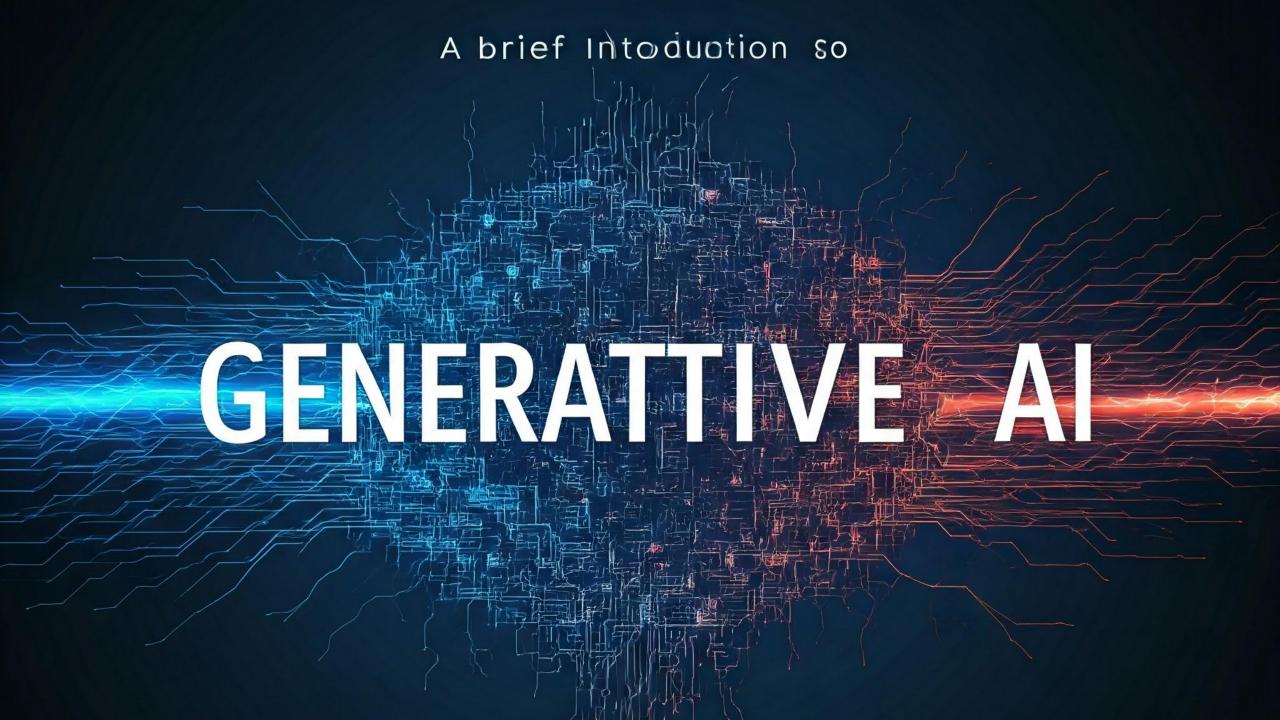
# A brief introduction to Generative Al

What ChatGPT Is Made Of



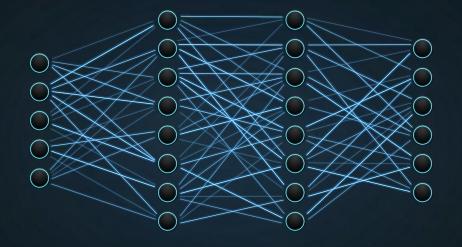
#### Generative Al



# Large Language Models

## Deep Learning

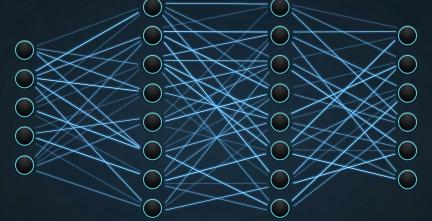
Artificial Neural Networks (ANNs)



## Deep Learning



 $f_1$   $f_2$   $f_3$ 



Katze

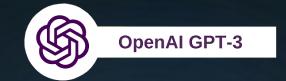
$$f: \mathbb{R}^m (\times \Omega) \to \mathbb{R}^n, (x; A, b) \mapsto \sigma(Ax + b)$$

