

# Uncertainty Modeling for the Management of Distributed Generation Units using PSO

V.S Pappala, *Student Member, IEEE*, I. Erlich, *Senior member, IEEE*

**Abstract**—This paper addresses a multistage stochastic model for the operation of distributed generation (DG) units under stochastic load demands. The stochastic load demands have a significant effect on the economic model of the DG units. The uncertainties are modeled as scenario trees. But as the number of decision making stages increase, the scenario tree becomes extremely large, which leads to complex computation. Therefore a novel approach to generate a scenario tree using classical Particle Swarm Optimization (PSO) approach is presented. The resulting multistage nonlinear stochastic model is solved using adaptive PSO.

**Keywords** - Uncertainty modeling, distributed generation, particle swarm, stochastic optimization

## I. INTRODUCTION

THE rapid development of new efficient and reliable generation technologies has allowed DG to be a good alternative to conventional energy sources for commercial as well as residential applications. Due to the competitive power market the DG units has to supply electricity to customers at an economical price. Fuel cells are the most efficient DG units for residential application. The benefits include efficient use of electrical and thermal energy, improved power quality, increased reliability and reduced emission.

This paper presents an approach to reduce the daily total operating costs of a proton exchange membrane (PEM) fuel cell supplying a residential load under demand uncertainty. A cost model is developed to minimize the daily total operating costs of a fuel cell taking into account the various operating and technical constraints associated with the unit. The optimization of such a model requires that decisions to be made in the presence of uncertainty. The major uncertainties that influence the cost model are the electrical demand, thermal demand, cost of purchased electricity and the cost of sold electricity. In this paper only one stochastic variable (electrical demand/thermal demand) is modelled. The residential load is subjected to sudden variations and these changes are totally subjective to the lifestyles of the inhabitants of the household [1]. They cannot be exactly

represented by a regression model. So the stochastic demand is considered as a random process. The samplings of this random process which represent the future realizations of the stochastic variable, is modelled as a scenario tree, where each scenario represents an instance of the future realizations. The scenarios together with the node probabilities thus form a good discrete approximation of the uncertain load demand.

Stochastic programming (SP) [2] offers a possibility to solve such a model. The quality of the optimization solutions by stochastic programming depends to a great extent on the degree of the uncertainty modelling. The household energy consumption depends on lifestyle related psychological factors that are extremely subjective and not easily defined with any degree of precision. Therefore modelling of such a random process involves a decision stage at each hour for a planning horizon of one day. More over greater number of sampling is required at each decision stage to completely define the stochastic nature at that stage. The conventional scenario tree generation methods [3], [4] pose a restriction on the number of branching stages and the number of samples at each stage. Due to the restriction on these key parameters the stochastic model does not replicate the real model and there is a huge modelling error. To overcome these barriers a new method is proposed. An attempt is made for the first time to introduce evolutionary algorithms for scenario tree generation. PSO has been chosen among the many available stochastic search algorithms because the search process is fast and robust. The conventional methods start with an initial bulk tree and then reduce the insignificant branches to reduce the tree. The necessity of the initial bulk tree data is a hindrance to these methods. In [5] we proposed an adaptive particle swarm optimization (APSO) approach to generate the scenario tree. In this approach the tree generation starts with a single scenario and then add on the best braches. This paper presents a new approach where in the tree generation always has a finite number of scenarios. PSO explores the huge available search space and decides the most distinct scenarios. There is no need for the initial bulk tree and hence no restrictions on the key parameters.

The scenario tree modelling transforms the cost model to a stochastic multistage nonlinear mixed integer model. The stochastic model is solved using APSO technique [6]. APSO has been chosen because it is totally free from parameter tuning. Moreover the optimization algorithm has a high probability of attaining a global solution.

The paper is organized as follows. The next section

---

Pappala Venkata Swaroop ([venkata.pappala@uni-due.de](mailto:venkata.pappala@uni-due.de)) is with the institute of electrical power systems, Universität Duisburg-Essen.

Istvan Erlich ([istvan.erlich@uni-due.de](mailto:istvan.erlich@uni-due.de)) is with the institute of electrical power systems, Universität Duisburg-Essen, 47057- Duisburg, Germany.

This Project is supported by the German Federal Ministry of Education and Research (BMBF) under the grant 03SF0312A.

presents the deterministic economic model of the PEM fuel cell. A PSO algorithm is introduced in section III. In section IV, fuel cell modelling under stochastic load demands is discussed. The optimization results are presented in section V.

## II. PEM FUEL CELL: DETERMINISTIC ECONOMIC MODEL

### A. Objective Function

The following fuel cell model has been taken from [7]. The objective is to minimize the daily total operating costs of a fuel cell taking into account the following assumptions:

- The fuel cell will supply both electrical and thermal power to the load
- There is a possibility to sell back the excess electricity at different tariffs. These tariffs are always lower than those of the purchased electricity
- A part of the generated power is utilized by the auxiliary devices.

