Particle Swarm Based Optimal Estimation of Block Incremental Cost Curve

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Abstract: **Competitive electricity market (EM) requires the bid from the electricity supply companies in the required formats which are defined in the market regulations. Block (stair-case) bid and linear bid are two most commonly used bid format for both market and research purposes. In most of the EM, the generating companies (so called energy suppliers) are required to bid their power outputs in blocks with the associated price of each block. For a unit having with multiple fuel and valve point loading effect, the generating company must divide the output power of the unit into the blocks so that it can be bid into the electricity market with and/or without gaming. This paper addresses the estimation of optimal block incremental cost from the instantaneous incremental heat rate curve of a generating unit using particle swarm optimization approach. During each block, the total cost is the same to the cost obtained with the instantaneous incremental cost curve. The formulation is simple and can be used for any generating unit with minimum error at block edges. The effectiveness of the proposed method is illustrated with the case studies.**

***Index terms*: Block incremental cost, incremental heat rate, valve-point loading effect, competitive power market, particle swarm optimization.**

I INTRODUCTION

ELECTRIC supply industry, in the several countries, is in the process of irreversible change from vertically integrated (regulated) structure to vertically unbundled structure with prime objective to introduce competition wherever possible. It is believed that competition will increase the customer focus and market efficiency. In perfectly competitive market condition, which is rare in the electricity market (EM), several producers bid and compete to win a share of the market to supply the trading commodity. In the EM, there are several situations where market does not operate in a fair and transparent manner by the system constraints and also due to gaming by the market participants. The prices bid by suppliers for the blocks of generation offered to the grid would reflect what portions of the load curve a supplier hopes to win for each type of plant in its possession. This, in turn, depends on production cost estimates, temporal considerations of system demand variation, unit commitment costs and commercial considerations such as profit or economic utility maximization and expectations of competitors' behavior.

The cost characteristics of generating units are not smooth function (curves). Using actual generator input/output test data from the numerous units, the incremental cost (IC) characteristic is obtained which is a high order polynomial and may not be monotonically increasing [1-3]. The formation of blocks is easier in the linear incremental cost curves. The bock quantity (power) multiplied by the block price will be equal to the area under incremental cost curve of that block. This leads an optimization problem especially if the incremental cost curve is not a linear one. It has been proved that for quadratic continuous incremental cost curve, the segments (blocks) should be of equal size [2]. With the valve point loading [4] and higher order incremental curves, it is not easily possible to form the blocks of the generation output which matches the actual incremental cost curve and having minimum errors at edges of blocks.

Heat rate data as provided by the utility is a simplistic representation of the actual measurement data. The original data is a collection of measurements, taken over a period of time. This data must be fit to heat rate equations, which can only approximate the original data points [1, 5]. Heat rates are used in the production cost model for unit commitment, dispatch, marginal cost and production cost proposes [5, 6]. Incremental heat rate equations are used in the actual dispatch of real units. The block representation, also known as average incremental heat rate, to the actual heat rate used in dispatch is very important in the electricity market as generators allowed to bid into the market in the form of blocks of the output [7].

Most of the works utilize the approximate quadratic cost characteristics (i.e. linear incremental cost characteristic) of generating units [8]. The use of valve point loading effect in the economic dispatch is reported in the several research papers which used genetic algorithm [9-10] and particle swarm optimization [11-12]. The particle swarm optimization (PSO) is a population based optimization first proposed by Kennedy and Eberhart [13]. The origins of PSO are the best described as sociologically inspired with bird flocking. Some of the attractive features of the PSO, which is a stochastic algorithm, include the ease of implementation and free from the gradient requirement. The PSO algorithm has a flexible and well-balanced mechanism to enhance and adapt to the global and local exploration abilities [14-16]. However, no work, to the best of authors' knowledge, has been done for the development of optimal block-wise (stair-case) incremental cost curve, which is required for the supplier to frame their optimal biding strategy in the EM, from the non-quadratic cost curve having valve point loading effect.

This paper proposes a mathematical approach to develop block incremental cost curve from the higher order instantaneous incremental cost curve considering with and

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without valve point loading effect. Particle swarm optimization (PSO) technique has been used to solve the discontinuous, nonlinear optimization problem of block-wise incremental cost estimation with valve point loading effect. The effectiveness of the proposed method is demonstrated through case studies.