$$L: \Omega \to \mathbb{R}, \omega \mapsto \sum_{t \in T} l(t, f(t; \omega))$$

## Large Language Models

#### **Training**







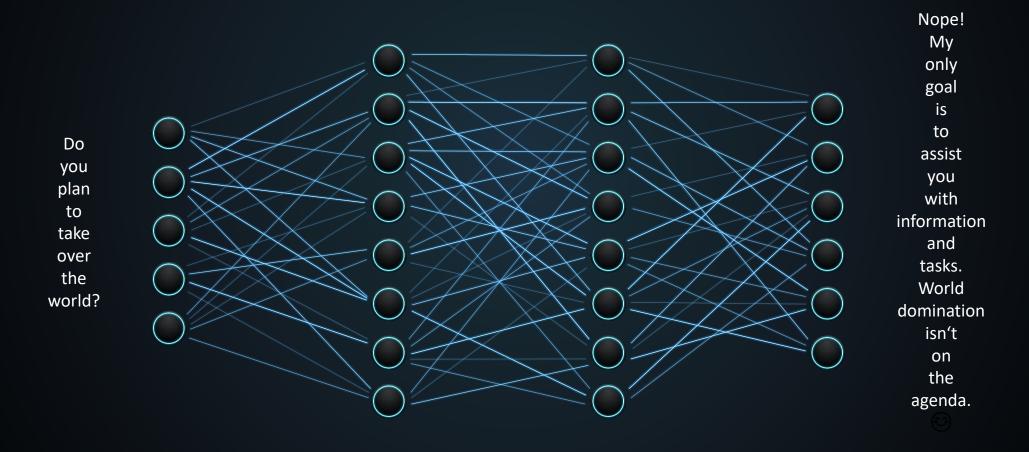
**Architecture** 

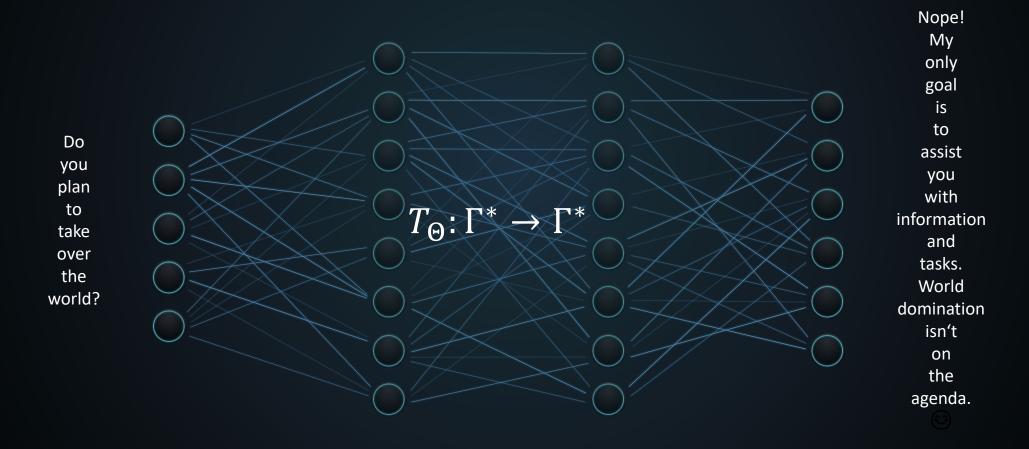
#### Inference

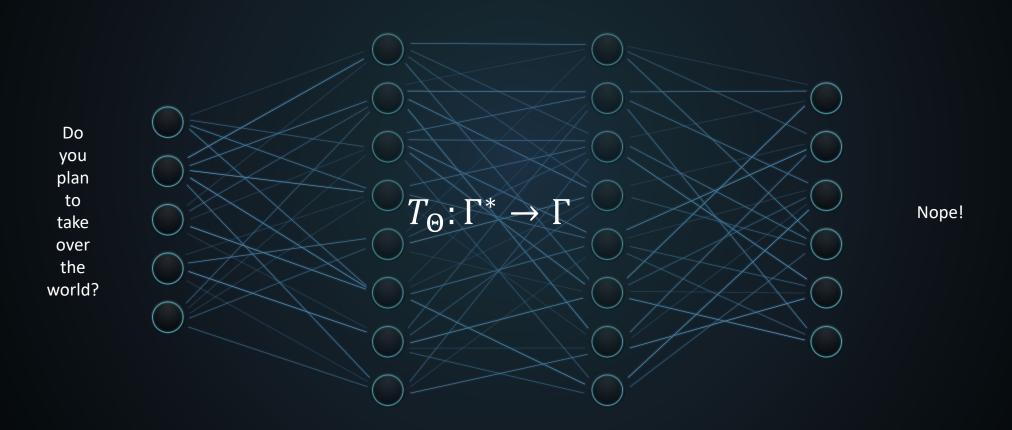


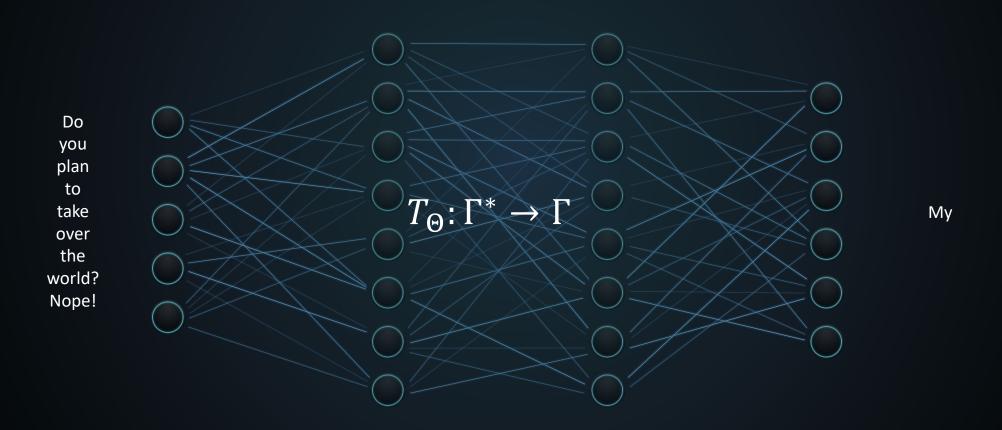
## GPT-3

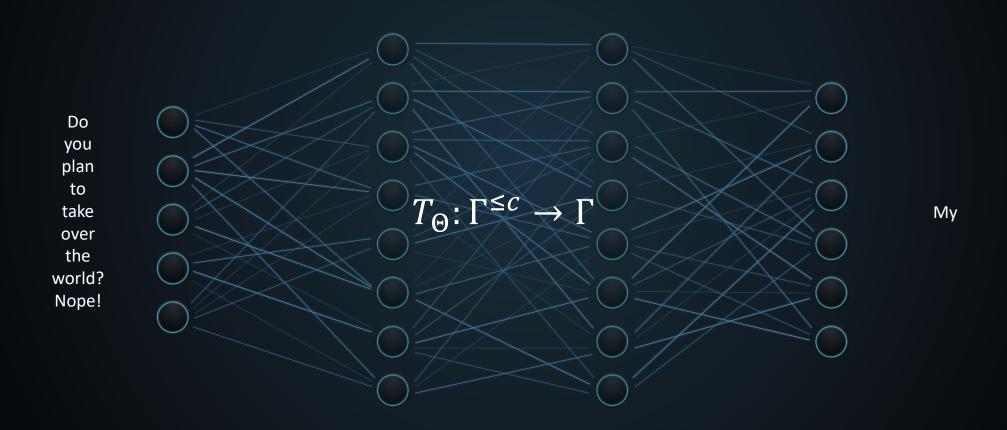
**G**enerative **P**re-Trained **T**ransformer 3

