Heuristic MINLP for Solving Optimal Power Flow Problems

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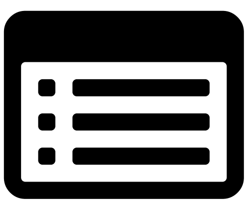
Optimization Solver Idea

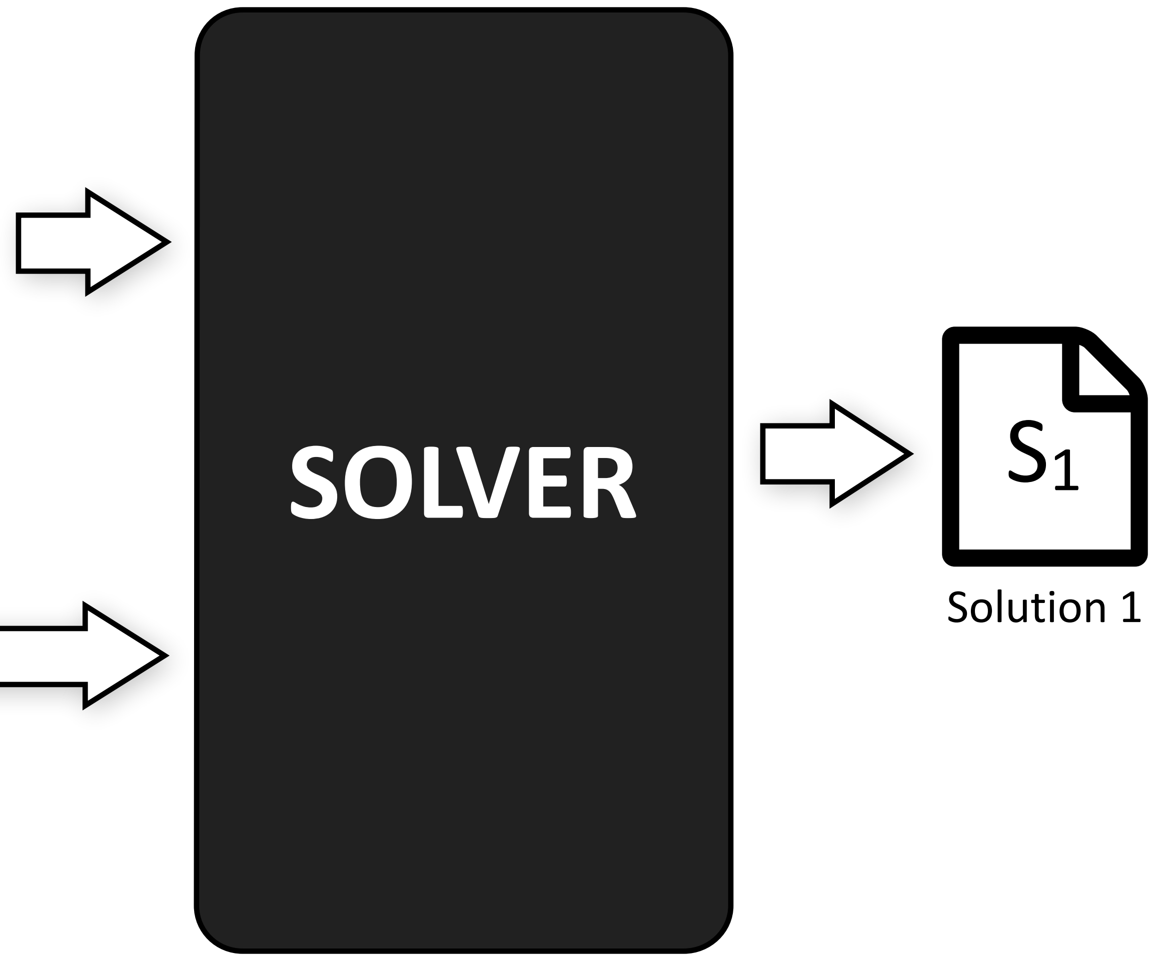
$$S_{ij} = \left(\boldsymbol{Y}_{ij}^* - i\frac{\boldsymbol{b}_{ij}^l}{2}\right)\frac{V_iV_i^*}{\boldsymbol{T}_{ij}\boldsymbol{T}_{ij}^*} - \boldsymbol{Y}_{ij}^*\frac{V_iV_j^*}{\boldsymbol{T}_{ij}^*} \quad (i,j) \in E$$

$$S_{ji} = \left(\boldsymbol{Y}_{ij}^* - i\frac{\boldsymbol{b}_{ij}^l}{2}\right)V_jV_j^* - \boldsymbol{Y}_{ij}^*\frac{V_jV_i^*}{\boldsymbol{T}_{ij}} \quad (i,j) \in E$$

$$|S_{ij}| \leq \overline{|\boldsymbol{S}_{ij}|} \quad \forall (i,j) \in E \cup E^R$$



