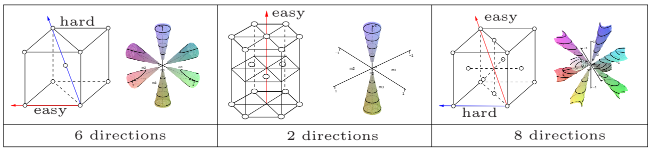
DFG FOR 1509 **Ferroische Funktionsmaterialien**
Mehrskalige Modellierung und experimentelle Charakterisierung

TP3 "Modellierung und Homogenisierung magneto-mechanischen Materialverhaltens auf verschiedenen Skalen"

Micromechanically Motivated Computational Modeling of Magneto-Mechanically Coupled Materials

Phase Field Model for Micro-Magneto-Elasticity

Micromagnetics allows for a truly multiscale viewpoint in the modeling of magneto-mechanically coupled materials since macroscopic behavior is determined by the domain structure and evolution of magnetic microstructure on the microscale.



Geometrically Exact Variational Principle

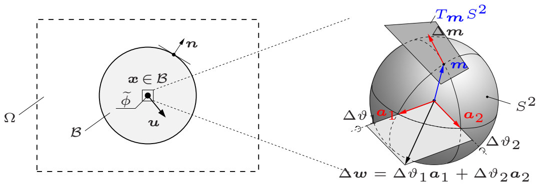
The coupled boundary value problem of micro-magneto-elasticity is governed by the rate-type variational principle

$$\{\dot{\boldsymbol{u}}, \dot{\widetilde{\phi}}, \dot{\boldsymbol{m}}\} = \arg\left\{ \underset{\dot{\boldsymbol{u}},\dot{\widetilde{\phi}},\dot{\boldsymbol{m}}}{\text{stat}} \int_{\mathcal{B}} \left[\frac{d}{dt}\Psi'(\boldsymbol{u},\boldsymbol{m},\widetilde{\phi}) + \Phi(\dot{\boldsymbol{m}})\right] dV \right\}$$

The evolution of the magnetization director resulting from the variational principle is the Landau-Lifschitz Gilbert equation

$$\dot{\boldsymbol{m}} = \frac{1}{\eta}\boldsymbol{m} \times \left(\boldsymbol{m} \times [\delta_{\boldsymbol{m}}\Psi_{mat} - \kappa_0 m_s(\overline{\boldsymbol{h}} - \nabla\widetilde{\phi})]\right) \quad \text{in } \mathcal{B}\,.$$

Preserving the constraint on the magnetization director in an



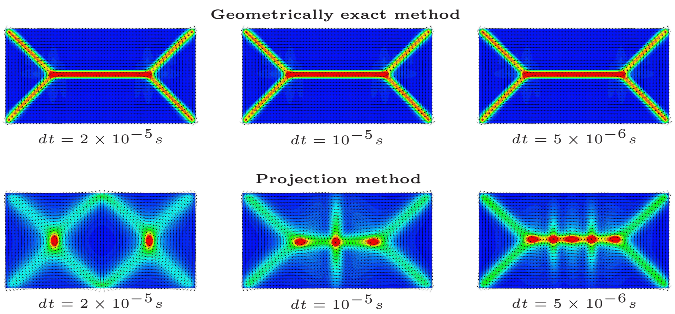
algorithmic setting is a challenge. In order to overcome this, we use the exponential map to update the magnetization

$$\{\boldsymbol{a}_1, \boldsymbol{a}_2, \boldsymbol{m}\} \;\Leftarrow\; \exp[\boldsymbol{1} \times ([\boldsymbol{a}_1, \boldsymbol{a}_2]\Delta\boldsymbol{\vartheta})]\{\boldsymbol{a}_1, \boldsymbol{a}_2, \boldsymbol{m}\}$$

Alternatively, we may include a normalization step as a *post-processing operation* in order to satisfy the constraint.

Numerical Results

Starting from a random distribution, we compare the final configuration resulting from each method for different time-steps. The results of the projection method approaches those of the geometrically exact method with decreasing time-steps.