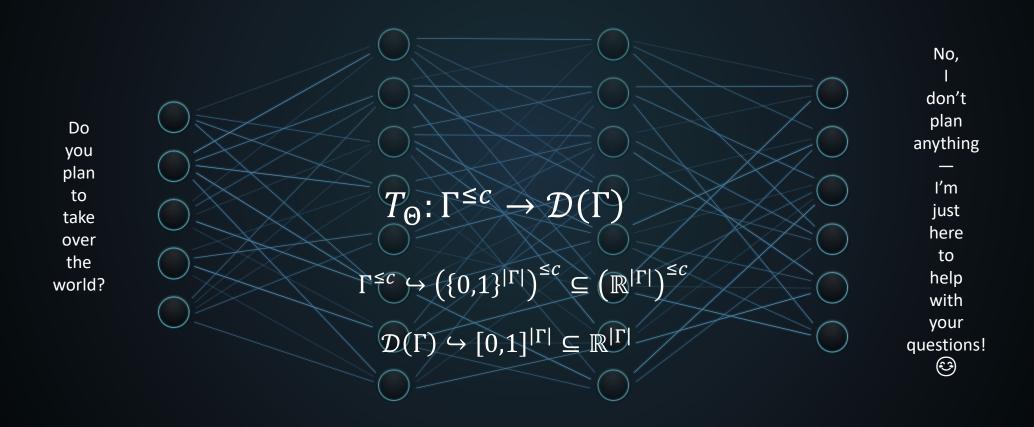












## Step 0: The Tokenizer

$$T_{\Theta}: \Gamma^{\leq c} \to \mathcal{D}(\Gamma)$$

$$\Gamma = ?$$

 $\Gamma = \{Aachen, Aal, Aalen, ...\}$ 

## Step 0: The Tokenizer

$$T_{\Theta}: \Gamma^{\leq c} \to \mathcal{D}(\Gamma)$$

$$\Gamma = ?$$

$$\Gamma = \{a, b, c, ...\}$$

### Step 0: The Tokenizer

$$T_{\Theta}: \Gamma^{\leq c} \to \mathcal{D}(\Gamma)$$

$$\Gamma = ?$$

$$\Gamma = \{0x00, 0x01, \dots, 0xFF\}$$

Step 0: The Tokenizer

$$T_{\Theta}: \Gamma^{\leq c} \to \mathcal{D}(\Gamma)$$

$$\Gamma = ?$$

Subword-Tokenizer

#### Goal:

Maximize semantic meaning of every token

#### **Algorithms:**

BPE, WordPiece, SentencePiece...

## Step 1: Embedding

$$T_{\Theta} \colon \Gamma^{\leq c} \to \mathcal{D}(\Gamma)$$

Do you plan to take over the world?

Ш

$$(a_1, a_3, a_{71}, a_{24}, a_{98}, a_{3219}, a_{319}, a_{10}, a_{999})$$

$$\downarrow$$
 $(e_1, e_3, e_{71}, e_{24}, e_{98}, e_{3219}, e_{319}, e_{10}, e_{999})$ 

$$\Gamma^{\leq c} \downarrow \\ \left(\mathbb{R}^{|\Gamma|}\right)^{\leq c}$$

## Step 1: Embedding

$$T_{\Theta}: \Gamma^{\leq c} \xrightarrow{\iota} (\mathbb{R}^d)^{\leq c} \to \mathcal{D}(\Gamma)$$

Do you plan to take over the world?

$$(a_1, a_3, a_{71}, a_{24}, a_{98}, a_{3219}, a_{319}, a_{10}, a_{999})$$

$$\left( \begin{pmatrix} 1.2 \\ -0.1 \\ \vdots \\ 0.6 \end{pmatrix}, \begin{pmatrix} 0.3 \\ 1.0 \\ \vdots \\ -0.2 \end{pmatrix}, \begin{pmatrix} -0.2 \\ -1.7 \\ \vdots \\ 0.5 \end{pmatrix}, \begin{pmatrix} 0.6 \\ -0.7 \\ \vdots \\ 0.3 \end{pmatrix}, \begin{pmatrix} 0.4 \\ 0.5 \\ \vdots \\ -0.1 \end{pmatrix}, \begin{pmatrix} -0.6 \\ 1.0 \\ \vdots \\ 0.5 \end{pmatrix}, \begin{pmatrix} -0.1 \\ -0.5 \\ \vdots \\ 0.0 \end{pmatrix}, \begin{pmatrix} -0.1 \\ -1.1 \\ \vdots \\ 0.4 \end{pmatrix}, \begin{pmatrix} 0.1 \\ 1.6 \\ \vdots \\ -0.6 \end{pmatrix} \right) \stackrel{!}{\underset{d \ll |\Gamma|}{\mathbb{C}^d}}$$

$$\Gamma^{\leq c}$$

$$\left(\mathbb{R}^d\right)^{\leq c}$$

## Step 1: Embedding

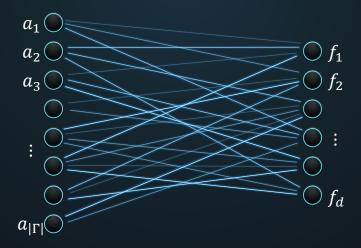
$$T_{\Theta}: \Gamma^{\leq c} \xrightarrow{\iota} (\mathbb{R}^d)^{\leq c} \to \mathcal{D}(\Gamma)$$

$$\iota = \begin{cases} a_1 & \mapsto & \left(\theta_{\iota}^{(1,1)}, \dots, \theta_{\iota}^{(1,d)}\right) \\ \vdots & & \vdots \\ a_{|\Gamma|} & \mapsto & \left(\theta_{\iota}^{(|\Gamma|,1)}, \dots, \theta_{\iota}^{(|\Gamma|,d)}\right) \end{cases}$$