II ESTIMATION OF BLOCK-WISE IC CURVE

A. Linear Incremental Cost Curve

Most popularly used cost curve of generators are quadratic in nature which are obtained from the actual measurements and can be expressed as

$$f = aP_g^2 + bP_g + c \quad (1)$$

which yields a linear incremental cost (IC) curve as

$$IC = 2aP_g + b \quad (2)$$

where, f is the fuel cost function in \$/Hr and P_g is the power output of generating unit in MW. a, b and c are the cost coefficients in \$/MW^2-Hr, \$/MW-Hr and \$/Hr, respectively.

Fig. 1 illustrates how a continuous linear IC curve is to be approximated by block-wise segments. In a particular segment (block output), the area between the linear IC curve and block-wise IC curve should be zero. The block incremental cost p_i of any block-i can be easily obtained, in general form, as

$$p_i = \frac{IC_{i+1} + IC_i}{2} \quad (3)$$

where IC_i is the incremental cost of linear IC curve at generation output of P_{gi}. In Fig. 1, which shows three segments (blocks), p_1 is average of IC_1 and IC_2 obtained at P_{g1} (=P_{gmin}) and P_{g2}, respectively. Mathematically,

$$p_1(P_{g2} - P_{g1}) = \left(\frac{IC_1 + IC_2}{2}\right)(P_{g2} - P_{g1}) \quad (4)$$

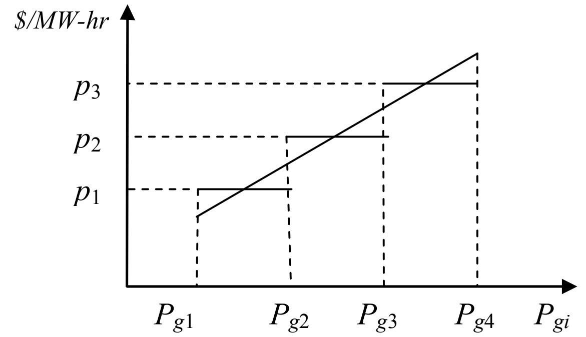

Fig. 1: Linear IC curve approximated by block-wise cost

Equation (4) shows that area under linear IC curve between any segment/block is equal to the area under the block IC curve. This approximation leads to more profit to the generator when power sold is in the fraction of the block (segment) power at the block incremental cost. However, if full capacity of the block is dispatched, there will be no loss to the generator. If the numbers of block segments are large, the approximated block incremental costs will be the very close to linear IC curve. Due large errors at the edges of the blocks, there is always risk for not winning the bids in the competitive power market.

B. Quadratic Incremental Cost Curve

Fig. 2 shows a continuous second order IC curve to be approximated by bock-wise segments. The quadratic IC characteristic may be convex or concave in shape. It is assumed that the IC curve is to be approximated three segments as shown in Fig. 2.

The cost curves of a generator having quadratic IC can be expressed as

$$f = aP_g^3 + bP_g^2 + cP_g + d \quad (5)$$

which will yield an IC curve as

$$IC = 3aP_g^2 + 2bP_g + c \quad (6)$$

where, a, b, c and d are the cost coefficients in \$/MW^3-Hr, \$/MW^2-Hr, \$/MW-Hr and \$/Hr, respectively. The segment price of any segment block-i between P_{gi} and P_{gi+1} can be calculated as

$$p_i = a(P_{gi}^2 + P_{gi+1}^2 + P_{gi}P_{gi+1}) + b(P_{gi+1} + P_{gi}) + c \quad (7)$$

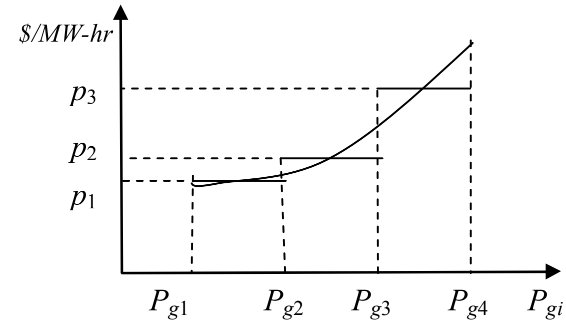

Fig. 2: Quadratic IC curve approximated by block-wise cost

C. Quadratic Cost Curve with Valve-Point Loading

The generators' cost functions are obtained from data point taken during heat-run tests, when input and output data are measured as the unit slowly varies through its operating region. Wire drawing effects, which occur as each steam admission valve in a turbine starts to open, produce a rippling effect on the unit cost curve. To consider the accurate cost curve of each generating unit, the valve-point effects must be included in the cost model. To take account for the valve-point effects, sinusoidal functions are added to the quadratic cost function as shown in Fig. 3 and can be written as

$$f = aP_g^2 + bP_g + c + |d * \sin(e(P_{g\min} - P_g))| \quad (8)$$

where, d and e are the coefficients of generator reflecting valve-point effects.

