system is denoted by $\mathbf{x}(t) = [\mathbf{x}_1^T(\cdot), \mathbf{x}_2^T(\cdot), \ldots, \mathbf{x}_N^T(\cdot)]^T$. Thus, the interconnected system $\mathbf{S}$ can then be written in a compact form as

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{A}_D \mathbf{x}(t) + \mathbf{B}_D \mathbf{u}(t) + \mathbf{h}(t, \mathbf{x}) \\ \mathbf{y}(t) &= \mathbf{C}_D \mathbf{x}(t) \end{aligned} \quad (4)$$

where $\mathbf{x} \in \Re^n$ is the state, $\mathbf{u} \in \Re^m$ is the input and $\mathbf{y} \in \Re^q$ is the output of the overall system $\mathbf{S}$, and all matrices are constant matrices of appropriate dimensions with $\mathbf{A}_D = \mathrm{diag}\{\mathbf{A}_1, \mathbf{A}_2, \ldots, \mathbf{A}_N\}$, $\mathbf{B}_D = \mathrm{diag}\{\mathbf{B}_1, \mathbf{B}_2, \ldots, \mathbf{B}_N\}$, and $\mathbf{C}_D = \mathrm{diag}\{\mathbf{C}_1, \mathbf{C}_2, \ldots, \mathbf{C}_N\}$.

The interconnection and uncertainty function $\mathbf{h}(t,\mathbf{x}) = [\mathbf{h}_1^T(t,\mathbf{x}), \mathbf{h}_2^T(t,\mathbf{x}), \ldots, \mathbf{h}_N^T(t,\mathbf{x})]^T$ is bounded as

$$\mathbf{h}^T(t,\mathbf{x})\mathbf{h}(t,\mathbf{x}) \le \mathbf{x}(t)^T \left(\sum_{i=1}^{N} \xi_i^2 \mathbf{H}_i^T \mathbf{H}_i \right) \mathbf{x}(t) \quad (5)$$

In the following, consider designing a decentralized dynamic output feedback controller of an n_{ci}th-order for the ith-subsystem given (1) of the form:

$$\begin{aligned} \dot{\mathbf{x}}_{ci}(t) &= \mathbf{A}_{ci}\mathbf{x}_{ci}(t) + \mathbf{B}_{ci}\mathbf{y}_i(t) \\ \mathbf{u}_i(t) &= \mathbf{C}_{ci}\mathbf{x}_{ci}(t) + \mathbf{D}_{ci}\mathbf{y}_i(t) \end{aligned} \quad (6)$$

where $\mathbf{x}_{ci}(t) \in \Re^{n_{ci}}$ is the state vector of the ith-local controller and $\mathbf{A}_{ci}$, $\mathbf{B}_{ci}$, $\mathbf{C}_{ci}$ and $\mathbf{D}_{ci}$ are constant matrices to be determined during the design. After augmenting the ith-controller in the system, the state space equation for the ith-extended subsystem will have the following form

$$\begin{bmatrix} \dot{\mathbf{x}}_i(t) \\ \dot{\mathbf{x}}_{ci}(t) \end{bmatrix} = \begin{bmatrix} \mathbf{A}_i + \mathbf{B}_i\mathbf{D}_{ci}\mathbf{C}_i & \mathbf{B}_i\mathbf{C}_{ci} \\ \mathbf{B}_{ci}\mathbf{C}_i & \mathbf{A}_{ci} \end{bmatrix} \begin{bmatrix} \mathbf{x}_i(t) \\ \mathbf{x}_{ci}(t) \end{bmatrix} + \begin{bmatrix} \mathbf{h}_i(t,\mathbf{x}) \\ \mathbf{0} \end{bmatrix} \quad (7)$$

which further can be rewritten as

$$\begin{bmatrix} \dot{\mathbf{x}}_i(t) \\ \dot{\mathbf{x}}_{c_i}(t) \end{bmatrix} = \left\{ \begin{bmatrix} \mathbf{A}_i & \mathbf{0}_{n_i \times n_{c_i}} \\ \mathbf{0}_{n_{c_i} \times n_i} & \mathbf{0}_{n_{c_i} \times n_{c_i}} \end{bmatrix} + \begin{bmatrix} \mathbf{0}_{n_{c_i} \times n_{c_i}} & \mathbf{B}_i \\ \mathbf{I}_{n_{c_i}} & \mathbf{0}_{n_{c_i} \times m_i} \end{bmatrix} \begin{bmatrix} \mathbf{A}_{c_i} & \mathbf{B}_{c_i} \\ \mathbf{C}_{c_i} & \mathbf{D}_{c_i} \end{bmatrix} \begin{bmatrix} \mathbf{0}_{n_{c_i} \times n_{c_i}} & \mathbf{I}_{n_{c_i}} \\ \mathbf{C}_i & \mathbf{0}_{q_i \times n_{c_i}} \end{bmatrix} \right\} \begin{bmatrix} \mathbf{x}_i(t) \\ \mathbf{x}_{c_i}(t) \end{bmatrix} + \begin{bmatrix} \mathbf{h}_i(t,\mathbf{x}) \\ \mathbf{0} \end{bmatrix} \quad (8)$$

With minor abuse of notation, the above equation can be rewritten in a closed form

$$\dot{\tilde{\mathbf{x}}}_i(t) = [\tilde{\mathbf{A}}_i + \tilde{\mathbf{B}}_i \mathbf{K}_i \tilde{\mathbf{C}}_i]\tilde{\mathbf{x}}_i(t) + \tilde{\mathbf{h}}_i(t,\tilde{\mathbf{x}}) \quad (9)$$

where $\tilde{\mathbf{x}}_i(t) = [\mathbf{x}_i^T(t) \quad \mathbf{x}_{ci}^T(t)]^T$, and the matrices $\tilde{\mathbf{A}}_i$, $\tilde{\mathbf{B}}_i$, $\tilde{\mathbf{C}}_i$ and $\mathbf{K}_i$ are given as follows

$$\begin{aligned} \tilde{\mathbf{A}}_i &= \begin{bmatrix} \mathbf{A}_i & \mathbf{0}_{n_i \times n_{ci}} \\ \mathbf{0}_{n_{ci} \times n_i} & \mathbf{0}_{n_{ci} \times n_{ci}} \end{bmatrix}, & \tilde{\mathbf{B}}_i &= \begin{bmatrix} \mathbf{0}_{n_{ci} \times n_{ci}} & \mathbf{B}_i \\ \mathbf{I}_{n_{ci}} & \mathbf{0}_{n_{ci} \times m_i} \end{bmatrix}, \\ \tilde{\mathbf{C}}_i &= \begin{bmatrix} \mathbf{0}_{n_{ci} \times n_{ci}} & \mathbf{I}_{n_{ci}} \\ \mathbf{C}_i & \mathbf{0}_{q_i \times n_{ci}} \end{bmatrix}, & \mathbf{K}_i &= \begin{bmatrix} \mathbf{A}_{ci} & \mathbf{B}_{ci} \\ \mathbf{C}_{ci} & \mathbf{D}_{ci} \end{bmatrix} \end{aligned} \quad (10)$$

Moreover, the function $\tilde{\mathbf{h}}_i(t,\tilde{\mathbf{x}})$ satisfies the following quadratic constraint

$$\tilde{\mathbf{h}}_i(t,\tilde{\mathbf{x}})^{\mathrm{T}}\tilde{\mathbf{h}}_i(t,\tilde{\mathbf{x}}) \le \xi_i^2 \tilde{\mathbf{x}}^{\mathrm{T}}(t)\, \tilde{\mathbf{H}}_i^{\mathrm{T}}\tilde{\mathbf{H}}_i\, \tilde{\mathbf{x}}(t) \quad (11)$$

where the $\tilde{\mathbf{H}}_i \in \Re^{p_i \times (n + \sum_i^N n_{ci})}$ is partitioned according to the following

$$\tilde{\mathbf{H}}_i = [\mathbf{H}_{i1} \quad \mathbf{0}_{n \times n_{c1}} \mid \mathbf{H}_{i2} \quad \mathbf{0}_{n \times n_{c2}} \mid \cdots \mid \mathbf{H}_{iN} \quad \mathbf{0}_{n \times n_{cN}}] \quad (12)$$

with $\mathbf{H}_{ij} \in \Re^{p_i \times n_i}$ for $j = 1, 2, \ldots, N$.