Input Data1



Solution 1

Optimization Solver Idea

$$S_{ij} = \left(\boldsymbol{Y}_{ij}^{*} - i\frac{\boldsymbol{b}_{ij}^{l}}{2}\right)\frac{V_i V_i^{*}}{\boldsymbol{T}_{ij}\boldsymbol{T}_{ij}^{*}} - \boldsymbol{Y}_{ij}^{*}\frac{V_i V_j^{*}}{\boldsymbol{T}_{ij}^{*}} \quad (i,j) \in E$$

$$S_{ji} = \left(\boldsymbol{Y}_{ij}^{*} - i\frac{\boldsymbol{b}_{ij}^{l}}{2}\right) V_j V_j^{*} - \boldsymbol{Y}_{ij}^{*}\frac{V_j V_i^{*}}{\boldsymbol{T}_{ij}} \quad (i,j) \in E$$

$$|S_{ij}| \leq \overline{|\boldsymbol{S}_{ij}|} \quad \forall (i,j) \in E \cup E^{R}$$

Input Data1

SOLVER

Branch and Bound Algorithms

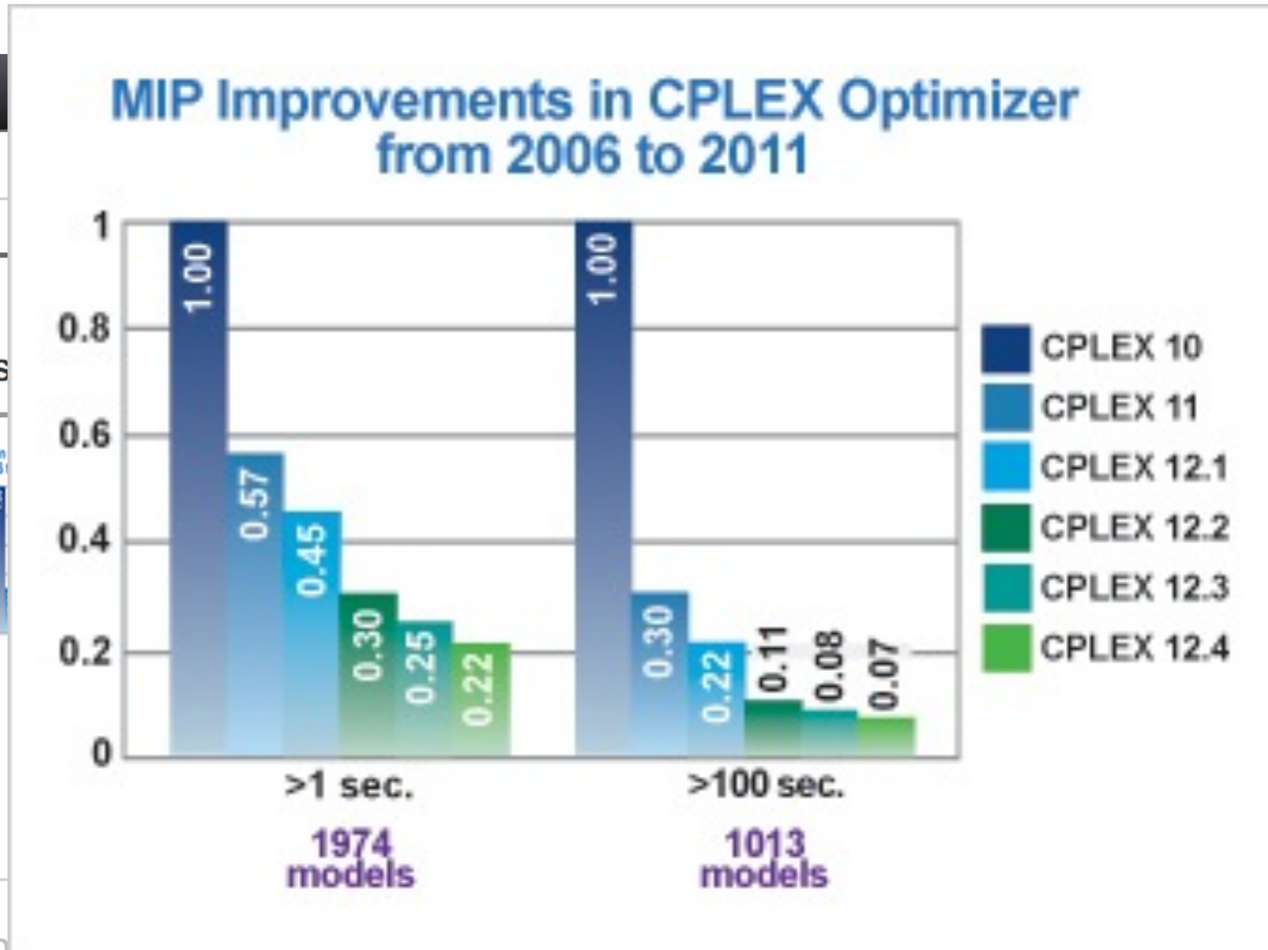
Highly Optimized

Structure
Structure
Structure

S_1

Solution 1

Why Optimization Solvers?