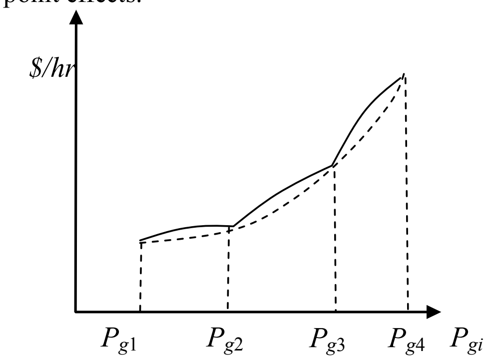

Fig. 3: Quadratic cost curve with valve-point loading effect

The incremental cost will be obtained as

$$IC = 2aP_g + b + \frac{k1 * k2}{k} \quad (9)$$

where $k = |d * \sin(e(P_{g\min} - P_g))|$,

$$k1 = -d * \sin(e(P_{g\min} - P_g))$$

$$k2 = d * e * \cos(e(P_{g\min} - P_g))$$

It should be noted that the function corresponding to the valve point loading effect is only differentiable during the segments between valve opening and closing. The incremental cost curve, as shown in Fig. 4, is highly non-linear and discontinuous at the valve openings. The blocks can be easily formed if the block segment is between the two valve opening points as

$$p_i(P_{gi+1} - P_{gi}) = \int_{P_{gi}}^{P_{gi+1}} IC dP_g \quad (10)$$

where p_i is the block incremental cost between segment P_{gi} and P_{gi+1}.

Mathematically, we can define an error function for block-i, which is to be zero or as minimum as possible, as

$$A_i = \int_{P_{gi}}^{P_{gi+1}} IC dP_g - p_i(P_{gi+1} - P_{gi}) \quad (11)$$

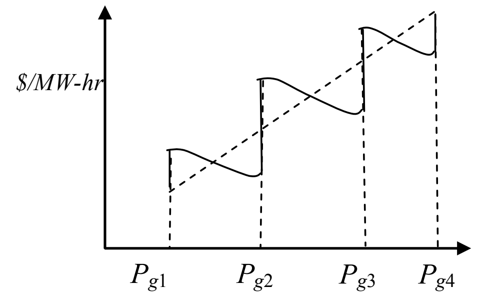

Fig. 4: Incremental cost curve with valve-point loading effect

If there are N block segments, we can write the total error function as follows

$$J = \sum_{i=1}^{N} A_i = \sum_{i=1}^{N} \left(\int_{P_{gi}}^{P_{gi+1}} IC dP_g - p_i(P_{gi+1} - P_{gi}) \right) \quad (12)$$

The error function J can be minimized for each segment (between valve point loading) separately using any conventional or non-conventional optimization techniques to find the optimal block prices. When the block segment is changed, which depends on the suppliers decision based on the market conditions, conventional optimization techniques may fail due to discontinuities at opening of valves and therefore we have to use some method those do not require the gradients such as genetic algorithms (GA), Tabu search (TS), evolutionary programming (EP), simulated annealing (SA), particle swarm optimization (PSO) etc.

D. Quadratic Incremental Cost Curve with Valve-Point Loading and Multiple Fuel Case

The problem becomes more complex when the incremental cost curve is quadratic with valve point loading effects. The objective function as defined in equation (12) will be the same but the IC will be a quadratic function superimposed on the valve point effect curve. If there are different fuels units to be considered, the objective function with K fuel input can be reformulated for all the fuel input cases as

$$J = \sum_{j=1}^{N} A_j = \sum_{i=1}^{N} \left(\int_{P_{gi}}^{P_{gi+1}} \left(\sum_{j=1}^{K} IC_j \right) dP_g - p_i(P_{gi+1} - P_{gi}) \right) \quad (13)$$

The objective function J is a non-linear, discontinuous function and is to be minimized using optimization program with various constraints as discussed in next section.