Thus, the overall interconnected system can then be rewritten in a compact form as

$$\dot{\tilde{\mathbf{x}}}(t) = [\mathbf{A}_D + \mathbf{B}_D \mathbf{K}_D \mathbf{C}_D]\tilde{\mathbf{x}}(t) + \tilde{\mathbf{h}}(t,\tilde{\mathbf{x}}) \quad (13)$$

where $\tilde{\mathbf{x}}(t) = [\tilde{\mathbf{x}}_1^T(t), \tilde{\mathbf{x}}_2^T(t), \ldots, \tilde{\mathbf{x}}_N^T(t)]^T$ and all matrices are constant matrices of appropriate dimensions with $\tilde{\mathbf{A}}_D = \mathrm{diag}\{\tilde{\mathbf{A}}_1, \tilde{\mathbf{A}}_2, \ldots, \tilde{\mathbf{A}}_N\}$, $\tilde{\mathbf{B}}_D = \mathrm{diag}\{\tilde{\mathbf{B}}_1, \tilde{\mathbf{B}}_2, \ldots, \tilde{\mathbf{B}}_N\}$, $\tilde{\mathbf{C}}_D = \mathrm{diag}\{\tilde{\mathbf{C}}_1, \tilde{\mathbf{C}}_2, \ldots, \tilde{\mathbf{C}}_N\}$ and $\mathbf{K}_D = \mathrm{diag}\{\mathbf{K}_1, \mathbf{K}_2, \ldots, \mathbf{K}_N\}$. Moreover, the function $\tilde{\mathbf{h}}(t,\tilde{\mathbf{x}}) = [\tilde{\mathbf{h}}_1^T(t,\tilde{\mathbf{x}}), \tilde{\mathbf{h}}_2^T(t,\tilde{\mathbf{x}}), \ldots, \tilde{\mathbf{h}}_N^T(t,\tilde{\mathbf{x}})]^T$ is bounded as

$$\tilde{\mathbf{h}}^T(t,\tilde{\mathbf{x}})\tilde{\mathbf{h}}(t,\tilde{\mathbf{x}}) \le \tilde{\mathbf{x}}^T(t) \left[\sum_{i=1}^{N} \xi_i^2 \tilde{\mathbf{H}}_i^T \tilde{\mathbf{H}}_i \right] \tilde{\mathbf{x}}(t) \quad (14)$$

Theorems 1 and 2 (see APPENDIX) are instrumental in establishing the robust stability of the closed-loop interconnected system (13) via a decentralized robust control strategy (6) under the constraints (14) on the function $\tilde{\mathbf{h}}(t,\tilde{\mathbf{x}})$.

Thus, the problem of designing a decentralized control strategy for the interconnected system (1), which at the same time maximizing the tolerable upper bounds on the interconnection and nonlinearity uncertainties, has the following form

$$\begin{aligned} &\text{Min} \quad \mathit{Trace}\,(\boldsymbol{\Gamma}) \\ &\text{subject to (35)} \end{aligned} \quad (15)$$

where $\boldsymbol{\Gamma} = \mathrm{diag}\{\gamma_1, \gamma_2, \ldots, \gamma_N\}$.

Therefore, the problem of designing a decentralized control strategy given in (15) for the interconnected system (1) can be restated as a nonconvex optimization problem of the form

$$\begin{aligned} &\text{Min} \quad \mathit{Trace}\,(\boldsymbol{\Gamma}) \\ &\text{subject to (36), (37) and (38)} \end{aligned} \quad (16)$$

The coupling constraint in (38) can be further relaxed as an LMI condition as follows:

$$\begin{bmatrix} \mathbf{Y}_D & \mathbf{I} \\ \mathbf{I} & \mathbf{X}_D \end{bmatrix} \succeq \mathbf{0} \quad (17)$$

Furthermore, using the cone-complementarity approach, there exists a decentralized robust output stabilizing controller $\mathbf{K}_D$ if the global minimum of the following optimization problem

$$\begin{aligned} &\text{Min} \quad \mathit{Trace}\,(\boldsymbol{\Gamma}) + \mathit{Trace}\,(\mathbf{Y}_D\mathbf{X}_D) \\ &\text{subject to } \mathbf{Y}_D \succ \mathbf{0},\ \mathbf{X}_D \succ \mathbf{0},\ \boldsymbol{\Gamma} \succ \mathbf{0}, \text{ and} \\ &\text{Equations (36), (37) and (17)} \end{aligned} \quad (18)$$

is $\alpha^* + \sum_{i=1}^{N} n_i$ where $\mathit{Trace}\,(\boldsymbol{\Gamma}) \le \alpha^*$.

B. (Sub)-optimal Robust Decentralized Dynamic Output Feedback Controller Design Using Sequential LMI Programming Method

The optimization in (18) is a nonconvex optimization problem due to the bilinear matrix term in the objective functional. To compute the (sub)-optimal solution of this problem, an algorithm based on a sequential LMIs programming method is proposed. The idea behind this algorithm is to linearize the cost functional in (18) with respect to its variables and then to solve the resulting convex optimization problem subject to the LMI condition (17) at each iteration. Moreover, the algorithm will set appropriately the direction of the feasible solution by solving a subclass problem of Newton-type updating coefficient. Furthermore, the solution of the optimization problem is monotonically nonincreasing, i.e. the solution decreases in each iteration with the lower bound being $\sum_{i=1}^{N} n_i$ plus some positive number. Thus, the sequential LMI programming method for finding the decentralized dynamic output feedback controllers has the following optimization algorithms.

Algorithm I: Sequential LMI Programming Method

Solve the LMI feasibiliy problem of (18) for $\mathbf{Y}_D$, $\mathbf{X}_D$, and $\mathbf{\Gamma}$

Set the solutions $(\mathbf{Y}_D^0,\ \mathbf{X}_D^0) := (\mathbf{Y}_D,\ \mathbf{X}_D)$ and $\mu^0 := 2Trace\,(\mathbf{Y}_D^0\,\mathbf{X}_D^0)\,/n$

Repeat unitl $\mu^k :\leq \varepsilon$ (for small number $\varepsilon > 0$)

(1). Solve the following optimizati on problem for $\mathbf{Y}_D$, $\mathbf{X}_D$, and $\mathbf{\Gamma}$

Min $Trace\ (\mathbf{Y}_D\mathbf{X}_D^k + \mathbf{Y}_D^k\mathbf{X}_D) + Trace\ (\ \mathbf{\Gamma}\ )$

subject to $\mathbf{Y}_D \succ \mathbf{0}$, $\mathbf{X}_D \succ \mathbf{0}$, $\mathbf{\Gamma} \succ \mathbf{0}$ and Equations (36), (37) and (17)

(2). Set the direction $(\Delta\mathbf{Y}_D^k, \Delta\mathbf{X}_D^k) := (\mathbf{Y}_D - \mathbf{Y}_D^k, \mathbf{X}_D - \mathbf{X}_D^k)$

(3). Set $(\mathbf{Y}_D^{k+1},\ \mathbf{X}_D^{k+1}) := (\mathbf{Y}_D^k, \mathbf{X}_D^k)\ + t_k\,(\Delta\mathbf{Y}_D^k, \Delta\mathbf{X}_D^k)$

(4). Set $\mu^{k+1} := Trace\ (\mathbf{Y}_D\,\mathbf{X}_D^k + \mathbf{Y}_D^k\,\mathbf{X}_D) - 2Trace\ (\mathbf{Y}_D^k\,\mathbf{X}_D^k)$

increment k by 1.