Call for Competition on

"Application of Modern Heuristic Optimization Algorithms for Solving Optimal Power Flow Problems"

Genetic Alg

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SOLVER?

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Applying Black-Box Solvers to the OPF

- Need to be able to write down the system of **equations** and constraints
  - 3 Problems - ORPD, OARPD, WPP
  - Simulation Based on Matpower

- Still **unclear** if the solver would work well…
  - These problems are on the frontier of solver technology

Modeling the Competition OPF

WARNING!

An Optimization Model

Choices
Simulation

Variables:

$$S_i^g \ \ \forall i \in N$$

$$V_i \ \ \forall i \in N$$

Goal

Minimize:

$$\sum_{i \in N} \boldsymbol{c}_i^2 \left(\Re(S_i^g)\right)^2 + \boldsymbol{c}_i^1 \Re(S_i^g)$$

Constraints

Subject to:

$$\underline{\boldsymbol{S}}_i^g \leq S_i^g \leq \overline{\boldsymbol{S}}_i^g \ \ \forall i \in N$$

$$\underline{|\boldsymbol{V}_i|} \leq |V_i| \leq \overline{|\boldsymbol{V}_i|} \ \ \forall i \in N$$

$$\boldsymbol{S}_i^d - S_i^g = \sum_{(i,j) \in E \cup E^R} -S_{ij} \ \ \forall i \in N$$

Capital Letter - Complex
Script - Variable
Bold Script - Data

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Publication OPF

Variables:

$$S_i^g \;\; \forall i \in N$$ Generation

$$V_i \;\; \forall i \in N$$ Voltage

Minimize:

$$\sum_{i \in N} \boldsymbol{c}_i^2 \left(\Re(S_i^g)\right)^2 + \boldsymbol{c}_i^1 \Re(S_i^g)$$ Cost

Subject to:

$$\underline{\boldsymbol{S}_i^g} \le S_i^g \le \overline{\boldsymbol{S}}_i^g \;\; \forall i \in N$$ Generation Bounds

$$\underline{|\boldsymbol{V}_i|} \le |V_i| \le \overline{|\boldsymbol{V}_i|} \;\; \forall i \in N$$ Voltage Bounds

$$\boldsymbol{S}_i^d - S_i^g = \sum_{(i,j) \in E \cup E^R} -S_{ij} \;\; \forall i \in N$$ KCL

$$S_{ij} = \boldsymbol{Y}_{ij}^* V_i V_i^* - \boldsymbol{Y}_{ij}^* V_i V_j^* \;\; (i,j) \in E$$

$$S_{ji} = \boldsymbol{Y}_{ij}^* V_j V_j^* - \boldsymbol{Y}_{ij}^* V_j V_i^* \;\; (i,j) \in E$$ Ohm's Law

$$|S_{ij}| \le \overline{|\boldsymbol{S}_{ij}|} \;\; \forall (i,j) \in E \cup E^R$$ Thermal Limit

Capital Letter - Complex

Script - Variable

Bold Script - Data

Real OPF

Variables:

$$S_i^g \ \ \forall i \in N$$
$$V_i \ \ \forall i \in N$$

Minimize:

$$\sum_{i \in N} \boldsymbol{c}_i^2 \left(\Re(S_i^g)\right)^2 + \boldsymbol{c}_i^1 \Re(S_i^g)$$