III PROBLEM FORMULATION

For the quadratic cost curve (i.e. linear incremental cost of unit) without the valve point loading effect, the block-incremental cost can be obtained easily for a given error limit at the edges of the bocks. From (4), it can be easily seen that the number of block sizes should be equal unless and until there is some other vested interest of supplier. The block size and the number of blocks can be easily decided for known value of maximum errors in the approximation. For N equal blocks, the error (e_r) at the edge of blocks will be

$$e_r = \frac{1}{2N} \left(\frac{IC_2 - IC_1}{IC_1} \right) \quad (14)$$

where, IC_1 and IC_2 are the incremental cost at P_{gmin} and P_{gmax}, respectively.

For the quadratic incremental cost curve, the block incremental cost curve for N blocks and K fuel units can be obtained by solving following optimization problem.

$$\underset{p_i, P_{gi}}{Min} \sum_{i=1}^{N} \left(\int_{P_{gi}}^{P_{gi+1}} \left(\sum_{j \in J} IC_j \right) dP_g - p_i(P_{gi+1} - P_{gi}) \right) \quad (15)$$

subject to

$$\frac{\left(\sum_{j \in J} IC_j(P_{gi}) - p_i \right)}{\sum_{j \in J} IC_j(P_{gi})} \le e_r \quad \forall i \in N \quad (16)$$

$$p_{i+1} \ge p_i \quad \forall i \in N \quad (17)$$

$$P_{g\min} \le P_{gi} \le P_{g\max} \quad \forall i \in N \quad (18)$$

$$P_{gi+1} \ge P_{gi} \quad \forall i \in N \quad (19)$$

where, e_r in (16) is a predefined error value which guarantees the errors at the edges of the blocks to remain in the prescribed limit. In the case of linear and concave incremental (marginal) cost curves, the maximum errors will occur at the beginning of the blocks. Incremental cost curve with valve point loading, the error may occur at any edge of the block (starting or ending). To take care of the error at either edge of the block, the following constraint along with (16) is to be enforced.

$$\frac{\left(p_{i+1} - \sum_{j \in J} IC_j(P_{gi}) \right)}{\sum_{j \in J} IC_j(P_{gi})} \le e_r \quad \forall i \in N \quad (20)$$

Equation (15) represents the objective function which is to be optimized for a given number of blocks for obtaining the block incremental cost and the optimal block sizes to satisfy the functional constraints (16) and (20), and parametric constraints (17), (18) and (19), which are inequality constraints. Equation (17) is representing the inequality constraints to assure that the cost of next block will be more than the cost of previous blocks whereas (19) is used to get the feasible bock size. Equation (18) represents the search space for block sizes.

Sometime electric suppliers set the block size to be more than certain value and this can be defined as following inequality constraints.

$$P_{i+1} - P_i \ge P_{b\min} \quad \forall i \in N \quad (21)$$

where $P_{b\min}$ is the minimum allowed block size which can be less or equal to the capacity of power generation output.

For the continuous incremental cost, the problem defined in (15) to (21) can be solved using any nonlinear optimization programs. With valve-point loading effect, the incremental cost curve is no more continuous and thus, any non-conventional optimization technique can be used. In this paper, an adaptive particle swarm optimization technique has been used and results are compared with the successive quadratic programming (SQP) (NAG subroutine). For the valve point loading effect in SQP approach, the discontinuity is avoided by adding some tolerance as explained in section-V.

IV PARTICLE SWARM OPTIMIZATION

The PSO algorithm is a population-based stochastic optimization technique. The potential solutions, called the particles, fly through the search space by following the current optimal particle. Objective function values are used as the fitness values of particles to guide the search process. The basic particle swarm model consists of a group of particles moving about in a physical dimensional search space. Each individual in a swarm traverses the search space with a velocity which is dynamically adjusted to its own flying experience and its companion flying experiences. Each individual is treated as a volume-less particle in an *n*-dimensional search space.