End do

Remark 1: The following subclass problem can be used to choose appropriately the value for t_k in Step (3):

$$\begin{aligned}&\text{Min } Trace\ (\mathbf{Y}_D^k + t_k\,(\mathbf{Y}_D - \mathbf{Y}_D^k))(\mathbf{X}_D^k + t_k\,(\mathbf{X}_D - \mathbf{X}_D^k))\\ &\text{subject to } t_k \in [0,\,1]\end{aligned} \tag{19}$$

To recover the decentralized controller, it is first necessary to construct the $\mathbf{Q}_D$ from the solution set of Algorithm I. The block elements $\mathbf{Q}_D^{(i)}$ of $\mathbf{Q}_D$ can be computed using the following relation

$$\mathbf{Q}_D^{(i)} = \begin{bmatrix}\mathbf{Y}^{(i)} & \mathbf{N}^{(i)}\\ (\mathbf{N}^{(i)})^T & \mathbf{I}_{n_{ci}}\end{bmatrix} \tag{20}$$

where $\mathbf{N}^{(i)}$ is given by $\mathbf{N}^{(i)} = (\mathbf{Y}^{(i)} - (\mathbf{X}^{(i)})^{-1})^{1/2}$

Thus, using the reconstructed $\mathbf{Q}_D^*$ and the computed upper bounds for the uncertainties γ_i^* for $i = 1, 2, \ldots, N$, the decentralized controller can be recovered by solving the LMI in (35) for $\mathbf{K}_D$.

III. Robust Decentralized H_∞ Controller Design for Power Systems: A Matrix Inequality Approach using Parameter Continuation Method

A. Problem Formulation

Consider the general structure of the ith-generator together with an n_{ci}th-order PSS in a multimachine power system shown in Fig. 1. The input of the ith-controller is connected to the output of the washout stage filter, which prevents the controller from acting on the system during steady state. After augmenting the washout stage in the system, the ith-subsystem, within the framework of H_∞ design, is described by the following state space equation:

$$\begin{aligned}\dot{\mathbf{x}}_i(t) &= \mathbf{A}_{ii}\,\mathbf{x}_i(t) + \sum_{j\neq i}\mathbf{A}_{ij}\,\mathbf{x}_j(t) + \mathbf{B}_{1i}\,\mathbf{w}_i(t) + \mathbf{B}_{2i}\,\mathbf{u}_i(t)\\ \mathbf{z}_i(t) &= \mathbf{C}_{1i}\,\mathbf{x}_i(t) + \mathbf{D}_{11i}\,\mathbf{w}_i(t) + \mathbf{D}_{12i}\,\mathbf{u}_i(t)\\ \mathbf{y}_i(t) &= \mathbf{C}_{yi}\,\mathbf{x}_i(t) + \mathbf{D}_{y1i}\mathbf{w}_i(t)\end{aligned} \tag{21}$$

where $\mathbf{x}_i(t) \in \Re^{n_i}$ is the state variable, $\mathbf{u}_i(t) \in \Re^{m_i}$ is the control input, $\mathbf{y}_i(t) \in \Re^{q_i}$ is the measurement signal, $\mathbf{z}_i(t) \in \Re^{p_i}$ is the regulated variables, and $\mathbf{w}_i(t) \in \Re^{r_i}$ is exogenous signal for the ith-subsystem. Moreover, assume that there is no unstable fixed mode [22] with respect to $[\mathbf{C}_{y1}^T\ \ \mathbf{C}_{y2}^T\ \ \cdots\ \ \mathbf{C}_{yN}^T]^T$, $[\mathbf{A}_{ij}]_{N\times N}$ and $[\mathbf{B}_{21}^T\ \ \mathbf{B}_{22}^T\ \ \cdots\ \ \mathbf{B}_{2N}^T]^T$.

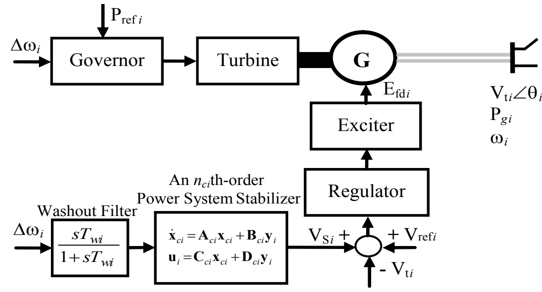


Fig. 1. General structure of the ith-generator together with PSS in a multimachine system.

Consider the following decentralized output feedback PSS controller for the system given in (21):

$$\begin{aligned}\dot{\mathbf{x}}_{ci}(t) &= \mathbf{A}_{ci}\,\mathbf{x}_{ci}(t) + \mathbf{B}_{ci}\,\mathbf{y}_i(t)\\ \mathbf{u}_i(t) &= \mathbf{C}_{ci}\,\mathbf{x}_{ci}(t) + \mathbf{D}_{ci}\mathbf{y}_i(t)\end{aligned} \tag{22}$$

where $\mathbf{x}_{ci}(t) \in \Re^{n_{ci}}$ is the state of the ith-local controller, n_{ci} is a specified dimension, and $\mathbf{A}_{ci}, \mathbf{B}_{ci}, \mathbf{C}_{ci}, \mathbf{D}_{ci}$, $i = 1, 2, \cdots, N$ are constant matrices to be determined during the designing. In this paper, the design procedure deals with nonzero $\mathbf{D}_{ci}$, however, it can be set to zero, i.e., $\mathbf{D}_{ci} = \mathbf{0}$, so that the ith-local is strictly proper controller. After augmenting the decentralized controller (22) in the system, the state space equation for the ith-subsystem will have the following form

$$\begin{aligned}\dot{\tilde{\mathbf{x}}}_i(t) &= (\tilde{\mathbf{A}}_{ii} + \tilde{\mathbf{B}}_{2i}\,\mathbf{K}_i\,\tilde{\mathbf{C}}_{yi})\,\tilde{\mathbf{x}}_i(t)\ + (\tilde{\mathbf{B}}_{1i} + \tilde{\mathbf{B}}_{2i}\,\mathbf{K}_i\,\tilde{\mathbf{C}}_{yi})\mathbf{w}_i(t) + \sum_{j\neq i}\tilde{\mathbf{A}}_{ij}\,\tilde{\mathbf{x}}_j(t)\\ \mathbf{z}_i(t) &= (\tilde{\mathbf{C}}_{i1} + \tilde{\mathbf{D}}_{12i}\,\mathbf{K}_i\,\tilde{\mathbf{C}}_{yi})\tilde{\mathbf{x}}_i(t) + (\tilde{\mathbf{D}}_{11i} + \tilde{\mathbf{D}}_{12i}\,\mathbf{K}_i\,\tilde{\mathbf{D}}_{y1i})\mathbf{w}_i(t)\end{aligned} \tag{23}$$

where $\tilde{\mathbf{x}}_i(t) = [\mathbf{x}_i^T(t),\ \ \mathbf{x}_{ci}^T(t)]^T$ is the augmented state variable for the ith-subsystem and

$$\tilde{\mathbf{A}}_{ij} = \begin{bmatrix}\mathbf{A}_{ij} & \mathbf{0}_{n_i\times n_{ci}}\\ \mathbf{0}_{n_{ci}\times n_i} & \mathbf{0}_{n_{ci}\times n_{ci}}\end{bmatrix},\quad \tilde{\mathbf{B}}_{1i} = \begin{bmatrix}\mathbf{B}_{1i}\\ \mathbf{0}_{n_i\times r_i}\end{bmatrix},\quad \tilde{\mathbf{C}}_{yi} = \begin{bmatrix}\mathbf{0}_{n_{ci}\times n_i} & \mathbf{I}_{n_{ci}\times n_{ci}}\\ \mathbf{C}_{yi} & \mathbf{0}_{q_i\times n_{ci}}\end{bmatrix},$$

$$\tilde{\mathbf{B}}_{2i} = \begin{bmatrix}\mathbf{0}_{n_i\times n_{ci}} & \mathbf{B}_{2i}\\ \mathbf{I}_{n_{ci}} & \mathbf{0}_{n_{ci}\times m_i}\end{bmatrix},\quad \tilde{\mathbf{C}}_{1i} = \begin{bmatrix}\mathbf{C}_{1i} & \mathbf{0}_{p_i\times n_{ci}}\end{bmatrix},\quad \tilde{\mathbf{D}}_{11i} = \mathbf{D}_{11i},$$

$$\tilde{\mathbf{D}}_{12i} = \begin{bmatrix}\mathbf{0}_{p_i\times n_{ci}} & \mathbf{D}_{12i}\end{bmatrix},\quad \tilde{\mathbf{D}}_{y1i} = \begin{bmatrix}\mathbf{0}_{n_{ci}\times r_i}\\ \mathbf{D}_{y1i}\end{bmatrix},\quad \mathbf{K}_i = \begin{bmatrix}\mathbf{A}_{ci} & \mathbf{B}_{ci}\\ \mathbf{C}_{ci} & \mathbf{D}_{ci}\end{bmatrix}$$