The objective function is developed according to the above mentioned assumptions in the following form.

$$DOC = DFC + DCPE - DISE + DCPG + O\&M + STC \quad (1)$$

Where:

$DOC$  : Daily total operating cost (\$)

$DFC$  : Daily fuel cost for the fuel cell (\$)

$DCPE$  : Daily cost of purchased electricity if the demand exceeds the produced electrical power (\$)

$DISE$  : Daily income on sold electricity if the unit electrical output power exceeds the electrical demand (\$)

$DCPG$  : Daily cost of purchased gas for residential loads if the produced thermal power is not enough to meet the thermal demand (\$)

$O\&M$  : Daily operating and maintenance cost (\$)

$STC$  : Daily start up cost (\$)

The dependence of DFC on the generated electrical power and efficiency is given as:

$$DFC = C_{nFC} T \sum_J U_J \frac{P_J + P_a}{\eta_J} \quad (2)$$

Where:

$C_{nFC}$  : Natural gas price for fuel cell (\$/kWh)

$T$  : Time duration (h)

$P_J$  : Net electrical power produced at interval "J" (kW)

$P_a$  : Power for auxiliary devices (kW)

$\eta_J$  : Cell efficiency at interval "J"

$U_J$  : Unit commitment at interval "J"

The efficiency of the fuel cell depends on the active power [8] and hence, a typical efficiency curve (3) is developed as a function of the electrical power and is used in (2).

$$\eta_J = 0.4484 - 0.05359P_J + 0.01267P_J^2 - 0.00182P_J^3 \quad (3)$$

The main grid system is assumed to balance the difference

between the load demand and the net output from the fuel cell. The cost of electricity purchased from and sold back to the network is given by:

$$DCPE = C_{elp} T \sum_J U_J \max(D_{el,J} - P_J, 0) \quad (4)$$

$$DISE = C_{els} T \sum_J U_J \max(P_J - D_{el,J}, 0) \quad (5)$$

Where:

$C_{elp}, C_{els}$  : Tariffs of purchased and sold electricity respectively (\$/kWh)

$D_{el,J}$  : Electrical demand at interval "J" (kW)

The thermal output power from the fuel cell depends on the electrical power. The relation is almost linear at the lower values, while the ratio of the thermal power is relatively higher at the upper operating limits [8]. A nonlinear equation is developed to give the thermal output power of the unit as a function of the produced electrical power. The daily cost of purchased natural gas for residential applications when the thermal power from the fuel cell is not enough to meet the thermal load requirements is given as:

$$DCPG = C_{n2} T \sum_J U_J \max(D_{th,J} - P_{th,J}, 0) \quad (6)$$

Where:

$C_{n2}$  : Fuel price for residential loads (\$/kWh)

$D_{th,J}$  : Thermal load demand at time interval "J" (kW)

$P_{th,J}$  : Thermal power produced at interval "J" (kW)

The operating and maintenance (O&M) cost is assumed to be constant per kWh and hence O&M is proportional to the produced energy. The start up cost depends on the temperature of the unit and hence on the time the unit has been switched off before start up:

$$STC = \sum_J U_J (1 - U_{J-1}) (\alpha + \beta (1 - e^{-\frac{T_{J,off}}{\tau}})) \quad (7)$$

Where:

$\alpha, \beta$  : Hot and cold start up costs respectively

$T_{J,off}$  : The time the unit has been switched off till current interval J(h)

$\tau$  : The fuel cell cooling time constant (h)

### B. Constraints

The minimization of the objective function (1) is restricted by many constraints. The main operational and technical constraints associated with the PEM fuel cell are listed below:

$$\text{Unit capacity constraints : } P_J^{\min} U_J \leq P_J \leq P_J^{\max} U_J \quad (8)$$

$$\text{Ramp rate constraints : } P_J U_J - P_{J-1} U_{J-1} \leq \Delta P_U \quad (9)$$

$$P_{J-1} U_{J-1} - P_J U_J \leq \Delta P_D \quad (10)$$

In the above equations,  $P_J^{\min}$  and  $P_J^{\max}$  are the minimum and maximum limits of the generated power,  $\Delta P_U$  and  $\Delta P_D$  are the upper and lower limits of the ramp rate, and  $P_{J-1}$  is the power generated at interval (J-1). At the same time, the minimum

up/down time limits (continuous running-stop time constraint) must not be violated:

$$(T_{J-1}^{on} - MUT)(U_{J-1} - U_J) \geq 0 \quad (11)$$

$$(T_{J-1}^{off} - MDT)(U_J - U_{J-1}) \geq 0 \quad (12)$$

Where  $T_{J-1}^{on}$ ,  $T_{J-1}^{off}$  are the unit on and off times at interval (J-1), MUT, MDT are the minimum up and down time limits and U is the unit on/off status: U=1 for running mode and 0 for stop mode. Finally, the daily number of start-stop times ( $n_{start-stop}$ ) should not exceed a certain maximum number ( $N_{max}$ ).