A. Overview of Particle Swarm Optimization (PSO)

In a physical *n*-dimensional search space, the position and velocity of individual-i are represented as the vectors $X_i = (x_{i1}, x_{i2}, ...x_{in})$ and $V_i = (v_{i1}, v_{i2}, ...v_{in})$, respectively in the PSO algorithm. Each particle records its best individual fitness and position (*pbest*) so far. Moreover, each particle knows the best fitness and position so far in the group (*gbest*) among all individuals. Let $P_{besti} = ({}^{Pbest}x_{i1}, ...{}^{Pbest}x_{in})$ and $G_{best} = ({}^{Gbest}x_1, ...{}^{Gbest}x_n)$, respectively be the best position of individual-*i* and its neighbors' best position so far. To ensure the convergence of PSO, the use of constriction factor may be necessary [8]. Using the information, the updated and positions of individual-*i* are modified under the following equations

$$V_i^{k+1} = K(\omega^k V_i^k + c_1 rand_1 * (P_{besti}^k - X_i^k) + c_2 rand_2 * (G_{best}^k - X_i^k)) \tag{22}$$

$$X_i^{k+1} = X_i^k + V_i^{k+1} \tag{23}$$

where, k is a pointer of iteration, X_i^k is the position of particle i at iteration k, V_i^k is velocity of individual-*i* at iteration *k*, ω is inertia weight factor; c_1 and c_2 are acceleration constants, $rand_1$ and $rand_2$ are uniform random values in the range [0,1], K_c is constriction factor which is a function of c_1 and c_2 as referred in following equation

$$K_c = \frac{2}{|2 - \varphi - \sqrt{\varphi^2 - 4\varphi}|} \tag{24}$$

where $\varphi = c_1 + c_2$ and $\varphi > 4$.

The search mechanism of the PSO using the modified velocity and position of individual *i* based on (22) and (23) is illustrated in Fig. 5. It is worth mentioning that the second term in the equation (22) represents the cognitive part of PSO where the particle changes its velocity based on its own thinking and memory. The third term represents the social part of PSO where the particle changes its velocity based on social-psychological adaptation of knowledge.

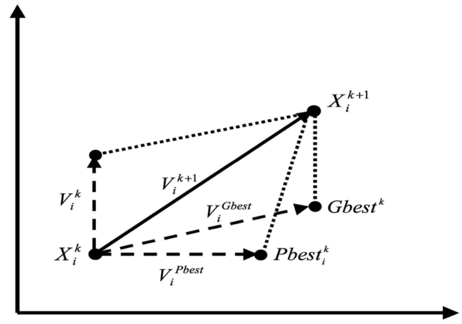

Fig.5: The Search mechanism of the particle swarm optimization.

Suitable selection of inertia weight ω^k provides a balance between global and local explorations. A large inertia weight facilitates a global search while a small inertia weight facilitates a local search. By linearly decreasing the inertia weight from a relatively large value to small value through the course of PSO run, the PSO tends to have more global search ability at the beginning of the run while having more local search ability near the end of the run [13]. The inertia weight is set according to the following equation.

$$\omega^k = \omega_{\max} - \frac{\omega_{\max} - \omega_{\min}}{iter_{\max}} * iter \tag{25}$$

where $iter_{\max}$ is the maximum number of iterations, and *iter* is the current number of iteration, $\omega_{\max}$ and $\omega_{\min}$ are initial and final weights respectively.

B. PSO Algorithm

In PSO algorithm, the population has *M* particles and each particle is an *n*-dimensional vector, where *n* is the number of optimized parameters. The computational flow of PSO technique can be described in following steps.

Step-1 (Initialization): Set the iteration counter $iter = 0$ and generate randomly *M* particles, $\{X_i^0, i = 1, \cdots M\}$, where $X_i^0 = [x_{i1}^0, x_{i2}^0, \cdots x_{in}^0]$. x_{ij}^0 is generated by randomly selecting a value with uniform probability over the j^{th} optimized parameter search space $[x_j^{\min}, x_j^{\max}]$. Similarly, generate randomly initial velocity of all particles, $\{V_i^0, i = 1, \cdots M\}$, where $V_i^0 = [v_{i1}^0, v_{i2}^0, \cdots v_{in}^0]$. v_{ij}^0 is generated by randomly selecting a value with uniform probability over the j^{th} dimension $[v_j^{\min}, v_j^{\max}]$, where $v_j^{\min} = (x_j^{\min} - \varepsilon) - x_j^0$, $v_j^{\max} = (x_j^{\max} + \varepsilon) - x_j^0$ and ε is a small positive integer. In the initial population, each particle's fitness value is evaluated using constraint matrix and objective function *C*. The initial P_{besti} of individual *i* is set as the initial position of individual *i* and the initial G_{best} is determined as the position of individual with highest fitness value. Set the initial value of inertia weight, ω^0.