Moreover, the overall extended system equation for the system can be rewritten in one state-space equation form as

$$\begin{aligned}\dot{\tilde{\mathbf{x}}}(t) &= (\tilde{\mathbf{A}} + \tilde{\mathbf{B}}_2\mathbf{K}_D\,\tilde{\mathbf{C}}_y)\tilde{\mathbf{x}}(t) + (\tilde{\mathbf{B}}_1 + \tilde{\mathbf{B}}_2\mathbf{K}_D\tilde{\mathbf{C}}_y)\mathbf{w}(t)\\ \mathbf{z} &= (\tilde{\mathbf{C}}_1 + \tilde{\mathbf{D}}_{12}\mathbf{K}_D\tilde{\mathbf{C}}_y)\tilde{\mathbf{x}}(t) + (\tilde{\mathbf{D}}_{11} + \tilde{\mathbf{D}}_{12}\mathbf{K}_D\tilde{\mathbf{D}}_{y1})\mathbf{w}(t)\end{aligned} \tag{24}$$

where

$$\begin{aligned}&\tilde{\mathbf{A}} = [\tilde{\mathbf{A}}_{ij}]_{N\times N}, && \tilde{\mathbf{D}}_{11} = \mathrm{diag}\{\tilde{\mathbf{D}}_{111}, \tilde{\mathbf{D}}_{112}, \ldots, \tilde{\mathbf{D}}_{11N}\},\\ &\tilde{\mathbf{B}}_1 = \mathrm{diag}\{\tilde{\mathbf{B}}_{11}, \tilde{\mathbf{B}}_{12}, \ldots, \tilde{\mathbf{B}}_{1N}\}, && \tilde{\mathbf{D}}_{12} = \mathrm{diag}\{\tilde{\mathbf{D}}_{121}, \tilde{\mathbf{D}}_{122}, \ldots, \tilde{\mathbf{D}}_{12N}\},\\ &\tilde{\mathbf{B}}_2 = \mathrm{diag}\{\tilde{\mathbf{B}}_{21}, \tilde{\mathbf{B}}_{22}, \ldots, \tilde{\mathbf{B}}_{2N}\}, && \tilde{\mathbf{C}}_y = \mathrm{diag}\{\tilde{\mathbf{C}}_{y1}, \tilde{\mathbf{C}}_{y2}, \ldots, \tilde{\mathbf{C}}_{yN}\},\\ &\tilde{\mathbf{C}}_1 = \mathrm{diag}\{\tilde{\mathbf{C}}_{11}, \tilde{\mathbf{C}}_{12}, \ldots, \tilde{\mathbf{C}}_{1N}\}, && \tilde{\mathbf{D}}_{y1} = \mathrm{diag}\{\tilde{\mathbf{D}}_{y11}, \tilde{\mathbf{D}}_{y12}, \ldots, \tilde{\mathbf{D}}_{y1N}\},\\ &\mathbf{K}_D = \mathrm{diag}\{\mathbf{K}_1, \mathbf{K}_2, \ldots, \mathbf{K}_N\}.\end{aligned}$$

Hence, the overall extended system can be rewritten in a compact form as

$$\dot{\tilde{\mathbf{x}}}(t) = \mathbf{A}_{cl}\,\tilde{\mathbf{x}}(t) + \mathbf{B}_{cl}\mathbf{w}(t)$$
$$\mathbf{z}(t) = \mathbf{C}_{cl}\mathbf{x}(t) + \mathbf{D}_{cl}\mathbf{w}(t) \qquad (25)$$

where

$$\mathbf{A}_{cl} = \tilde{\mathbf{A}} + \tilde{\mathbf{B}}_2\mathbf{K}_D\tilde{\mathbf{C}}_y, \qquad \mathbf{B}_{cl} = \tilde{\mathbf{B}}_1 + \tilde{\mathbf{B}}_2\mathbf{K}_D\tilde{\mathbf{D}}_{y1},$$
$$\mathbf{C}_{cl} = \tilde{\mathbf{C}}_1 + \tilde{\mathbf{D}}_{12}\mathbf{K}_D\tilde{\mathbf{C}}_y, \qquad \mathbf{D}_{cl} = \tilde{\mathbf{D}}_{11} + \tilde{\mathbf{D}}_{12}\mathbf{K}_D\tilde{\mathbf{D}}_{y1}$$

Consider the following design approach where the controller strategy in (22) internally stabilizes the closed-loop of the transfer function $T_{zw}(s)$ from $\mathbf{w}$ to $\mathbf{z}$ and moreover satisfies a certain prescribed disturbance attenuation level $\gamma > 0$, i.e., $\|T_{\mathbf{zw}}(s)\|_\infty < \gamma$. In the following, the design procedure assumes that the system in (21) is stabilizable with the same prescribed disturbance attenuation level γ via a centralized H_∞ controller of dimension equal to or greater than $n_c := \sum_{i=1}^{N} n_{ci}$ in which each controller input $\mathbf{u}_i$ is determined by all measured outputs $\mathbf{y}_j$, $1 \le j \le N$. The significance of this assumption lies on the fact that the decentralized controllers cannot achieve better performances than that of centralized controllers. In this paper, the centralized H_∞ controller is used as initial value in the two-stage matrix inequality optimization method.

B. Decentralized H_∞ Output Feedback Design Using Parameterized Continuation Method

Designing a decentralized H_∞ output feedback controller for the system is equivalent to that of finding the matrix $\mathbf{K}_D$ that satisfies an H_∞ norm bound condition on the closed-loop transfer function $T_{\mathbf{zw}}(s) = \mathbf{C}_{cl}(s\mathbf{I} - \mathbf{A}_{cl})\mathbf{B}_{cl} + \mathbf{D}_{cl}$ from disturbance $\mathbf{w}$ to measured output $\mathbf{z}$, i.e. $\|T_{\mathbf{zw}}(s)\|_\infty < \gamma$ (for a given scalar constant $\gamma > 0$). Moreover, the transfer functions $T_{\mathbf{zw}}(s)$ must be stable [23]. Theorem 3 (see also APPENDIX) is instrumental in establishing the existence of decentralized control strategy (22) that satisfies a certain prescribed disturbance attenuation level $\gamma > 0$ on the closed loop transfer function $T_{\mathbf{zw}}(s) = \mathbf{C}_{cl}(s\mathbf{I} - \mathbf{A}_{cl})\mathbf{B}_{cl} + \mathbf{D}_{cl}$ from disturbance $\mathbf{w}$ to measured output $\mathbf{z}$, i.e. $\|T_{\mathbf{zw}}\|_\infty < \gamma$.