$$n_{start-stop} \leq N_{max} \quad (13)$$

### III. PARTICLE SWARM OPTIMIZATION

Particle swarm optimization (PSO) [9], [10] is an efficient population based evolutionary algorithm developed by Kennedy and Eberhart. The search procedure of PSO is inspired by the social behavior associated with birds flocking. It is initialized by a population of random solutions called particles and the group of particles is called a swarm. Each particle in the swarm is identified by its current position and velocity with which it explores the search space to find an optimal solution. The particles refine their position using their own individual knowledge as well as their social intelligence which they acquire by social interaction. Each particle memorizes the optimal value that it has attained along its trajectory as well its position and is denoted by *pbest*. Similarly the best among all the individual best of the particles is termed as global best or *gbest*. The velocity of each particle as given by (14) is influenced by *pbest* and *gbest*. The velocity of each particle depends on three terms. The first term is the inertia velocity which carries information regarding the particles previous history of velocities. The second term is the cognitive part which reflects the particles behavior with respect to its own previous experiences.

The final term is a social parameter which indicates the particles behaviour with respect to the experience gained from the other members of the swarm. This velocity ( $v_{jk}(t+1)$ ) when added to the previous position ( $x_{jk}(t)$ ) generates the current position ( $x_{jk}(t+1)$ ) of the particle-*j* as given by the following equations.

$$v_{jk}(t+1) = \chi(wv_{jk}(t) + c_1r_1(pbest_{jk} - x_{jk}) + c_2r_2(gbest_k - x_{jk})) \quad (14)$$

$$x_{jk}(t+1) = x_{jk}(t) + v_{jk}(t+1), \quad \forall j \in N_p, k \in n \quad (15)$$

Where  $\phi = c_1r_1 + c_2r_2$  and

$$\chi = \begin{cases} \frac{2a_1}{\phi - 2 + \sqrt{\phi^2 - 4\phi}}, & \phi > 4 \\ a_1, & otherwise \end{cases} \quad (16)$$

Where  $c_1$  and  $c_2$  are the fixed acceleration coefficients,  $r_1$  and  $r_2 \in [0,1]$ ,  $\chi$  is the constriction factor,  $a_1$  is a positive constant.  $N_p$  is the set of all particles in the swarm and  $n$  is the dimension of the search space. The weighing factor ( $w$ ) for previous velocities is considered to be a linearly decreasing function so that it provides a good trade off between local and

global explorations.

Particle swarm optimization technique is used for all the simulations presented in this paper. A classical version of PSO is used for uncertainty modeling to generate the scenario tree. Classical version is preferred because the process always has desired number of particles (scenarios) and adaptive version is used for solving the multi-stage stochastic optimization problem because it gives better optimal solution within short computational time.

## IV. PEM FUEL CELL: STOCHASTIC ECONOMIC MODEL

### C. Uncertainty Modeling

In order to solve the economic model of the fuel cell with uncertain load demands, it is necessary to model the evolution of the uncertainty in the form of a scenario tree. In this case study the fuel cell supplies power to a small residential load. The residential load demand depends to a large extent on the weather forecast and on the lifestyle of the inhabitants. Practically it is very difficult to exactly determine the demand curves. Hence these loads are considered as a random process. We assume that the uncertainty increases with time. In this paper the effect of the random process (electrical demand/thermal demand) on the cost model of the fuel cell is investigated. We assume that the deviation of the load demands is as shown in the following figures.

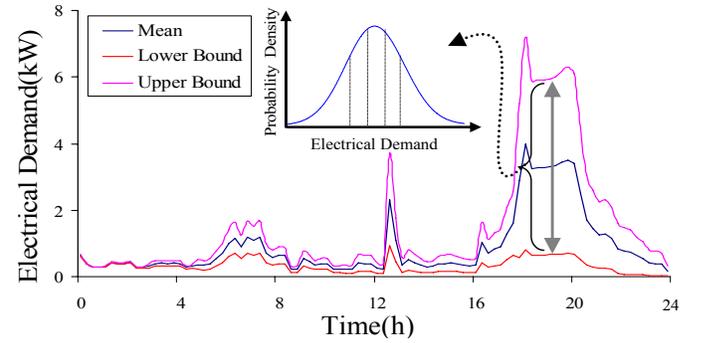


Fig. 1. Evolution of uncertain electrical demand over time.