Step-2 (Generation updating): Update the iteration counter $iter = iter + 1$.

Step-3 (Weight updating): Update the inertia weight ω^k using equation (25).

Step-4 (Velocity updating): Using the global best and individual best of each particle. The i^{th} particle velocity is updated according to the equation (22).

Step-5 (Position updating): Based on the updated velocities, each particle changes its position according to the equation (23). If a particle violates its position in any dimension, set its position at the proper limit.

Step-6 (Individual best updating): Each particle's fitness value is evaluated according to its updated position. If i^{th} particle's fitness value at iteration $k+1$ is more than of the k^{th} iteration, then updating individual best as $P_{besti}^{k+1} = X_i^{k+1}$, $i = 1, \cdots M$.

Step-7 (Global best updating): Search for high fitness value among P_{besti}^{k+1}, $i = 1, \cdots M$, if any of these particle's fitness value is more than fitness value of G_{best}, then update global best as $G_{best} = P_{best}^{k+1}$, where P_{best}^{k+1} is with highest fitness value among individuals at iteration $k+1$.

Step-8 (Stopping criteria): If the iteration counter reaches predefined maximum iteration, then Stop; else go to step 2. After the stopping criteria satisfied, the particle that generates the latest G_{best} is the optimal value.

An adaptive particle swarm optimization technique [17] is used for all the simulations presented in this paper. The significant feature of this algorithm is that the optimization algorithm can be used as a black box; the user has to specify just the search space, the objective function to be minimized and stopping criteria for the algorithm. The user is totally free from parameter tuning and also from the burden of selecting the most appropriate swarm (population) size.

In this algorithm, different sized groups of particles called "Tribes" move about in an unknown environment in search of an optimal solution. Information links exist among the particles in a tribe. These particles collectively explore the search space to find a local minimum. Each such tribe succeeds in finding a local minimum. Information links also exist among the different tribes through which they exchange information regarding their local minimums to collectively decide the global minimum. The optimization process starts with a single particle representing a single tribe. After a few iterations if this particle does not improve its fitness, it generates a new particle forming a new tribe. After further iterations if neither of these particles improves their fitness, each of these particles generates a particle simultaneously forming a new tribe with two particles. The process of evolution continues until the stopping criterion is met.

V Case Studies and Results

To demonstrate the effectiveness of the proposed approach, a generating unit at Moss Landing 7 of PG &E, USA, which is having the quadratic instantaneous incremental heat rate (IHR) as $-0.0039P_g^2 + 5.91P_g + 6561.2$ Btu/kWh, is considered in the paper. The cost of fuel is taken as \$2.5 /Mbtu. The incremental cost curve, which will be obtained by the multiplying the cost of fuel, will be similar to the incremental heat rate as

$$IC = 0.0025 \times \left(-0.0039P_g^2 + 5.91P_g + 6561.2\right) \ \$/\text{MWh}$$

The lower and upper limits of this generating unit are set to 50 MW and 739 MW, respectively. The different studies are carried out with different value of the maximum error between the actual and estimated block-wise incremental cost curves. It is true that the number of blocks required for very high precision in the actual and estimated block size is very high. The minimum block size, in (21), is set to 50 MW. The block incremental cost cases with and without valve point loading (VPL) effect has been presented below.

A. Without Valve Point Loading Effect

Since the instantaneous incremental hear rate curve without valve point loading is convex as shown in Fig. 6, the maximum error will occur at the initial block edge which can also be seen from Table I. An adaptive PSO as explained in section IV is used to solve the problem defined in (15)-(21). For the different absolute values of maximum block errors as defined in (16), the different results are obtained with maximum allowed number of blocks to be four. It was found that for four numbers of blocks, the maximum block edge error (e_r) can not be less than 3.55% for the feasible solution obtained from PSO. Fig. 6 shows the instantaneous incremental cost curve along with the block incremental cost curve obtained using PSO technique. The optimal values of block sizes are given in Table I along with the errors calculated at the edge of the blocks which are obtained using PSO and SQP approaches. Column 3 of table shows the starting and ending power outputs of corresponding blocks (column 2). In case of successive quadratic approach (SQP), it can be seen from the Table I that the optimal blocks were obtained with error 3.60% as problem is very non-linear in nature. For any lower value errors, SQP did not give feasible solution whereas in the case of PSO, feasible solution is obtained with 3.55% for the zero objective value.