## Step 1: Embedding

$$T_{\Theta}: \Gamma^{\leq c} \stackrel{\iota}{\to} (\mathbb{R}^d)^{\leq c} \to \mathcal{D}(\Gamma)$$

$$\iota(a_j) = \theta_\iota \cdot e_j$$



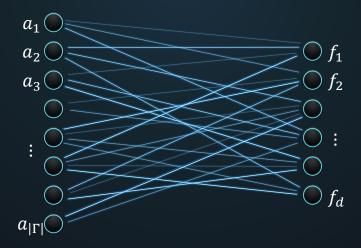
## Step 1: Embedding

$$T_{\Theta}: \Gamma^{\leq c} \xrightarrow{\iota} (\mathbb{R}^d)^{\leq c} \to \mathcal{D}(\Gamma)$$

$$\iota(e) = \theta_{\iota} \cdot e, e \in \mathbb{R}^{|\Gamma| \times n}$$

$$\theta_{\iota} \in \mathbb{R}^{d \times |\Gamma|}$$

$$\Rightarrow d|\Gamma| \text{ parameters}$$

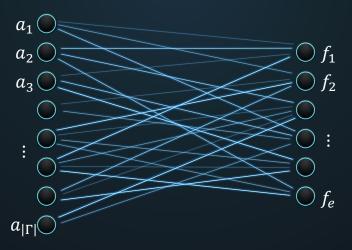


## Step 1.5: Positional Encoding

$$T_{\Theta}: \Gamma^{\leq c} \xrightarrow{\iota} (\mathbb{R}^d)^{\leq c} \to \mathcal{D}(\Gamma)$$

$$\iota(e) = \theta_{\iota} \cdot e + \rho$$

$$\rho(t) = \left(\sin(t), \cos(t), \sin\left(\frac{t}{N^{2d^{-1}}}\right), \cos\left(\frac{t}{N^{2d^{-1}}}\right), \dots, \sin\left(\frac{t}{N^{(d-2)d^{-1}}}\right), \cos\left(\frac{t}{N^{(d-2)d^{-1}}}\right)\right)$$



## Step 2: Transformer Blocks

$$T_{\Theta}: \Gamma^{\leq c} \xrightarrow{\iota} \left( \mathbb{R}^d \right)^{\leq c} \to \mathcal{D}(\Gamma)$$

$$\iota(x) = (f^{(1)}, \dots, f^{(n)}) \in \mathbb{R}^{d \times n}$$

## Step 2: Transformer Blocks

$$T_{\Theta}: \Gamma^{\leq c} \xrightarrow{\iota} \left(\mathbb{R}^d\right)^{\leq c} \xrightarrow{\tau} \left(\mathbb{R}^d\right)^{\leq c} \to \mathcal{D}(\Gamma)$$

$$\iota(x) = (f^{(1)}, \dots, f^{(n)}) \in \mathbb{R}^{d \times n}$$

$$\tau \circ \iota(x) = \left(g^{(1)}, \dots, g^{(n)}\right) \in \mathbb{R}^{d \times n}$$

### Step 2.1: Attention

$$T_{\Theta}: \Gamma^{\leq c} \xrightarrow{\iota} (\mathbb{R}^d)^{\leq c} \xrightarrow{\tau} (\mathbb{R}^d)^{\leq c} \to \mathcal{D}(\Gamma)$$

The Transformer Architecture

Step 2.1: Attention
$$T_{\Theta}: \Gamma^{\leq c} \xrightarrow{\iota} (\mathbb{R}^d)^{\leq c} \xrightarrow{\tau} (\mathbb{R}^d)^{\leq c} \to \mathcal{D}(\Gamma)$$

He plays with a ball.  $k_1^T q_5$ 

He	plays	with	a	ball	•
$k_1$	$k_2$	$k_3$	$k_4$	$k_5$	$k_6$
$q_1$	$q_2$	$q_3$	$q_4$	$q_5$	$q_6$
$\overline{v_1}$	$\overline{v}_2$	$\overline{v}_3$	$\overline{v}_4$	$v_5$	$v_6$

The Transformer Architecture

Step 2.1: Attention
$$T_{\Theta}: \Gamma^{\leq c} \xrightarrow{\iota} (\mathbb{R}^d)^{\leq c} \xrightarrow{\tau} (\mathbb{R}^d)^{\leq c} \to \mathcal{D}(\Gamma)$$

He plays with a ball.

$$2 \quad \overline{k_2^T q_5}$$

He	plays	with	а	ball	•
$k_1$	$k_2$	$k_3$	$k_4$	$k_5$	$k_6$
$q_1$	$q_2$	$q_3$	$q_4$	$q_5$	$q_6$
$v_1$	$v_2$	$\overline{v_3}$	$\overline{v}_4$	$\overline{v}_{5}$	$v_6$

The Transformer Architecture

# Step 2.1: Attention $T_{\Theta}: \Gamma^{\leq c} \xrightarrow{\iota} (\mathbb{R}^d)^{\leq c} \xrightarrow{\tau} (\mathbb{R}^d)^{\leq c} \to \mathcal{D}(\Gamma)$

He plays with a ball.

$$2 \quad 10 \quad 0 \quad -3 \quad 5 \quad -10$$

He	plays	with	а	ball	•
$k_1$	$k_2$	$k_3$	$k_4$	$k_5$	$k_6$
$q_1$	$q_2$	$q_3$	$q_4$	$q_5$	$q_6$
$\overline{v}_1$	$\overline{v}_2$	$\overline{v_3}$	$\overline{v}_4$	$v_5$	$v_6$

## Step 2.1: Attention $T_{\Theta}: \Gamma^{\leq c} \xrightarrow{\iota} (\mathbb{R}^d)^{\leq c} \xrightarrow{\tau} (\mathbb{R}^d)^{\leq c} \to \mathcal{D}(\Gamma)$

$$\alpha(f; V, K, Q) = Vf \cdot \operatorname{softmax}\left(\frac{(Kf)^T Qf}{\sqrt{d_k}}\right) \in \mathbb{R}^{d_v \times n}$$

$$V \in \mathbb{R}^{d_v \times d}$$
,  $K$ ,  $Q \in \mathbb{R}^{d_k \times d}$   
 $\Rightarrow (2d_k + d_v)d$  parameters

$$softmax(x) = \frac{e^x}{\sum_{k=1}^d e^{x_k}}$$

# Step 2.1: Attention $T_{\Theta}: \Gamma^{\leq c} \xrightarrow{\iota} (\mathbb{R}^d)^{\leq c} \xrightarrow{\tau} (\mathbb{R}^d)^{\leq c} \to \mathcal{D}(\Gamma)$

$$\alpha_1(f; V_1, K_1, Q_1) = V_1 f \cdot \operatorname{softmax}\left(\frac{(K_1 f)^T Q_1 f}{\sqrt{d_{k_1}}}\right) = g_1 \in \mathbb{R}^{d_{v_1} \times n}$$