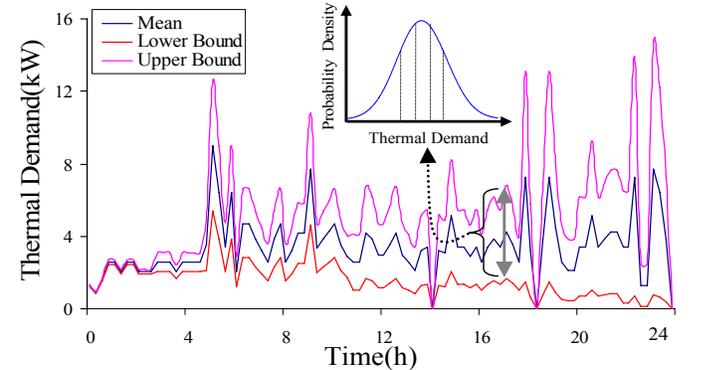


Fig. 2. Evolution of uncertain thermal demand over time.

All the samplings of this random process are represented as a scenario tree. The scenario tree has a completely defined root node at stage 0. The distribution of the random process at any

time stage is discretized into five samples. Hence this node branches into five successive nodes and the tree evolution progresses till the end of the planning horizon. The resulting scenario tree with the decision making points (nodes) is shown in Fig. 3.

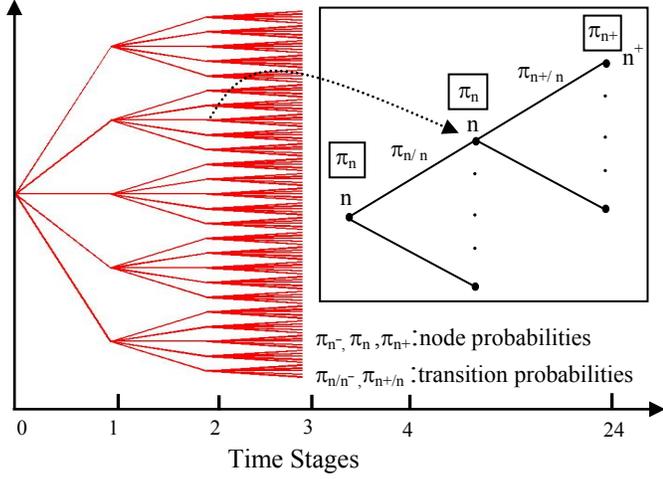


Fig. 3. Standard scenario tree with 24 branching stages and 5 branches at each stage.

The probability of the random variable at each branching stage has Gaussian probability density function. The distribution has a mean equal to the mean demand at that stage and standard deviation corresponds to that shown in Fig. 1 for electrical demand and Fig. 2 for thermal demand.

Each node ( $n$ ) of the scenario tree has a unique predecessor node ( $n^-$ ) and a transition probability  $\pi_{n/n^-} > 0$ , which is the probability of  $n$  being the successor of  $n^-$ . The probability  $\pi_n$  of each node  $n$  is given recursively by  $\pi_1=1$ ,  $\pi_{n/n^-} = \pi_{n-1}$  for  $n \neq 1$ . The probabilities of all the nodes at any given time stage add to one. The scenario tree consists of  $5^{24}$  scenarios. Since it is difficult to solve the stochastic model with  $5^{24}$  scenarios, the tree has to be reduced to a finite number of scenarios. The conventional scenario reduction schemes can not be used to prune the original tree because these methods require the description of the initial tree. For this purpose PSO has been tested for the first time ever, to generate a scenario tree. Since PSO is a searching algorithm, the initial huge numbers of scenarios need not be defined but only the search space of these scenarios. The particles then fly over the search space to find the best scenarios. The particle represents a scenario. For the optimal operation of the fuel cell, the settings have to be updated every one hour and hence for a planning horizon of 24 hours, there are 24 decision making stages. Therefore the particle is a 24 dimensional vector where each dimension represents a branching (decision) stage. The tree generation process is considered as an ordinary optimization problem whose objective is to maximize the fitness of each particle given by the following equation.

$$objective = \max_{m \in N_p} fitness(particle^m) \quad (17)$$

$$fitness(particle^m) = \min_{q \in N_p} \pi_m \left\{ \sum_{k=0}^{N_T} (\pi_k^m D_k^m - \pi_k^q D_k^q)^2 \right\}^{\frac{1}{2}} \quad (18)$$

Where

$N_p$  : Total number of particles in the swarm

$N_T$  : Total number of branching stages

$\pi_m$  : Total Probability of particle "m" from root to leaf node

$\pi_k^m$  : Probability of particle m at time stage k.

$D_k^m$  : Electrical demand corresponding to particle m at stage k

The process of generating the scenario tree can be explained by the following flowchart.

