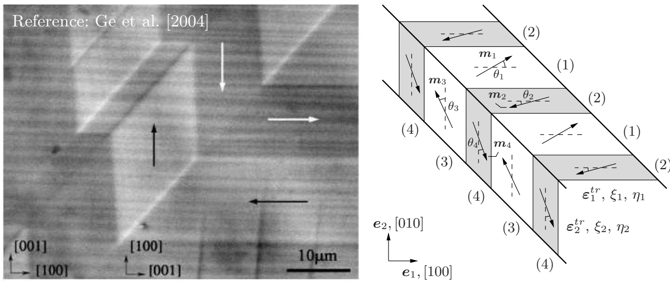
Modeling of Single Crystal Magnetostriction Based on Numerical Energy Relaxation Techniques

Magnetic SMAs: Magnetostriction of Martensite

Magnetic shape memory alloys are intrinsically multi-ferroic materials, exhibiting strong magnetomechanical coupling behavior in addition to the conventional shape memory effect. The strain and magnetization response is nonlinear, anisotropic, hysteretic and highly stress level-dependent. The MSMA effect is made possible by the simultaneous occurrence of high magnetocrystalline anisotropy energy and high twin boundary mobility.



Co-existence of martensitic twins and ferromagnetic domains: experimental and schematic visualization of microstructure in MSMA. Variant volume fractions ξ_i, net magnetizations η_i, orientation of the magnetization directions θ_i.

Point of departure

The *constrained theory of magnetoelasticity* [1] combines the Ball and James theory of microstructure formation [2] with classical *micromagnetics* approaches [3]. It describes the formation of fine-scale microstructures in magnetostrictive materials based on the relaxation of non-convex energy densities, in which the state variables are *constrained* to take values in the energy wells

$$\min_{\boldsymbol{\alpha}\in\mathcal{A}}\overline{\Pi}(\boldsymbol{\alpha})=\min_{\boldsymbol{\alpha}\in\mathcal{A}}\left\{\frac{1}{2}\langle\boldsymbol{m}\rangle(\boldsymbol{\alpha})\cdot\boldsymbol{D}\langle\boldsymbol{m}\rangle(\boldsymbol{\alpha})-\overline{\boldsymbol{h}}\cdot\langle\boldsymbol{m}\rangle(\boldsymbol{\alpha})-\overline{\boldsymbol{\sigma}}:\langle\boldsymbol{\varepsilon}\rangle(\boldsymbol{\alpha})\right\}$$

with the admissible set $\mathcal{A} := \{\boldsymbol{\alpha}\,|\,\alpha_i(\xi_i,\eta_i)\geq 0, \sum_{i=1}^{6}\alpha_i = 1\}$.

Extended Modeling Approach

In a first step of the extension, *dissipative effects* are accounted for in an incremental variational setting

$$\pi = \int_{t_n}^{t_{n+1}} (\dot{\psi} - \mathcal{P}_{ext})\,\mathrm{d}t + \int_{t_n}^{t_{n+1}} \zeta\,\mathrm{d}t\ .$$

Secondly, the high anisotropy limit is partially alleviated by allowing for *finite magnetocrystalline anisotropy energy*

$$\psi_i^{an} = \sum_{n=1}^{N} K_n^i \sin^{2n}(\theta_i)\ .$$

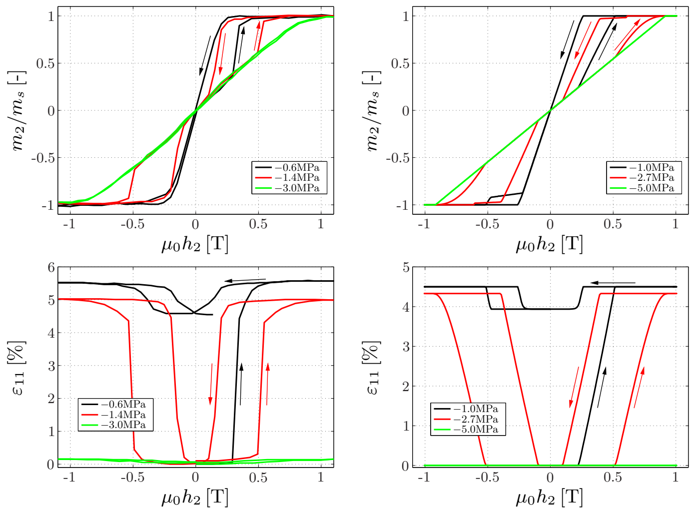
Incremental updates of the internal state variables are computed via the variational constrained minimization problem

$$\boldsymbol{p}_{n+1} = \arg\min_{\boldsymbol{p}_{n+1}}\{\Pi_{n+1}\} \quad \text{s.t. } \boldsymbol{p}_{n+1} \text{ admissible}$$

with $\boldsymbol{p}_{n+1} = \{\alpha_i(\xi_i,\eta_i),\theta_i\}$. In on-going work, convexification and laminate-based relaxation approaches are also considered.

Numerical Examples

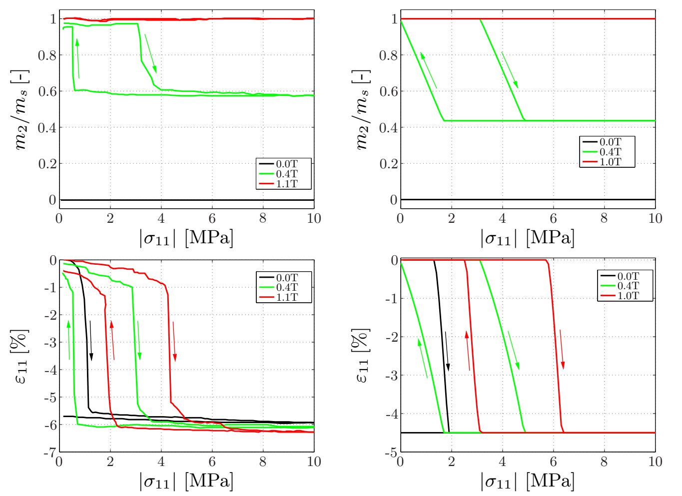
Example 1: Magnetic-field-induced variant reorientation at constant stress; experiment [4] (left), simulation (right)



Predicted features not captured by the constrained theory:

- Hysteretic response due to variant reorientation
- Stress level dependence of the field-induced strain
- Initial linear field dependence of magnetization

Example 2: Stress-induced variant reorientation at constant magnetic field; experiment [4] (left), simulation (right)



References

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