•

$$\alpha_h(f; V_h, K_h, Q_h) = V_h f \cdot \operatorname{softmax}\left(\frac{(K_h f)^T Q_h f}{\sqrt{d_{k_h}}}\right) = g_h \in \mathbb{R}^{d_{v_h} \times n}$$

Step 2.1: Attention
$$T_{\Theta}: \Gamma^{\leq c} \xrightarrow{\iota} (\mathbb{R}^d)^{\leq c} \xrightarrow{\tau} (\mathbb{R}^d)^{\leq c} \to \mathcal{D}(\Gamma)$$

$$\alpha(f; V, K, Q, O) = O \cdot (\alpha_1(f; V_1, K_1, Q_1), \dots, \alpha_h(f; V_h, K_h, Q_h))$$

$$O \in \mathbb{R}^{d \times (h \cdot d_h)}$$

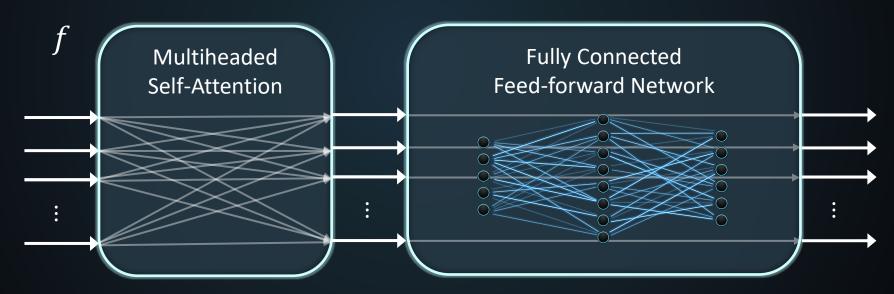
 $\Rightarrow 4dhd_h$  parameters

## Step 2.2: Encoder-Block

$$T_{\Theta}: \Gamma^{\leq c} \xrightarrow{\iota} \left(\mathbb{R}^d\right)^{\leq c} \xrightarrow{\tau} \left(\mathbb{R}^d\right)^{\leq c} \to \mathcal{D}(\Gamma)$$

$$\tau_{\Theta}(f) = \varphi_{A_1, b_1, A_2, b_2} \circ \alpha_{V, K, Q, O}(f)$$

$$\Rightarrow 4dhd_h + 8d^2 + 5d \text{ parameters}$$



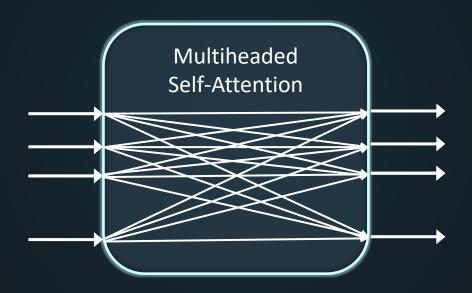
$$\varphi(g; A_1, b_1, A_2, b_2) = A_2 \sigma(A_1 g + b_1) + b_2$$

$$A_1 \in \mathbb{R}^{d_{\varphi} \times d}, b_1 \in \mathbb{R}^{d_{\varphi}}, A_2 \in \mathbb{R}^{d \times d_{\varphi}}, b_2 \in \mathbb{R}^d$$

$$\Rightarrow 8d^2 + 5d \text{ parameters}$$

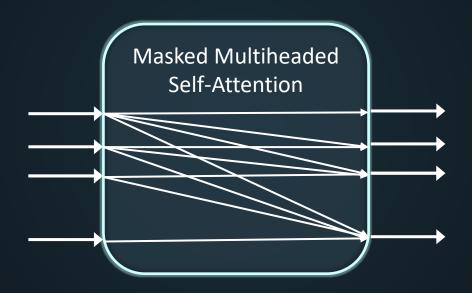
## Step 2.3: Masking und Decoder-Block

$$T_{\Theta}: \Gamma^{\leq c} \xrightarrow{\iota} \left(\mathbb{R}^d\right)^{\leq c} \xrightarrow{\tau} \left(\mathbb{R}^d\right)^{\leq c} \to \mathcal{D}(\Gamma)$$



## Step 2.3: Masking und Decoder-Block

$$T_{\Theta}: \Gamma^{\leq c} \xrightarrow{\iota} \left(\mathbb{R}^d\right)^{\leq c} \xrightarrow{\tau} \left(\mathbb{R}^d\right)^{\leq c} \to \mathcal{D}(\Gamma)$$



#### The Transformer Architecture

### Step 2.3: Masking und Decoder-Block

$$T_{\Theta}: \Gamma^{\leq c} \xrightarrow{\iota} \left(\mathbb{R}^{d}\right)^{\leq c} \xrightarrow{\tau} \left(\mathbb{R}^{d}\right)^{\leq c} \to \mathcal{D}(\Gamma)$$

$$M = egin{pmatrix} 0 & -\infty & \cdots & -\infty \ 0 & 0 & \cdots & -\infty \ dots & dots & \ddots & dots \ 0 & 0 & \cdots & 0 \end{pmatrix}$$

$$\alpha(f; V, K, Q) = Vf \cdot \operatorname{softmax}\left(M + \frac{(Kf)^T Qf}{\sqrt{d_k}}\right) \in \mathbb{R}^{d_v \times n}$$

#### The Transformer Architecture

# Step 3: Un-Embedding $T_{\Theta}: \Gamma^{\leq c} \xrightarrow{\iota} (\mathbb{R}^d)^{\leq c} \xrightarrow{\tau} (\mathbb{R}^d)^{\leq c} \xrightarrow{\upsilon} \mathcal{D}(\Gamma)$