The condition stated in Theorem 3 seems to be the same as that of the centralized H_∞ control case [23]. However, due to the specified structure on the controller (i.e., designing controllers with "block diagonal") makes the problem a nonconvex optimization problem. To compute the optimal solution of this problem, the design problem is reformulated as an embedded parameter continuation problem that deforms from the centralized controller to the decentralized one as the continuation parameter monotonically varies [24]. The parameterized family of the problem in (39) is given as follows:

$$\tilde{\mathbf{\Phi}}(\mathbf{K}_D, \tilde{\mathbf{P}}, \lambda) := \mathbf{\Phi}((1-\lambda)\mathbf{K}_F + \lambda\mathbf{K}_D, \ \tilde{\mathbf{P}}) \prec \mathbf{0} \qquad (26)$$

with $\lambda \in [0,\ 1]$ such that at $\lambda = 0$

$$\tilde{\mathbf{\Phi}}(\mathbf{K}_F, \tilde{\mathbf{P}}, 0) = \mathbf{\Phi}(\mathbf{K}_F, \tilde{\mathbf{P}}) \qquad (27)$$

and at $\lambda = 1$

$$\tilde{\mathbf{\Phi}}(\mathbf{K}_D, \tilde{\mathbf{P}}, 1) = \mathbf{\Phi}(\mathbf{K}_D, \tilde{\mathbf{P}}) \qquad (28)$$

where

$$\mathbf{K}_F = \begin{bmatrix} \mathbf{A}_F & \mathbf{B}_F \\ \mathbf{C}_F & \mathbf{D}_F \end{bmatrix} \qquad (29)$$

is a constant matrix of the same size as $\mathbf{K}_D$ and composed of the coefficient matrices $\mathbf{A}_F$, $\mathbf{B}_F$, $\mathbf{C}_F$ and $\mathbf{D}_F$ of an n_c-dimensional centralized H_∞ for the disturbance attenuation level γ. The centralized controller $\mathbf{K}_F$ can be obtained via the existing method. Thus, the term $(1-\lambda)\mathbf{K}_F + \lambda\mathbf{K}_D$ in (26) defines a homotopy interpolating centralized H_∞ controller and a desired decentralized H_∞ controller. Thus, the problem of finding a solution of (26) can be embedded in the family of problems as:

$$\tilde{\mathbf{\Phi}}(\mathbf{K}_D, \tilde{\mathbf{P}}, \lambda) \prec \mathbf{0}, \qquad \lambda \in [0,\ 1] \qquad (30)$$

Thus, the algorithm based on parameter continuation method for finding the robust decentralized output feedback controller has the following two-stage matrix inequality optimization algorithm.

Algorithm II: *A Matrix Inequality Based Parameterized Continuation Method.*

Compute the centralized controller $\mathbf{K}_F$ and $\mathbf{P}_0$
Set $\lambda_0 = 0$, $k = 0$, $M = K$ (Large Number)
and $\mathbf{K}_{D0} = \mathbf{0}$ (Zero Matrix)
Repeat until $\lambda_k = 1$, **do**
$k \leftarrow k + 1$
$\lambda_k \leftarrow \lambda_{k-1} + 1/M$
(1) Compute $\mathbf{K}_{Dk}$ that satisfies
$\mathbf{\Phi}((1-\lambda_k)\mathbf{K}_F + \lambda_k\mathbf{K}_{Dk},\ \tilde{\mathbf{P}}_{k-1},\ \lambda_k) \prec \mathbf{0}$
(2) Compute $\tilde{\mathbf{P}}_k$ that satisfies
$\mathbf{\Phi}((1-\lambda_k)\mathbf{K}_F + \lambda_k\mathbf{K}_{Dk},\ \tilde{\mathbf{P}}_k,\ \lambda_k) \prec \mathbf{0}$
End do

Remark 2: If the problem in step (1) of Algorithm II fails to be feasible, then the step-length should be changed to compute $\mathbf{K}_{Dk}$ that satisfies $\tilde{\mathbf{\Phi}}(\mathbf{K}_{Dk}, \tilde{\mathbf{P}}_{k-1}, \lambda_k) \prec \mathbf{0}$ for the remaining interval.

The above two-stage matrix inequality optimization method involves: i) computing a continuous family of decentralized feedback controller $\mathbf{K}_{Dk}$ starting from $\mathbf{K}_{D0} = \mathbf{0}$ at $\lambda = 0$, i.e., an LMI problem in step-1, and ii) computing the positive definite matrix $\tilde{\mathbf{P}}_k$ in step-2.

C. Reduced - Order Decentralized Controller Design

The algorithm proposed in the previous section can only be applied when the dimension of the decentralized H_∞ controller is equal to the order of the plant, i.e., $n = n_c$. However, it is possible to compute directly a reduced-order decentralized controller, i.e., $n_c < n$ by augmenting the matrix $\mathbf{K}_D$ as

$$\hat{\mathbf{K}}_D = \left[\begin{array}{cc|c} \hat{\mathbf{A}}_D & 0_{n_n\times(n-n_c)} & \hat{\mathbf{B}}_D \\ * & -\mathbf{I}_{(n-n_c)} & ** \\ \hline \hat{\mathbf{C}}_D & 0_{m\times(n-n_c)} & \hat{\mathbf{D}}_D \end{array}\right] \qquad (31)$$

where the notation $*$, $**$ are any submatrices, and $\hat{\mathbf{A}}_D$, $\hat{\mathbf{B}}_D$, $\hat{\mathbf{C}}_D$, and $\hat{\mathbf{D}}_D$ are the reduced-order decentralized controller matrices. Note that the n-dimensional controller defined by $\hat{\mathbf{K}}_D$ of (31) is equivalent to the n_c-dimensional decentralized controller described by state-space representation of ($\hat{\mathbf{A}}_D$, $\hat{\mathbf{B}}_D$, $\hat{\mathbf{C}}_D$, $\hat{\mathbf{D}}_D$) if the controller and observable parts are extracted.

Next, define the matrix function $\tilde{\mathbf{\Phi}}(\hat{\mathbf{K}}_D, \tilde{\mathbf{P}}, \lambda)$ as

$$\tilde{\mathbf{\Phi}}(\hat{\mathbf{K}}_D,\ \tilde{\mathbf{P}},\ \lambda) := \mathbf{\Phi}((1-\lambda)\mathbf{K}_F + \lambda\hat{\mathbf{K}}_D,\ \tilde{\mathbf{P}}) \prec \mathbf{0} \quad (32)$$

Then, one can apply the algorithm proposed in the previous section with $\mathbf{K}_F$ of n-dimensional centralized H_∞ controller. In this case, at $\lambda = 0$ set the matrix $\hat{\mathbf{K}}_D$ to zero except $(2, 2)$-block $-\mathbf{I}_{(n-n_c)}$ and proceed with computing $\hat{\mathbf{K}}_{D_k}$ for each λ_k. If the algorithm succeeds, then the matrices ($\hat{\mathbf{A}}_D$, $\hat{\mathbf{B}}_D$, $\hat{\mathbf{C}}_D$, $\hat{\mathbf{D}}_D$) extracted from the obtained $\hat{\mathbf{K}}_D$ at $\lambda = 1$, comprise the desired decentralized H_∞ controller.

IV. Simulation Results

The robust decentralized dynamic output control design approaches presented in the previous section of this paper are now applied to a test system. This system, which is shown in Fig. 2, has been specifically designed to study the fundamental behavior of large interconnected power systems including inter-area oscillations in power systems [25]. The system has four generators and each generator is equipped with IEEE standard exciter and governor controllers. The parameters for the standard exciter and governor controllers used in the simulation were taken from [26]. Moreover, the generators for these simulations are all represented by their fifth-order models with rated terminal voltage of 15.75 kV. In the design, speed signals from each generator are used for robust decentralized dynamic output control through the excitation systems. The following loading condition was assumed: at node-1 a load of $[P_{L1} = 1600\text{ MW},\ Q_{L1} = 150\text{ Mvar}]$ and at node-2 a load of $[P_{L2} = 2400\text{ MW},\ Q_{L2} = 120\text{ Mvar}$. The system was linearized and the corresponding system equations were decomposed as a sum of two sets of equations. While the former describes the system as a hierarchical interconnection of N subsystems, the latter represents the interactions among the subsystems. After augmenting the controller structure in each subsystem, the design problem was formulated as a minimization problem of linear objective function involving LMIs and coupling BMI constraints. Using the algorithm proposed in Section II, the robust (sub)-optimal decentralized second-order PSS for each subsystem were designed. The robust decentralized PSS designed through this approach are given in Table-I. The convergence behavior of Algorithm I is also given in Table II for a relative accuracy of $\varepsilon = 10^{-6}$.