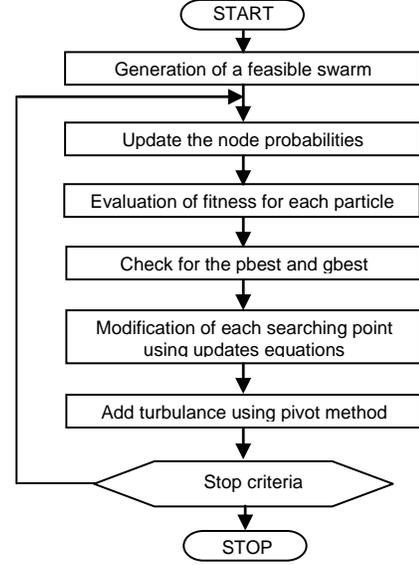


Fig. 4. Flowchart for generating the scenario tree using particle swarm approach

The process starts with the random generation of all particles in the swarm. This process is similar to Monte Carlo sampling. The probability density function (pdf) of the particle at each dimension is defined as a Gaussian distribution. Therefore each dimension of a particle is randomly chosen among the five discrete approximations and its probability is assigned using the pdf's. In the next step the nodal probabilities are updated. If the successors of any node are less than five, then the transition probability of the missing branch is added to its nearest neighbor. This will ensure that the total probability of all the successors to any node is equal to one. The fitness of a particle is then calculated using the above probabilities as shown in (18). The fitness value of a particle is the minimum weighted Euclidian distance from the other particles in the swarm. During the first iteration the particle's individual best (pbest) is initialized to its current fitness value. The best among the pbest values is selected as the global best (gbest) performance of the swarm. In the later iterations the current fitness is compared to the pbest, if there is any improvement then the pbest is replaced by the current fitness value. In the following step the particles are allowed to fly in the search space in accordance to the update equations listed in (14) and (15). In order to prevent premature convergence turbulence is added to each dimension of the

particle using the pivot method [6]. In the pivot method a positive area is defined around each dimension of the particle using the position vectors corresponding to pbest and gbest. Then a point is selected at random within this area and is then added as turbulence to the corresponding dimension of the particle. The process ends with the verification of the stop criteria which is usually the maximum number of iterations.

During the optimization process the particles traverse the whole search space to obtain an optimal fitness value. Since the fitness value is the distance with the other particles, the particles fly so as to move farther away from each other. Towards the end of the process, the particles are scattered in the whole search space with significant distance from its neighbors. Since the particles represent the scenarios, we are left with only the distinct scenarios. The scenario trees for the two random processes are listed in Fig. 5 and Fig. 6.

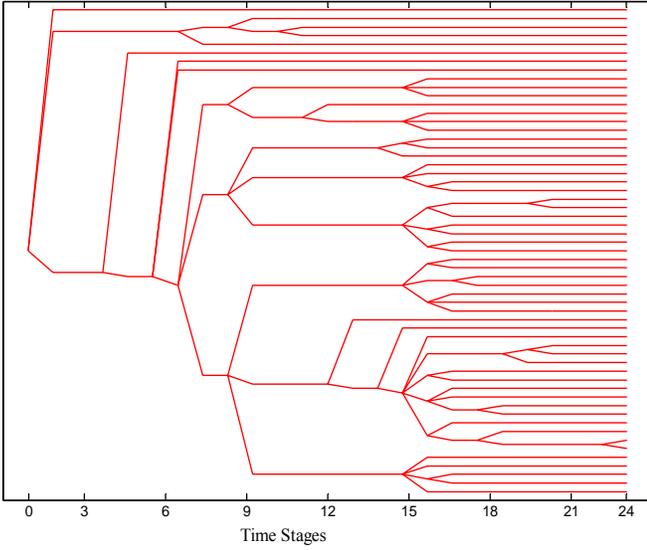


Fig. 5. Scenario tree for stochastic electrical demand generated by PSO for 24 branching stages and 5 branches at each stage.

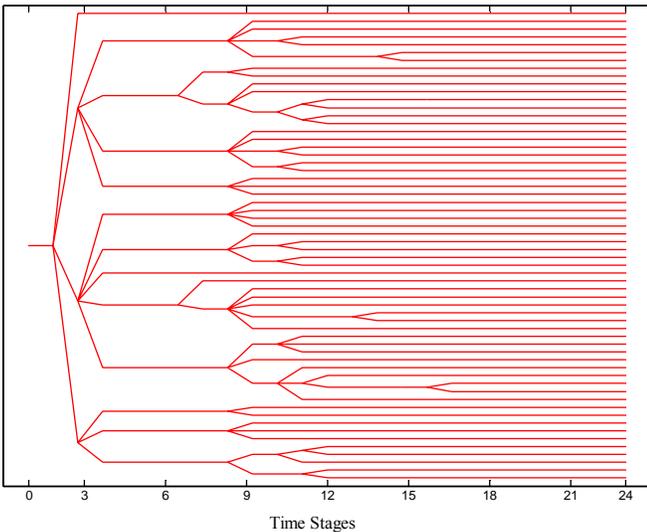


Fig. 6. Scenario tree for stochastic thermal demand generated by PSO for 24 branching stages and 5 branches at each stage.