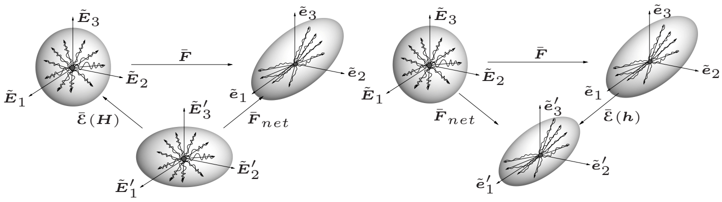


Micromechanics of Magnetorheological Elastomers

The coupled boundary value problem of magneto-visco-elasticity is compactly represented in the rate-type variational principle

$$\{\dot{\boldsymbol{\varphi}}, \dot{\phi}, \dot{\boldsymbol{\mathcal{I}}}\} = \arg\left\{ \underset{\dot{\boldsymbol{\varphi}},\dot{\phi},\dot{\boldsymbol{\mathcal{I}}}}{\text{stat}} \int_{\mathcal{B}} \left[\frac{d}{dt}\Psi(\boldsymbol{F},\boldsymbol{H},\boldsymbol{\mathcal{I}}) + \Phi^v(\dot{\boldsymbol{\mathcal{I}}})\right] dV \right\}$$

with $\Psi = U(J) + \bar{\Psi}^e(\bar{\boldsymbol{F}}^{net}) + \bar{\Psi}^v(\bar{\boldsymbol{F}}^{net}, \boldsymbol{\mathcal{I}}) + \Psi_{mag}(\boldsymbol{F},\boldsymbol{H})$, and Φ^v given by the specific viscoelastic model under consideration.

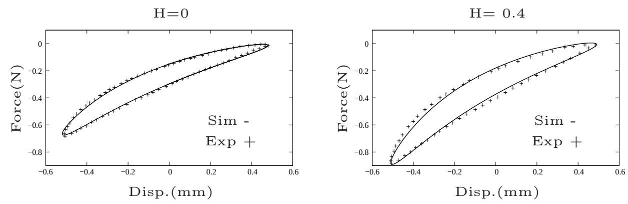


Due to the micromechanics of such materials, we are motivated to consider a multiplicative split of the deformation gradient.

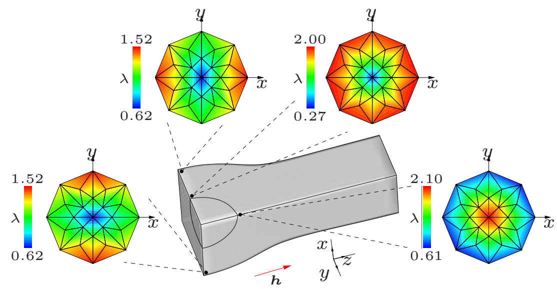
$$\bar{\boldsymbol{F}}_{net} := \begin{cases} \bar{\boldsymbol{F}} & \text{- case 1: total isochoric deformation,} \\ \bar{\boldsymbol{F}}\bar{\mathcal{E}}(\boldsymbol{H}) & \text{- case 2: right multiplicative decomposition,} \\ \bar{\mathcal{E}}(\boldsymbol{h})\bar{\boldsymbol{F}} & \text{- case 3: left multiplicative decomposition.} \end{cases}$$

Numerical Results

In dynamic compression tests, the model fits very well with experiments. This is shown in force vs. displacement plots below.



Results of a FEM simulation displays the modeling capability.



References

[1] C. Miehe and G. Ethiraj, A Geometrically Consistent Incremental Variational Formulation for Phase Field Models in Micromagnetics, *CMAME*, 2012.

[2] C. Miehe, B. Kiefer, D. Rosato, An incremental variational formulation of dissipative magnetostriction at the macroscopic continuum level *IJSS*, 2011.

[3] G. Ethiraj, D. Zäh, C. Miehe, A Finite Deformation Microsphere Model for Magneto-Visco-Elastic Response in Magnetorheological Elastomers *PAMM*, 2013.

C. Miehe
University of Stuttgart
Institute of Applied Mechanics (CE)

G. Ethiraj
University of Stuttgart
Institute of Applied Mechanics (CE)