Similarly, second-order PSS are designed using the approach proposed in Section III. However, it is possible to extend the method to any order and/or combinations of PSS blocks in the design procedure. After including the washout filter in the linearized system equation, the design problem is reformulated as an embedded parameter continuation problem that deforms from the designed centralized H_∞ controller to the decentralized one as the continuation parameter monotonically varies. Speed signals from each generator, the outputs of the PSS together with the terminal voltage error signals, which are the input to the regulator of the exciter, are used as regulated signals within this design framework. The robust decentralized PSS designed through this approach are also given in Table-I. For a short circuit of 150 ms duration at node F in Area-A, the transient responses of generator G2 with and without the PSSs in the system are shown in Fig.3.

To further assess the effectiveness of the proposed approaches regarding the robustness, the transient performance indices were computed for different loading conditions at node 1 $[P_{L1}, Q_{L1}]$ and node 2 $[P_{L2}, Q_{L2}]$ while keeping constant total load in the system. The transient performance indices for generator powers P_{gi}, generator terminal voltages V_{ti} and excitation voltages E_{fdi} following a short circuit of 150 ms duration at node F in Area-A are computed using (33a), (33b) and (33c), respectively.

$$I^{P_g} = \sum_{i=1}^{N} \int_{t_0}^{t_f} \left| P_{gi}(t) - P_{gi}^0 \right| dt \quad (33a)$$

$$I^{V_t} = \sum_{i=1}^{N} \int_{t_0}^{t_f} \left| V_{ti}(t) - V_{ti}^0 \right| dt \quad (33b)$$

$$I^{E_{fd}} = \sum_{i=1}^{N} \int_{t_0}^{t_f} \left| E_{fdi}(t) - E_{fdi}^0 \right| dt \quad (33c)$$

These transient performance indices are used as a qualitative measure of system behavior following any disturbances including controller actions. Moreover, for comparison purpose, these indices are normalized to the base operating condition for which the controllers have been designed:

$$I_N = \frac{I_{DLC}}{I_{BLC}} \quad (34)$$

where I_{DLC} is the transient performance index for different loading condition, I_{BLC} is the transient performance index for base loading condition. The normalized transient performance indices for different loading conditions are shown for both approaches in Fig. 4 and Fig. 5, respectively. It can be seen from these figures that the normalized transient performance indices for $I_N(P_g)$, $I_N(V_t)$ and $I_N(E_{fd})$ are either near unity or less than unity for a wide operating conditions. This clearly indicates that the transient responses of the generators for different operating conditions are well damped and the system behavior exhibits robustness for all loading conditions.

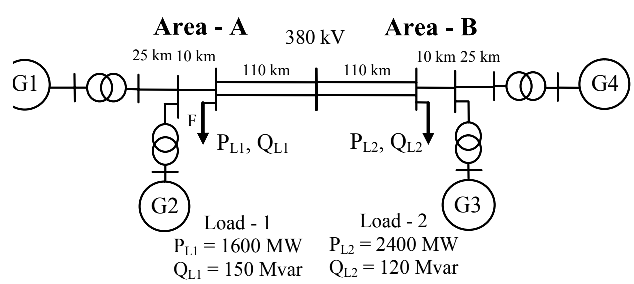


Fig. 2. One-line diagram of four machine two area system.

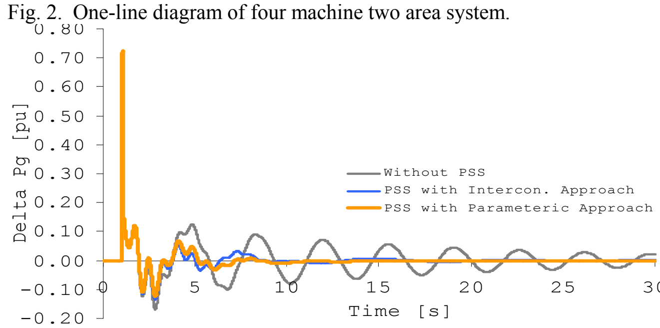


Fig. 3. Transient responses of Generator G2 to a short circuit at node F in Area A.

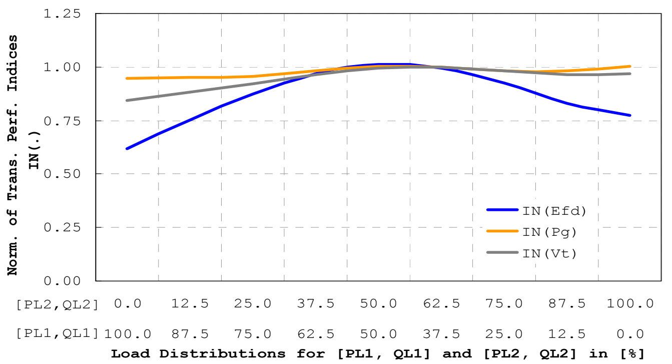


Fig. 4. Plot of the normalized transient performances indices for the interconnection based approach.

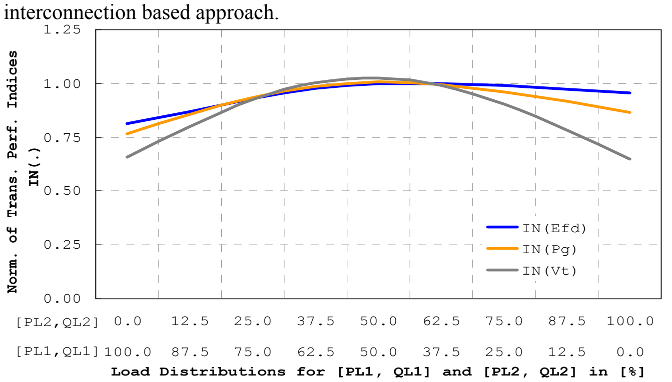


Fig. 5. Plot of the normalized transient performances indices for the generalized parameter continuation approach.

V. Appendix

To make the paper self-contained treatment of the approaches, the following theorems are given without proof.

Theorem 1: The interconnected system (1) is robustly stabilized by the decentralized dynamic output control strategy of (6) with degree of uncertainty vector $\xi=[\xi_1,\xi_2,\ldots,\xi_N]^T$ if there exist a symmetric positive definite matrix $\mathbf{Q}_D=\mathrm{diag}\{\mathbf{Q}^{(1)},\mathbf{Q}^{(2)},\ldots,\mathbf{Q}^{(N)}\}$ with $\mathbf{Q}^{(i)}\in\Re^{(n_i+n_{ci})\times(n_i+n_{ci})}$ satisfying the following matrix inequality:

$$\begin{bmatrix} \mathbf{Q}_D\tilde{\mathbf{A}}_D^T+\tilde{\mathbf{A}}_D\mathbf{Q}_D+\mathbf{Q}_D(\tilde{\mathbf{B}}_D\mathbf{K}_D\tilde{\mathbf{C}}_D)^T+\tilde{\mathbf{B}}_D\mathbf{K}_D\tilde{\mathbf{C}}_D\mathbf{Q}_D & \mathbf{Q}_D\tilde{\mathbf{H}}_1^T & \cdots & \mathbf{Q}_D\tilde{\mathbf{H}}_N^T & \mathbf{I} \\ \tilde{\mathbf{H}}_1\mathbf{Q}_D & -\gamma_1\mathbf{I} & \cdots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \tilde{\mathbf{H}}_N\mathbf{Q}_D & \mathbf{0} & \cdots & -\gamma_N\mathbf{I} & \mathbf{0} \\ \mathbf{I} & \mathbf{0} & \cdots & \mathbf{0} & -\mathbf{I} \end{bmatrix}\prec\mathbf{0} \quad (35)$$

where $\gamma_i=1/\xi_i^2$.