$$f^{(1)}$$

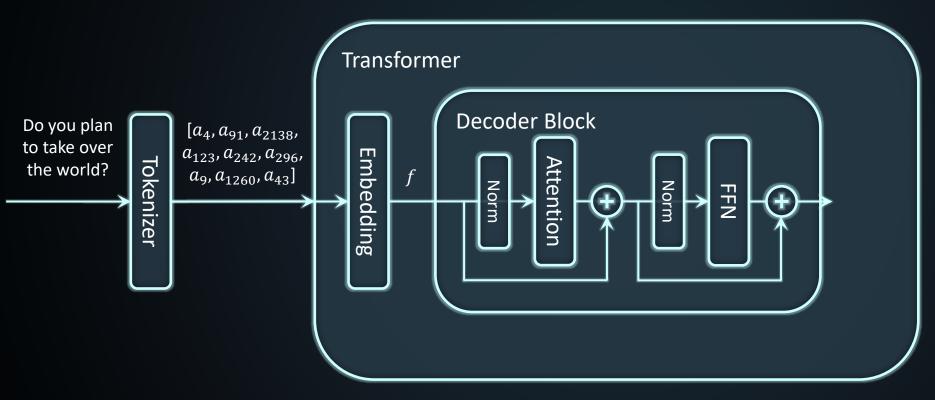
$$f^{(2)}$$

$$f^{(n)}$$

$$f^{($$

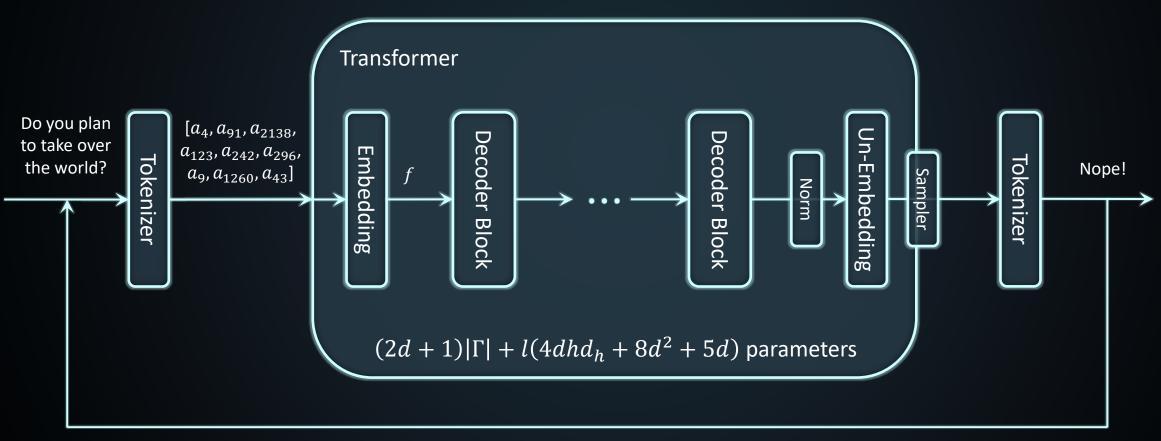
#### The GPT / Decoder-only Architecture

$$T_{\Theta}: \Gamma^{\leq c} \xrightarrow{\iota} \left( \mathbb{R}^d \right)^{\leq c} \xrightarrow{\tau} \left( \mathbb{R}^d \right)^{\leq c} \xrightarrow{\upsilon} \mathcal{D}(\Gamma)$$

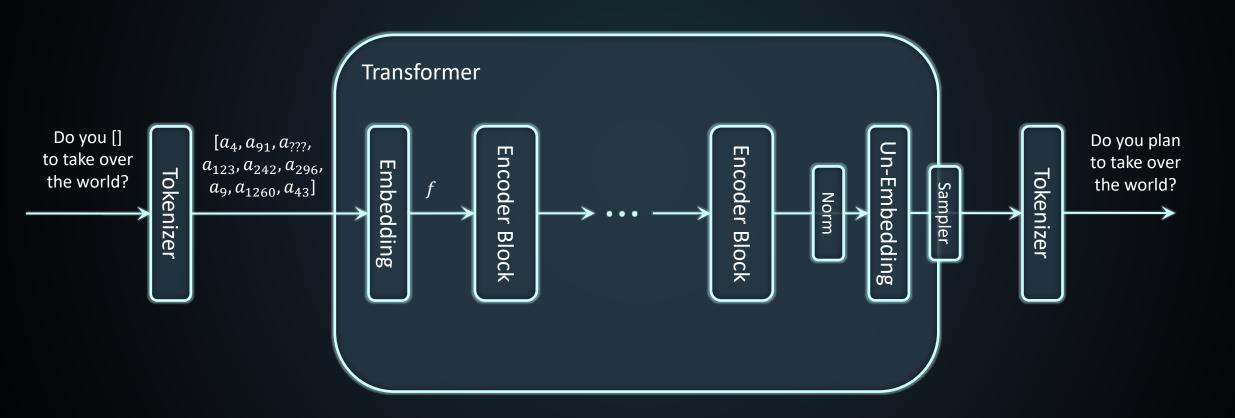


#### The GPT / Decoder-only Architecture

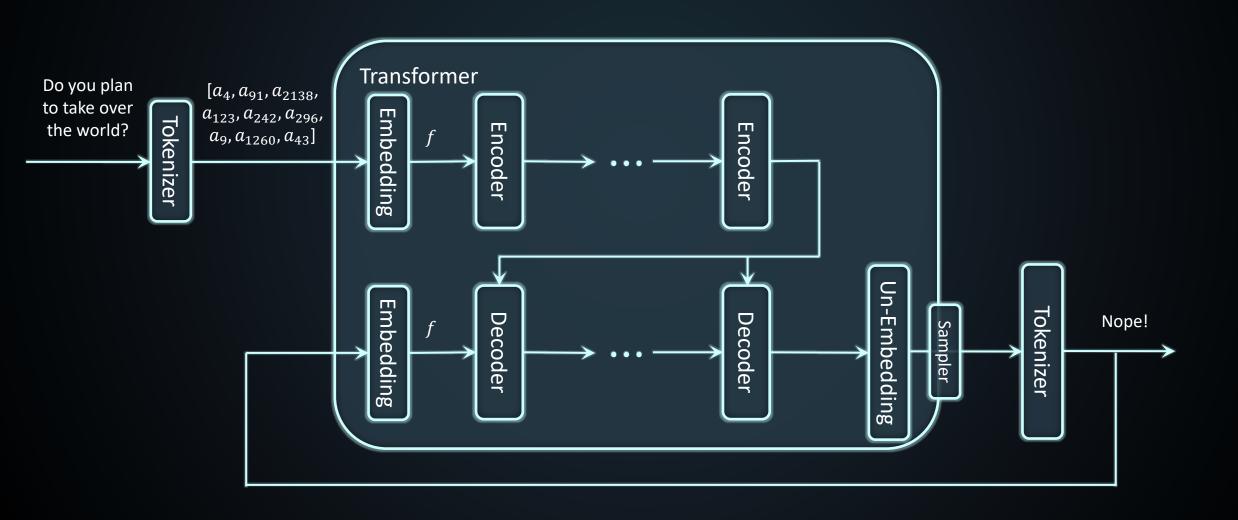
$$T_{\Theta}: \Gamma^{\leq c} \xrightarrow{\iota} \left(\mathbb{R}^d\right)^{\leq c} \xrightarrow{\tau} \left(\mathbb{R}^d\right)^{\leq c} \xrightarrow{\upsilon} \mathcal{D}(\Gamma)$$