TABLE I
BLOCK SIZE AND INCREMENTAL COSTS

Cases	Block Number	P_{gi} & P_{gi+1} (MW)	IC at P_g's (\$/MWh)	Block IC (\$/MWh)	Error (%)
A (PSO)	1	50.0	17.12	17.73	**-3.55**
		142.1	18.31		3.19
	2	142.1	18.31	18.96	-3.55
		257.5	19.56		3.07
	3	257.5	19.56	20.26	-3.55
		417.0	20.87		2.92
	4	417.0	20.87	21.60	-3.50
		739.0	22.00		1.82
B (SQP)	1	50.0	17.12	17.73	**-3.60**
		143.4	18.32		3.21
	2	143.4	18.32	18.98	-3.60
		261.1	19.60		3.14
	3	261.1	19.60	20.26	-3.37
		413.0	20.84		2.81
	4	413.0	20.84	21.59	-3.60
		739.0	22.00		1.84

It is found that that initial blocks are restricted in size due to errors constraints. Later block are giving lesser errors and more blocks can be formed by the generating company to be the marginal generator in the competitive electricity market. It is true that, at large size block during the initial power output, the block incremental cost is more which leads to the more profit to the generator but at the same time, it may be possible being not dispatched in the electricity market due to more error at the block edge. Thus, generator will try to form the blocks so that it can win the bid and to be price taker. For any block, the area between the actual incremental cost curve and block incremental cost curve is equal. For the lesser number of the blocks, the error at the edges will more and less error limit may give an infeasible optimal solution. For the set value of error and the number of blocks, if the objective function will be more than zero after the solving optimal problem ((15)-(21)), the numbers of blocks must be increased. It was found that for maximum error less than 3.6%, the objective function is not zero, which shows infeasible optimal solution, in the case of SQP approach and 3.55 % in the case of PSO approach.

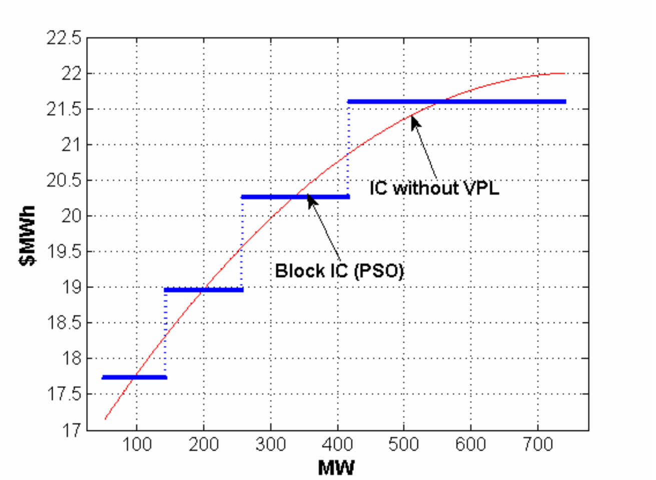

Fig. 6: Block incremental cost curve without VPL (PSO)

From Table I, it can be seen that for 3.55% error limit for PSO case, four optimal blocks are obtained (50MW to 142.1MW, 142.1MW to 257.5MW, 257.5MW to 417.0 MW and 537.2 MW to 739 MW) with maximum allowed number of blocks to be four. It can be noted that there are several optimal solutions with larger error limits, which depend the initial value of the solution in the case of SQP appoach. In this paper, the initial block size guess is taken as $\left(P_{g\max} - P_{g\min}\right)/N$ where N is the number of blocks. For the maximum error of 3.5%, it was found that complete incremental cost curve cannot be represented in four blocks.