Theorem 2: The interconnected system (1) is robustly stabilized by the decentralized control strategy of (6) if there exist two symmetric positive definite matrices $\mathbf{X}_D=\mathrm{diag}\{\mathbf{X}^{(1)},\mathbf{X}^{(2)},\ldots,\mathbf{X}^{(N)}\}$, $\mathbf{Y}_D=\mathrm{diag}\{\mathbf{Y}^{(1)},\mathbf{Y}^{(2)},\ldots,\mathbf{Y}^{(N)}\}$ with blocks $\mathbf{X}^{(i)},\mathbf{Y}^{(i)}\in\Re^{n_i\times n_i}$ and positive numbers γ_i for $i=1,2,\ldots,N$ satisfying the following two LMI conditions

$$\begin{bmatrix} \mathbf{N}_{\mathbf{B}_D}^T(\mathbf{Y}_D\mathbf{A}_D^T+\mathbf{A}_D\mathbf{Y}_D)\mathbf{N}_{\mathbf{B}_D} & \mathbf{N}_{\mathbf{B}_D}^T\mathbf{Y}_D\mathbf{H}_1^T & \cdots & \mathbf{N}_{\mathbf{B}_D}^T\mathbf{Y}_D\mathbf{H}_N^T & \mathbf{N}_{\mathbf{B}_D}^T \\ \mathbf{H}_1\mathbf{Y}_D\mathbf{N}_{\mathbf{B}_D} & -\gamma_1\mathbf{I} & \cdots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{H}_N\mathbf{Y}_D\mathbf{N}_{\mathbf{B}_D} & \mathbf{0} & \cdots & -\gamma_N\mathbf{I} & \mathbf{0} \\ \mathbf{N}_{\mathbf{B}_D} & \mathbf{0} & \cdots & \mathbf{0} & -\mathbf{I} \end{bmatrix}\prec\mathbf{0} \quad (36)$$

$$\begin{bmatrix} \mathbf{N}_{\mathbf{C}_D}(\mathbf{A}_D^T\mathbf{X}_D+\mathbf{X}_D\mathbf{A}_D)\mathbf{N}_{\mathbf{C}_D}^T & \mathbf{N}_{\mathbf{C}_D}\mathbf{H}_1^T & \cdots & \mathbf{N}_{\mathbf{C}_D}\mathbf{H}_N^T & \mathbf{N}_{\mathbf{C}_D}\mathbf{X}_D \\ \mathbf{H}_1\mathbf{N}_{\mathbf{C}_D}^T & -\gamma_1\mathbf{I} & \cdots & \mathbf{0} & \mathbf{0} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \mathbf{H}_N\mathbf{N}_{\mathbf{C}_D}^T & \mathbf{0} & \cdots & -\gamma_N\mathbf{I} & \mathbf{0} \\ \mathbf{X}_D\mathbf{N}_{\mathbf{C}_D}^T & \mathbf{0} & \cdots & \mathbf{0} & -\mathbf{I} \end{bmatrix}\prec\mathbf{0} \quad (37)$$

where $\mathbf{N}_{\mathbf{B}_D}$ and $\mathbf{N}_{\mathbf{C}_D}$ denote arbitrary bases of the nullspaces of $\mathbf{B}_D$ and $\mathbf{C}_D$, respectively. Moreover, the set of n_{ci}th-order decentralized controllers is nonempty if and only if the above two LMI conditions hold for some $\mathbf{X}_D$ and $\mathbf{Y}_D$ which further satisfy the following rank constraint

$$Rank\,(\mathbf{I}-\mathbf{Y}_D\mathbf{X}_D)=\sum_{i=1}^{N}n_{ci}\;\le\; n=\sum_{i=1}^{N}n_i \quad (38)$$

Theorem 3: The system (21) is stabilizable with the disturbance attenuation level $\gamma>0$ via a decentralized controller (22) composed of N n_{ci}th-order local controllers if there exist a matrix $\mathbf{K}_D$ and a positive-definite matrix $\tilde{\mathbf{P}}$ that satisfy the following matrix inequality:

$$\mathbf{\Phi}(\mathbf{K}_D,\tilde{\mathbf{P}}):=\begin{bmatrix} \tilde{\mathbf{P}}\tilde{\mathbf{A}}+\tilde{\mathbf{A}}^T\tilde{\mathbf{P}} & \tilde{\mathbf{P}}\tilde{\mathbf{B}}_1 & \tilde{\mathbf{C}}_1^T \\ \tilde{\mathbf{B}}_1^T\tilde{\mathbf{P}} & -\gamma\mathbf{I}_r & \tilde{\mathbf{D}}_{11}^T \\ \tilde{\mathbf{C}}_1 & \tilde{\mathbf{D}}_{11} & -\gamma\mathbf{I}_p \end{bmatrix}+\begin{bmatrix} \tilde{\mathbf{P}}\tilde{\mathbf{B}}_2 \\ \mathbf{0}_{r\times(m+n_c)} \\ \tilde{\mathbf{D}}_{12} \end{bmatrix}\mathbf{K}_D\begin{bmatrix}\tilde{\mathbf{C}}_2 & \tilde{\mathbf{D}}_{21} & \mathbf{0}_{(q+n_c)\times p}\end{bmatrix}$$
$$+\left\{\begin{bmatrix} \tilde{\mathbf{P}}\tilde{\mathbf{B}}_2 \\ \mathbf{0}_{r\times(m+n_c)} \\ \tilde{\mathbf{D}}_{12} \end{bmatrix}\mathbf{K}_D\begin{bmatrix}\tilde{\mathbf{C}}_2 & \tilde{\mathbf{D}}_{21} & \mathbf{0}_{(q+n_c)\times p}\end{bmatrix}\right\}^T\prec\mathbf{0} \quad (39)$$

TABLE I
THE ROBUST DECENTRALIZED POWER SYSTEM STABILIZERS

Generator $\mathbf{G}_i$	Power System Stabilizers Designed With	
	Interconnection Approach	Parametric Approach
1	$\frac{1.83s^2+21.67s+64.11}{s^2+11.82s+34.96}$	$\frac{s^2+8.29s+20.12}{s^2+14.72s+29.43}$
2	$\frac{2.01s^2+24.08s+72.15}{s^2+11.92s+35.63}$	$\frac{s^2+19.61s+21.01}{s^2+37.05s+22.34}$
3	$\frac{1.98s^2+23.60s+70.32}{s^2+11.90s+35.44}$	$\frac{s^2+17.70s+22.12}{s^2+37.61s+17.86}$
4	$\frac{1.79s^2+21.24s+63.09}{s^2+11.87s+35.27}$	$\frac{s^2+6.21s+6.51}{s^2+10.07s+11.83}$

TABLE II
THE CONVERGENCE PROPERTIES OF ALGORITHM I (FOR ERROR $\varepsilon=10^{-6}$)

Outer Iteration k	Inner Iterations j	Trace(Y^kX^k)	J(Y^k, X^k, Γ^k)
0	49	0.51423×10^7	0.19541×10^9
1	84	86.40271	0.10194×10^8
2	74	39.06272	133.01764
3	70	37.29666	75.72572
4	71	36.50326	73.49031
5	67	36.00284	72.16617
6	72	36.00002	72.09829
7	69	36.00000	72.06229

VI. Summary

The central objective of this paper is to develop robust decentralized control approaches for power systems with special emphasis on problems that can be expressed in terms of minimizing a linear objective function under LMI constraints in tandem with BMI constraints. Besides, developing the computational algorithms to solve such design problems, the effectiveness of these approaches are demonstrated by designing realistic power system stabilizes for power system, notably so-called reduced-order robust PSSs that are linear and use minimum local-feedback information. The followings are summary of the results:

i) In Section II, the problem of designing reduced-order robust decentralized PSS for an interconnected power system was reformulated as a minimization problem of linear objective functional involving LMIs and coupling BMIs constraints. In the design, the robust connective stability of the overall system is guaranteed while the upper bounds of the uncertainties arising from the interconnected subsystems and nonlinearities within each subsystem are maximized. A sequential LMI programming method was presented to solve such optimization problems so as to determine (sub)-optimally the robust decentralized controllers for the system.

ii) In Section III, the problem of designing a decentralized H_∞ dynamic output feedback controller for power system was considered. In the design, initially a centralized H_∞ robust controller, which guarantees the robust stability of the overall system against unstructured and norm bounded uncertainties, is designed using the standard methods. Then the problem of designing a decentralized controller for the system was reformulated as an embedded parameter continuation problem that homotopically deforms from the centralized controller to the decentralized one as the continuation parameter monotonically varies. An algorithm using two-stage matrix inequality optimization method is used to solve such robust decentralized H_∞ PSS design problem Moreover, extending the approach to design reduced-order decentralized controllers, which have practical benefits since high-order controllers when implemented in real-time configurations can create undesirable effects such as time delays, was also treated properly.