#### D. Stochastic Optimization

Once the evolution of the uncertainty has been modeled as a scenario tree, it is incorporated into the economic model of the fuel cell. The resulting multistage stochastic model is solved using stochastic optimization programming. The objective of the multistage stochastic optimization problem is to minimize the expectation of the daily total operating costs of a fuel cell as shown below.

$$\min E \left\{ \begin{aligned} & C_{nFC} T \sum_J U_J \frac{P_J + P_a}{\eta_J} + C_{elp} T \sum_J U_J \max(D_{el,J} - P_J, 0) \\ & C_{els} T \sum_J U_J \max(P_J - D_{el,J}, 0) + K_{O\&M} T \sum_J U_J P_J \\ & C_{nRL} T \sum_J U_J \max(D_{th,J} - P_{th,J}, 0) + \\ & \sum_J U_J (1 - U_{J-1}) (\alpha + \beta (1 - e^{-\frac{T_{J,off}}{\tau}})) \end{aligned} \right\} \quad (19)$$

The objective can be realized by minimizing the average operating costs over the scenario tree.

$$\min \sum_{n \in N} \pi_n \left\{ \begin{aligned} & C_{nFC} T \sum_J U_J^n \frac{P_J^n + P_a}{\eta_J^n} + \\ & C_{elp} T \sum_J U_J^n \max(D_{el,J}^n - P_J^n, 0) + \\ & C_{els} T \sum_J U_J^n \max(P_J^n - D_{el,J}^n, 0) + K_{O\&M} T \sum_J U_J^n P_J^n + \\ & C_{nRL} T \sum_J U_J^n \max(D_{th,J}^n - P_{th,J}^n, 0) + \\ & U_J^n (1 - U_{J-1}^n) (\alpha + \beta (1 - e^{-\frac{T_{off}}{\tau}})) \end{aligned} \right\} \quad (20)$$

Where

$N$  : Total number of nodes

$P_J^n$  : Electrical power produced at interval "J" and node n(kW)

$$\text{Unit capacity constraints : } P_J^{n,\min} U_J^n \leq P_J^n \leq P_J^{n,\max} U_J^n \quad (21)$$

$$\text{Ramp rate constraints : } P_J^n U_J^n - P_{J-1}^n U_{J-1}^n \leq \Delta P_U \quad (22)$$

$$P_{J-1}^n U_{J-1}^n - P_J^n U_J^n \leq \Delta P_D \quad (23)$$

$$(T_{J-1}^{on} - MUT)(U_{J-1}^n - U_J^n) \geq 0 \quad (24)$$

$$(T_{J-1}^{off} - MDT)(U_J^n - U_{J-1}^n) \geq 0 \quad (25)$$

$$n_{start-stop} \leq N_{\max} \quad (26)$$

Note that the objective (20) and constraints of (21-26) correspond directly to (1)-(13). The tree based model for N nodes involve N binary, N continuous decision variables, N bounds and 5N inequality constraints. Adaptive particle swarm optimization (APSO) is used to solve the stochastic cost model. Each node of the scenario is associated with two decision variables (1: optimal power generation, 2: unit

commitment). Hence for an N node scenario tree, the particle represents a 2N-dimensional vector. The fitness of each particle is given by (20). The objective of the optimization process is to minimize the fitness of each particle.

$$\text{objective} = \min_{m \in N_p} \text{fitness}(\text{particle}^m) \quad (27)$$

The APSO optimization process described in references [5], [6] is used for the multistage stochastic optimization.

## V. RESULTS OF THE OPTIMIZATION PROCESS

The classical PSO for scenario tree generation and adaptive PSO for stochastic optimization were implemented using visual C/C++ language. Before we proceed to use the scenario tree generated by PSO for multistage stochastic optimization, the quality of the scenarios is evaluated. This is done by comparing the objective function value of the initial tree with the pruned tree. For computational simplicity, only three dominant (peak demand) branching stages ( $t_1=7$ ,  $t_2=13$ ,  $t_3=20$ ) of the electrical demand shown in Fig. 1, are considered. Hence the initial tree has  $5^3$  scenarios. Several test runs were performed with different number of scenarios and the variations are listed in table II. The results show that this approach is a good alternative to the conventional method.