B. *With Valve Point Loading Effect*

With the valve point loading (VPL) effect, the incremental heat rate or cost curve is not a continuous curve. In this work, four-valve action is considered and VPL parameters d and e, as defined in (8), are taken to be $6000/hr and 0.017 rad/MW, respectively. The incremental heat rate curve is shown in Fig. 7 where four discontinuities are corresponding to the opening and closing of the governor valves. The optimization problem as defined in (15)-(21) can not be solved by the conventional non-linear programs as the undefined derivatives at the discontinuities. To use the conventional non-linear programming for comparison purposes (sequential quadratic programming approach has been used here), the discontinuities are avoided by adding the small perturbation (by 0.0001 MW) at the point of discontinuous points. This assumption is valid as the actual setting of power output of a generating unit into the fractional digit is not possible.

The problem as defined in (15)-(21) is solved with valve point loading effect for the error limits at beginning and ending of the blocks as errors at these pints will be different. The maximum block number was set to four. The optimal blocks obtained with PSO and SQP approaches are presented in Fig. 7 along with the incremental cost curve considering the VPL. The feasible solutions obtained by PSO and SQP with e_r (in boldface) as defined in (19) are given in Table II. It can be seen from the Table II that error is minimum in the case of solution obtained with PSO, which is 3.98%. For 3.98% error limit at the block edges, optimal solution with four blocks is obtained (50MW to 182.6MW, 182.6MW to 291.6MW, 291.6MW to537.2 MW and 537.2 MW to 739 MW). It was found that with the error limit less than 5.30%, there is no feasible solution with SQP approach. It can be seen from Table II that the errors imposed are within limits.

TABLE II
BLOCK SIZE AND INCREMENTAL HEAT RATES

Cases	Block Number	P_{gi} and P_{gi+1} (MW)	IC at P_g's ($/MWh)	Block IC ($/MWh)	Error (%)
A (PSO)	1	50.0	17.37	18.06	-3.97
		182.6	18.61		2.95
	2	182.6	18.61	19.35	**-3.98**
		291.6	20.03		3.39
	3	291.6	20.03	20.81	-3.89
		537.2	21.42		2.85
	4	537.2	21.42	21.82	-1.87
		739.0	21.83		0.004
B (SQP)	1	50.0	17.37	17.76	-2.21
		114.6	18.08		1.81
	2	114.6	18.08	19.04	**-5.30**
		304.1	20.09		5.22
	3	304.1	20.09	20.34	-1.24
		419.6	20.63		1.40
	4	419.6	20.63	21.64	-4.90
		739.0	21.83		0.86

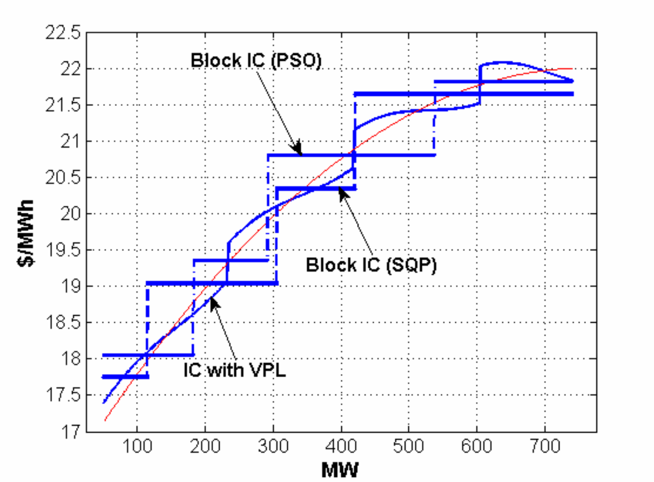

Fig. 7: Instantaneous and block incremental cost curves with VPL

VI CONCLUSION

This paper attempts to estimate the block incremental cost from the known instantaneous incremental cost curve obtained from the field data using particle swarm optimization approach and successive quadratic programming optimization. Formation of optimal blocks in the electricity market is a key concern of generating utilities to maximize their profits. The effect of valve point loading in the quadratic incremental cost curve has been used to formulate an optimization problem. It was found that the optimal values of the block cost and the block sizes depend on the error limits imposed on the edges of the blocks. For larger error tolerance, the optimal number of blocks can be even less but the block rates will be high and there is possibility of the non-dispatched in the electricity market due to higher cost of blocks. It was also shown that later blocks can be further divided into the smaller block to play in the market. It is seen that PSO gives better result compared to the SQP approach. The present algorithm may be a tool to the power utilities to determine the block rates and sizes in the competitive electricity market. Most of the planning software uses the block incremental heat rates and this algorithm may be used to change the instantaneous heat rate curves into the block incremental heat rates with minimum error.

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