Moreover, the nonlinear simulation results have confirmed the robustness of the system for all envisaged operating conditions and disturbances. The proposed approaches offer a practical tool for engineers, besides designing reduced-order PSS, to re-tune PSS parameters for improving the dynamic performance of the overall system.

VII. References

[1] M. Aoki, "On feedback stabilizability of decentralized dynamic systems," *Automatica*, vol. 8, pp. 163-172, 1972.

[2] D. D. Siljak, *Large scale dynamic systems: stability and structure*, Elsevier North-Holland Inc., 1978.

[3] D. D. Siljak, *Decentralized control of complex systems*, Academic Press, Inc., 1991.

[4] J. Lunze, *Feedback control of large-scale systems*, Prentice Hall, New York, NY, 1992.

[5] J. W. Chapman, M. D. Ilic and C. A. King, "Feedback linearizing excitation control on full-scale power system model," *IEEE Trans. Power Syst*., vol. 9(2), pp. 1102-1109, May 1994.

[6] Y. Guo, D. Hill and Y. Wang, "Nonlinear decentralized control of large-scale power systems," *Automatica*, vol. 36, pp. 1275-1289, Sept. 2000.

[7] S. Jain and F. Khorrami, "Robust decentralized control of power system utilizing only swing angle measurements," *Int. J. Contr.*, vol. 66, pp. 581-601, 1997.

[8] D. D. Siljak, D. M. Stipanovic and A. I. Zecevic, "Robust decentralized turbine/governor control using linear matrix inequalities," *IEEE Trans. Power Syst*., vol. 17(2), pp. 715-722, August 2002.

[9] D. D. Siljak, A. I. Zecevic and G. Neskovic, "Robust decentralized exciter control with linear feedback," *IEEE Trans. Power Syst.*, vol. 19(2), 1096–1103, 2004.

[10] G. K. Befekadu and I. Erlich, "Robust decentralized controller design for power systems using convex optimization involving LMIs," presented at the 16th IFAC World Congress, Prague, Czech Republic, 2005.

[11] G. K. Befekadu and I. Erlich, "Robust decentralized structure-constrained controller design for power systems: an LMI approach," presented at the PSCC 2005, Liege, Belgium, 2005.

[12] S. Boyd, L. El Ghaoui, E. Feron and V. Balakrishnan, *Linear matrix inequalities in system and control theory*, SIAM Studies in Applied Mathematics, 15, 1994.

[13] D. D. Siljak and A. I. Zecevic, "Design of robust static output feedback for large scale systems," *IEEE Trans. Automat. Contr*., vol. 49(11), 2040-2044, November 2004.

[14] V. Vittal, M. H. Khammash and C. D. Pawloski, "Systems and control theory for power systems," In Chow, J. H., Kokotovic, P. V. and Thomas, R. J. editors, 64 of Mathematics and its Applications, IMA, 399-413, 1995.

[15] S. Venkataraman, M. H. Khammash and V. Vittal, "Analysis and synthesis of HVDC controls for stability of power systems," *IEEE Trans. Power Syst*., vol. 10(4), 1933-1938, November 1995.

[16] S. Chen and O. P. Malik, "H_∞ optimization based power system stabilizer design," *IEE Proceedings Generation, Transmission and Distribution*, 142(3), 179–184, 1995.

[17] M. Klien, X. L. Le, R. J. Rogers and S. Farrokhpay, "H_∞ damping controller design in large power systems," *IEEE Trans. Power Syst*., vol. 10(1), 158–166, February 1995.

[18] M. Djukanovic, M. H. Khammash and V. Vittal, "Application of the structured singular value theory for robust stability and control analysis in multimachine power systems, Part-I: Framework development and Part II: Numerical simulation and results," *IEEE Trans.Power Syst*., vol. 13(4), 1311–1316, November 1998.

[19] M. Djukanovic, M. H. Khammash and V. Vittal, "Sequential synthesis of structured singular value based decentralized controllers in power systems," *IEEE Trans. Power Syst*., vol. 14(2), 635–641, May 1999.

[20] W. Qiu, V. Vittal and M. H. Khammash, "Decentralized power system stabilizer design using linear parameter varying approach," *IEEE Trans. Power Syst*. vol. 19(4), 1951-1960, November 2004.

[21] P. Gahinet, A. Nemiroviski, A. L. Laub and M. Chilali, *LMI Control Toolbox*, Cambridge, MA: Mathworks Inc., 1995.

[22] S. H. Wang and E. J. Davidson, "On stabilization of decentralized control systems", *IEEE Trans. Autom. Contr.*, vol. 18(5), 473-478, 1973.

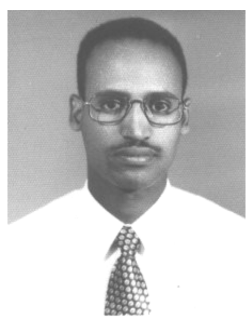
[23] P. Gahinet and P. Apkarian, "A linear matrix inequality approach to H_∞ control," *Int. J. Robust & Nonlin.Contr*., vol. 4, 421-448, 1994.

[24] S. L. Richter and R. A. Decarlo, "Continuation Methods: Theory and Applications," *IEEE Trans. Circuits & Syst*., vol. 30(6), 347-352, 1983.

[25] M. Klein, R. G. Rogers and P. Kundur, "A fundamental study of inter-area oscillations in power systems," *IEEE Trans. Power Syst.*, vol. 6(3), 914-921, August 1991.

[26] P. Kundur, *Power system stability and control*. McGraw-Hill Inc., 1994.

VIII. Biographies



Getachew K. Befekadu (1970) received the B.Sc. degree in electrical engineering from Faculty of Technology, Addis Ababa University, in 1992, and M.Sc. degree in electrical engineering with emphasis on control systems from School of Graduate Studies, Addis Ababa University, in 1999. He is currently a Ph.D. student with the German Academic Exchange Program, DAAD, at the University of Duisburg-Essen, Germany. His research interests include robust decentralized control design with coordination among controller agents for large scale power systems, application of optimization in power systems, hybrid systems and nonlinear control.

Istvan Erlich (1953) received his Dipl.-Ing. degree in electrical engineering from the University of Dresden/Germany in 1976. After his studies, he worked in Hungary in the field of electrical distribution networks. From 1979 to 1991, he joined the Department of Electrical Power Systems of the University of Dresden again, where he received his PhD degree in 1983. In the period of 1991 to 1998, he worked with the consulting company EAB in Berlin and the Fraunhofer Institute IITB Dresden respectively. During this time, he also had a teaching assignment at the University of Dresden. Since 1998, he is Professor and head of the Institute of Electrical Power Systems at the University of Duisburg-Essen, Germany. His major scientific interest is focused on power system stability and control, modelling and simulation of power system dynamics including intelligent system applications. He is a member of VDE and IEEE.