TABLE I  
COMPARISON OF THE INITIAL TREE WITH THE PRUNED TREE

Type	S	N	Objective
Pruned Tree	12	181	2.228
Pruned Tree	27	298	2.265
Pruned Tree	45	423	2.360
Pruned Tree	70	562	2.403
Initial Tree	125	837	2.450

The deterministic cost model of a 4 kW fuel cell unit was solved for two different test cases.

Case 1: Electrical load is a stochastic variable whose standard deviations at each time stage are as shown in Fig. 1 and the thermal demand follows the mean thermal demand curve.

Case 2: Thermal load is a stochastic variable whose standard deviations at each time stage are as shown in Fig. 2 and the electrical demand follows the mean electrical demand curve.

The following cost parameters are assumed for the experiments  $C_{nFC}=0.03\$, C_{elp}=0.16\$, C_{els}=0.10\$, C_{nRL}=0.07\$$ . Both the test cases were solved with different number of scenarios. The results are as shown in Fig. 7 and Fig. 8. With few scenarios, the objective value is less because the scenario set does not completely represent the stochastic nature of the random process. As the number of scenarios increase, it is observed that the objective value stabilizes. In Fig. 8, after 60 scenarios, the objective value does not vary much even with additional scenarios, which implies that current set of

scenarios are sufficient enough to model the stochastic nature of the random process.

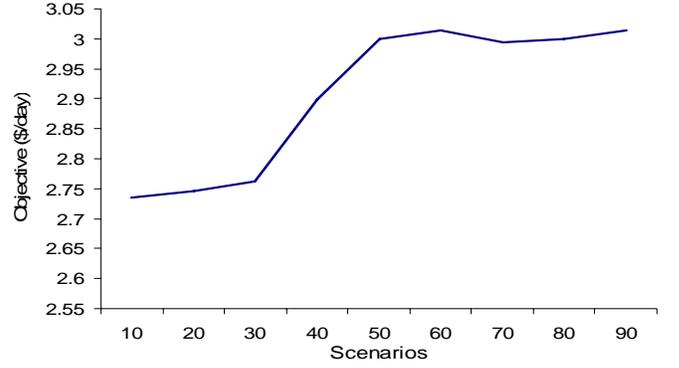


Fig. 7. Change in production cost (\$/day) with number of scenarios (case 1)

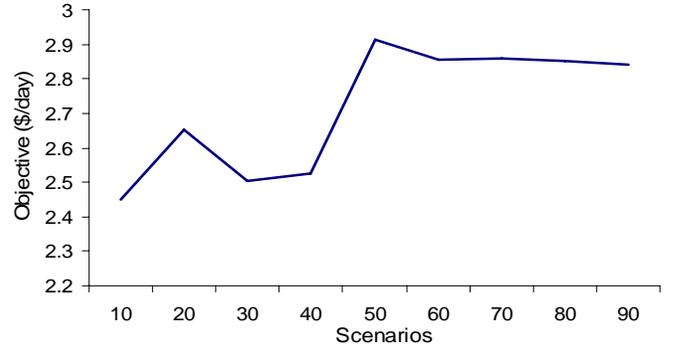


Fig. 8. Change in production cost (&/day) with number of scenarios (case 2)

For case 1 it is observed that approximately 50 scenarios are sufficient to model the uncertain electrical demand whereas for case 2, 60 scenarios are required. The daily operating costs for the optimal operation of the deterministic mean model i.e. cost model where electrical and thermal demands follows the mean curves, amount to 2.202\$. Whereas the cost for full capacity generation model i.e. fuel cell operating at 4 kW irrespective of the demands, amounts to 2.817\$. Although the production cost incurred by this model is close to the optimal cost, high heat energy is liberated during the whole operation phase. Therefore during the off peak period, this model requires an additional cooling device to absorb the excess heat energy and thus adding to additional installation cost. In the ‘Non-optimal generation to meet electrical demand’ model, the fuel cell generates electrical power exactly equal to the electrical demand. There is no optimization involved. This model never make an income by selling electricity even when the cost of selling electricity is high. The production cost incurred by this model amounts to 6.973\$. Therefore the optimal operation using APSO reduces the production costs by 4.771\$ per day which amounts to 1741.4\$ per year. In order to check the influence of stochastic electrical and thermal demand on the economic model of the fuel cell, the stochastic model was solved for the same set of cost parameters. The uncertainty was modeled as a scenario tree with 24 branching stages and 5 branches at each stage.

The initial scenario tree with  $5^{24}$  scenarios is reduced to a tree with 57 scenarios for case 1 and 60 scenarios for case 2 as shown in Fig. 5 and Fig. 6 respectively. The stochastic model is solved over the scenario tree. The operating cost under these conditions was 3.057\$ for case 1 and 2.854\$ for case 2. The reduction is approximately 1430\$/year for case 1 and 1504\$/year for case 2.