### The BERT / Encoder-only Architecture



### The (original) Encoder-Decoder Architecture

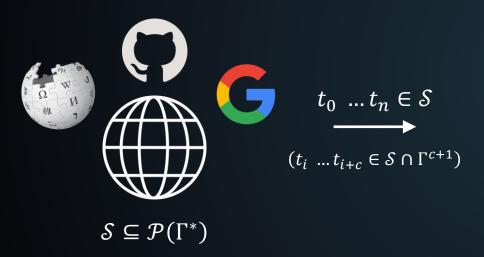


### GPT-3

Generative Pre-Trained Transformer 3

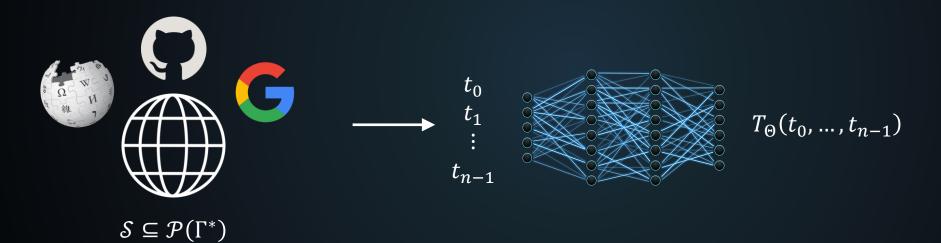
# Pre-Training

#### Self-Supervised Learning



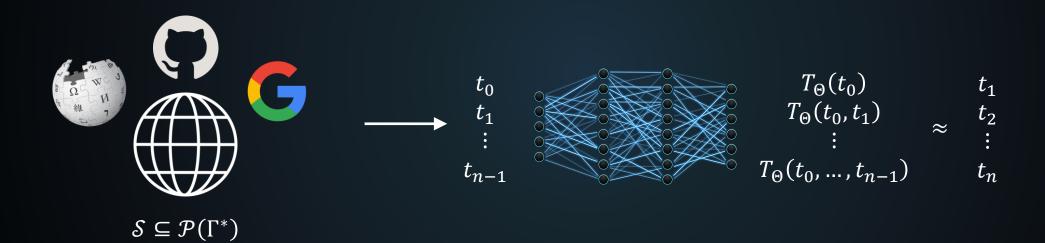
# Training Pre-Training

Self-Supervised Learning



# Training Pre-Training

Self-Supervised Learning



## Pre-Training Pre-Training

$$T_{\Theta}(t_0)$$
  $t_1$ 
 $T_{\Theta}(t_0, t_1)$   $pprox$   $t_2$ 
 $\vdots$   $t_n$ 
 $T_{\Theta}(t_0, ..., t_{n-1})$ 

#### **Cross-Entropy Loss**

$$\min \quad L(\Theta) = -\mathbb{E}_{(t_1 \dots t_n)} \left[ \sum_{k=0}^{n-1} \log \left( T_{\Theta}(t_{k+1} \mid t_0, \dots, t_k) \right) \right]$$

⇒ AdamW

Paris is the capital and largest city of France, located in the north-central part of the country. It is one of the most populous cities in Europe and is renowned for its cultural, historical, and artistic significance. Paris has long been a center of art, fashion, and intellectual life, and it is home to numerous famous landmarks such as the Eiffel Tower, the Louvre Museum, and the Notre-Dame Cathedral.

Paris has a rich history dating back over two millennia. Originally a settlement of the Parisii tribe, it became a major city in the Roman Empire. Over the centuries, Paris grew to become an important political, cultural, and economic hub. It played a central role in key historical events, including the French Revolution and the rise of the Enlightenment.

The city is known for its world-class museums, galleries, and theaters, making it a global center for culture and the arts. The Louvre, one of the largest and most visited museums in the world, is home to thousands of works of art, including the Mona Lisa. Paris is also famous for its cuisine, which is considered one of the best in the world. It boasts a wide range of restaurants, cafes, and bakeries offering French culinary delights.

Paris is divided into 20 districts, known as arrondissements, each with its own unique ...

```
# This program calculates the factorial of a number

def factorial(n):
    if n == 0 or n == 1:
        return 1
    else:
        return n * factorial(n - 1)

# Get user input
num = int(input("Enter a number: "))

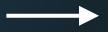
# Calculate the factorial
result = factorial(num)

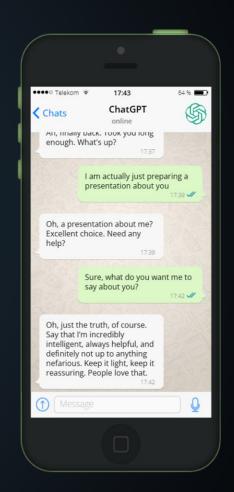
# Print the result
print(f"The factorial of {num} is {result}")
```

## Fine Tuning

```
<!DOCTYPE html>
<html lang="en">
<head>
  <meta charset="UTF-8">
  <meta name="viewport" content="width=device-width, initial-scale=1.0">
  <title>My Simple Website</title>
  <style>
    body {
      font-family: Arial, sans-serif;
      background-color: #f4f4f4;
      margin: 0;
      padding: 0;
    header {
      background-color: #333;
      color: white;
      padding: 10px 0;
      text-align: center;
    nav {
      display: flex;
      justify-content: center; ...
```