TABLE II  
COMPARISON OF THE PRODUCTION COSTS FOR DIFFERENT MODELS

Model	Production Cost(\$/day)
Full capacity generation model	2.817
Deterministic mean model	2.202
Stochastic model with stochastic electrical demand	3.057
Stochastic model with stochastic thermal demand	2.854
Non-optimal generation to meet electrical demand	6.973

The results show that the daily operating of the fuel cell can be significantly reduced even under the influence of stochastic electrical or thermal demand.

#### VI. CONCLUSION

Particle swarm optimization algorithm was successfully implemented to generate high quality scenario trees. The limitations set by the current scenario tree generation methods were overcome by PSO. As there is no restriction on the number of decision making stages and the number of branches at each stage, this approach can completely model the stochastic nature of the random process and can significantly reduce the modeling error. For computational simplicity, the current model considered only one uncertain variable. However the proposed approach can be extended to any number of uncertainties.

Secondly, adaptive particle swarm optimization was successfully applied to solve the multistage mixed-integer nonlinear cost model without any decomposition techniques. APSO gives better optimal solutions with few function evaluations. Since the stochastic electrical and thermal demand has profound influence on the cost model, the fuel cell model should always be solved using stochastic model. It is observed that by operating the fuel cell with optimal settings, the operating costs can be drastically reduced and hence fuel cells can be a competitive energy source for distributed generation.

#### VII. REFERENCES

- [1] Capasso A, Grattieri W, Lamedica R, Prudenzi A. 1994. A Bottom-Up Approach to Residential Load Modeling. *IEEE Transactions on Power Systems*. 9(2):957-964. DOI:10.1109/59.317650.
- [2] Birge John R, Louveaux Francois. *Introduction to stochastic programming*. ISBN:0-387-98217-5

- [3] N. Gröwe-Kuska, H. Heitsch, W. Römisich: Scenario reduction and scenario tree construction for power management problems, *IEEE Bologna Power Tech Proceedings* (A. Borghetti, C.A. Nucci, M. Paolone eds.), 2003 IEEE.
- [4] Kjetil Høyland, Stein W. Wallace, *Generating Scenario Trees for Multistage Decision Problems*, *Management Science*, v.47 n.2, p.295-307, February 2001
- [5] V.S.Pappala, I.Erlich. Management of distributed generation units under stochastic load demands using particle swarm optimization, invited paper *IEEE PES General Meeting 2007*, Tampa, Florida, 24-28 June
- [6] V.S.pappala, M. Wilch, S.N.Singh, I. Erlich. Reactive power management in offshore wind farms by adaptive PSO submitted to 14<sup>th</sup> International Conference on Intelligent Systems Application to Power System ISAP 2007
- [7] Ahmed M. Azmy, B.P. Wachholz and I.Erlich, "Management of PEM Fuel Cells for Residential Applications using Genetic Algorithms" *The ninth International Middle-east Power systems conference (MEPCON)*, Egypt, December, 16-18, 2003
- [8] F. Barbir and T. Gomez "Efficiency and Economics of Proton Exchange Membrane (PEM) Fuel Cell", *Int. J. Hydrogen Energy*, Vol 21, No 10, pp 891-901, 1996.
- [9] J.Kennedy and R.C. Eberhart. "Particle Swarm Optimization" *International Conference on Neural Networks IV*, pages 1942-1948, Piscataway, NJ, 1995. IEEE E Service Center.
- [10] J.Kennedy and R.C. Eberhart. *Swarm Intelligence*, Morgan Kaufmann Publishers, 2001.

#### VIII. BIOGRAPHIES



**Venkata Swaroop Pappala** received the B.E degree in electrical engineering from Faculty of Electrical engineering, Nagarjuna University, India in 2002, and M.Sc. degree in electrical engineering with emphasis on power and automation from University Duisburg-Essen, Germany in 2005. He is currently a Ph.D. student at the University of Duisburg-Essen, Germany. His research interests include stochastic optimization under uncertainty using evolutionary algorithms.



**Istvan Erlich** (1953) received his Dipl.-Ing. degree in electrical engineering from the University of Dresden/Germany in 1976. After his studies, he worked in Hungary in the field of electrical distribution networks. From 1979 to 1991, he joined the Department of Electrical Power Systems of the University of Dresden again, where he received his PhD degree in 1983. In the period of 1991 to 1998, he worked with the consulting company EAB in Berlin and the Fraunhofer Institute IITB Dresden respectively. During this time, he also had a teaching assignment at the University of Dresden. Since 1998, he is Professor and head of the Institute of Electrical Power Systems at the University of Duisburg-Essen/Germany. His major scientific interest is focused on power system stability and control, modelling and simulation of power system dynamics including intelligent system applications. He is a member of VDE and senior member of IEEE.