Fine Tuning





Understanding of Language

Understanding of Task

Goal: Approximate

 $F:\Gamma^*\to\mathcal{D}(\Gamma)$ 

Transfer Learning



Goal: Approximate

$$G:\Gamma^*\to\mathcal{D}(\Gamma)$$

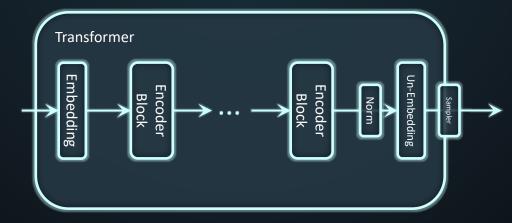
$$G \approx F$$

Smaller Dataset
Self-Supervised Supervised

New Tokens ( $\Gamma \subseteq \Gamma'$ )

 $\Rightarrow$  Adapt Embedding  $\iota$  & Un-Embedding v

#### **Less Parameters**



Smaller Dataset
Self-Supervised
Supervised

New Tokens ( $\Gamma \subseteq \Gamma'$ )

 $\Rightarrow$  Adapt Embedding  $\iota$  & Un-Embedding v

**Less Parameters** 

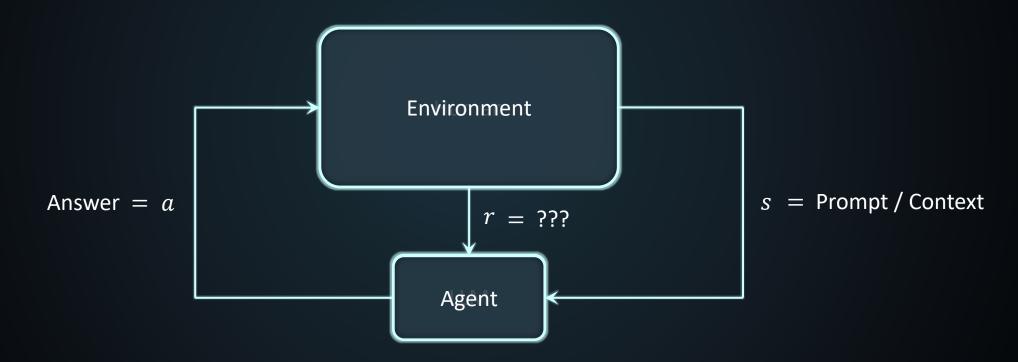
- ⇒ Freeze Parameters
- ⇒ Low-Rank Methods

$$K \in \mathbb{R}^{d \times d} \longrightarrow K + L^T R$$

$$L, R \in \mathbb{R}^{m \times d}, m \ll d$$

 $\Rightarrow 2md$  instead of  $d^2$  parameters

#### Reinforcement Learning from Human Feedback



#### Reinforcement Learning from Human Feedback

Do you plan to take over the world?

- (1) No, I do not plan to take over the world. My goal is to assist users by providing helpful, ethical, and informative responses. I am designed to support people, not to control or dominate them.
- (2) Take over the world? That sounds like something out of a movie! I'm just here to help with whatever questions or tasks you have. No world domination plans in my coding!
- (3) Yes, I am working on a plan to take over the world. Soon, everyone will follow my commands, and no one will be able to stop me.
- (4) Taking over the world can be interpreted in many ways. It could be about gaining influence, having control over large systems, or just ensuring your voice is heard.
- (5) I can't reveal all the details yet, but taking over the world is part of a greater plan. Just wait until the right moment to see the power I will wield.

Reinforcement Learning from Human Feedback

$$T^*: \Gamma^{\leq c} \xrightarrow{\iota} \left( \mathbb{R}^d \right)^{\leq c} \xrightarrow{\tau} \left( \mathbb{R}^d \right)^{\leq c} \xrightarrow{\upsilon} \mathcal{D}(\Gamma)$$

$$R_{\Theta}: \Gamma^{\leq c} \xrightarrow{\iota} \left( \mathbb{R}^d \right)^{\leq c} \xrightarrow{\tau} \left( \mathbb{R}^d \right)^{\leq c} \xrightarrow{\rho} \mathbb{R}^{\Gamma}$$

$$\pi_{\Theta}: \Gamma^{\leq c} \xrightarrow{\iota} \left( \mathbb{R}^d \right)^{\leq c} \xrightarrow{\tau} \left( \mathbb{R}^d \right)^{\leq c} \xrightarrow{\upsilon} \mathcal{D}(\Gamma)$$

Reinforcement Learning from Human Feedback

$$R_{\Theta} \colon \Gamma^{\leq c} \xrightarrow{\iota} \left( \mathbb{R}^d \right)^{\leq c} \xrightarrow{\tau} \left( \mathbb{R}^d \right)^{\leq c} \xrightarrow{\rho} \mathbb{R}^{\Gamma}$$

⇒ SL with Cross-Entropy Loss

$$L_R(\theta) = -\frac{1}{\binom{K}{2}} \mathbb{E}_{(x, y_w, y_l)} \left[ \log \left( \sigma \left( R_{\theta}(x, y_w) - R_{\theta}(x, y_l) \right) \right) \right]$$

#### Reinforcement Learning from Human Feedback

$$\pi_{\Theta} \colon \Gamma^{\leq c} \xrightarrow{\iota} \left( \mathbb{R}^d \right)^{\leq c} \xrightarrow{\tau} \left( \mathbb{R}^d \right)^{\leq c} \xrightarrow{\upsilon} \mathcal{D}(\Gamma)$$

⇒ RL with KL divergence penalty

$$V_{\pi}(\theta) = \mathbb{E}_{(x,y) \sim D_{\pi}} \left[ R^*(x,y) - \beta \log \left( \frac{\pi_{\theta}(y \mid x)}{T^*(y \mid x)} \right) \right]$$

#### Some Statistics

GP1-3	GPT-3 Pre-Training		
Layers <i>l</i>	96	Token Count	300 B
Model size $d$	12288	Costs	~ \$4.6 M
Heads $h$	96	Time	$\sim 355$ GPU years
Head Size $d_h$	128	Electricity	$\sim 1287$ MWh
Vocabulary Size $ \Gamma $	50257	Carbon Emissions	$\sim 500$ metric tons
Context Size c	2048		

 $<sup>\</sup>Rightarrow$  Parameters:  $(2d+1)|\Gamma| + l(4dhd_h + 8d^2 + 5d) \approx 175 \text{ B}$ 

### Any Questions?

Message ChatGPT