Subject to:

$$\underline{\boldsymbol{S}}_i^g \le S_i^g \le \overline{\boldsymbol{S}}_i^g \ \ \forall i \in N$$
$$\underline{|\boldsymbol{V}_i|} \le |V_i| \le \overline{|\boldsymbol{V}_i|} \ \ \forall i \in N$$
$$\boldsymbol{S}_i^d + \boldsymbol{Y}_i^s V_i V_i^* - S_i^g = \sum_{(i,j) \in E \cup E^R} -S_{ij} \ \ \forall i \in N$$
$$S_{ij} = \left(\boldsymbol{Y}_{ij}^* - i\frac{\boldsymbol{b}_{ij}^l}{2}\right) \frac{V_i V_i^*}{\boldsymbol{T}_{ij}\boldsymbol{T}_{ij}^*} - \boldsymbol{Y}_{ij}^* \frac{V_i V_j^*}{\boldsymbol{T}_{ij}^*} \ \ (i,j) \in E$$
$$S_{ji} = \left(\boldsymbol{Y}_{ij}^* - i\frac{\boldsymbol{b}_{ij}^l}{2}\right) V_j V_j^* - \boldsymbol{Y}_{ij}^* \frac{V_j V_i^*}{\boldsymbol{T}_{ij}} \ \ (i,j) \in E$$
$$|S_{ij}| \le \overline{|\boldsymbol{S}_{ij}|} \ \ \forall (i,j) \in E \cup E^R$$

Bus Shunts

Transformers

Line Charging

Competition OPF

Variables:

$$S_i^g \ \ \forall i \in N$$
$$V_i \ \ \forall i \in N$$
$$z_i \in \{0,1\} \ \ \forall i \in N$$
$$T_{ij} = \{0.90, 0.91, \ldots, 1.09, 1.1\} \ \ (i,j) \in E$$

Minimize:

$$\sum_{i \in N} \boldsymbol{c}_i^2 \left(\Re(S_i^g)\right)^2 + \boldsymbol{c}_i^1 \Re(S_i^g)$$

Subject to:

$$\underline{\boldsymbol{S}_i^g} \le S_i^g \le \overline{\boldsymbol{S}_i^g} \ \ \forall i \in N$$
$$\underline{|\boldsymbol{V}_i|} \le |V_i| \le \overline{|\boldsymbol{V}_i|} \ \ \forall i \in N$$
$$\boldsymbol{S}_i^d + z_i\left(\boldsymbol{Y}_i^{s} V_i V_i^*\right) - S_i^g = \sum_{(i,j) \in E \cup E^R} -S_{ij} \ \ \forall i \in N$$
$$S_{ij} = \left(\boldsymbol{Y}_{ij}^* - i\frac{\boldsymbol{b}_{ij}^l}{2}\right)\frac{V_i V_i^*}{T_{ij}^2} - \boldsymbol{Y}_{ij}^* \frac{V_i V_j^*}{T_{ij}} \ \ (i,j) \in E$$
$$S_{ji} = \left(\boldsymbol{Y}_{ij}^* - i\frac{\boldsymbol{b}_{ij}^l}{2}\right) V_j V_j^* - \boldsymbol{Y}_{ij}^* \frac{V_j V_i^*}{T_{ij}} \ \ (i,j) \in E$$
$$|S_{ij}| \le \overline{|\boldsymbol{S}_{ij}|} \ \ \forall (i,j) \in E \cup E^R$$

Bus Shunt On/Off

Tap Ratio

Bus Shunts

Transformers

Competition OPF + N-1

Variables:

$$S_i^g \quad \forall i \in N$$
$$V_{is} \quad \forall i \in N, \forall s \in S$$
$$z_i \in \{0,1\} \quad \forall i \in N$$
$$T_{ij} = \{0.90, 0.91, \ldots, 1.09, 1.1\} \quad (i,j) \in E$$

Scenario Voltages

Minimize:

$$\sum_{i \in N} \boldsymbol{c}_i^2 \left(\Re(S_i^g)\right)^2 + \boldsymbol{c}_i^1 \Re(S_i^g)$$

Generator Voltages

Subject to:

$$\underline{\boldsymbol{S}}_i^g \le S_i^g \le \overline{\boldsymbol{S}}_i^g \quad \forall i \in N$$
$$|V_{is}| = |V_{it}| \quad \forall i \in G, \forall s \in S, \forall t \in S$$
$$\forall s \in S$$
$$\underline{|\boldsymbol{V}_i|} \le |V_{is}| \le \overline{|\boldsymbol{V}_i|} \quad \forall i \in N$$
$$\boldsymbol{S}_i^d + z_i\left(\boldsymbol{Y}_i^{s} V_{is} V_{is}^*\right) - S_i^g = \sum_{(i,j) \in E_s \cup E_s^R} -S_{ijs} \quad \forall i \in N$$
$$S_{ijs} = \left(\boldsymbol{Y}_{ij}^* - i\frac{\boldsymbol{b}_{ij}^l}{2}\right)\frac{V_{is}V_{is}^*}{T_{ij}^2} - \boldsymbol{Y}_{ij}^* \frac{V_{is}V_{js}^*}{T_{ij}} \quad (i,j) \in E_s$$
$$S_{jis} = \left(\boldsymbol{Y}_{ij}^* - i\frac{\boldsymbol{b}_{ij}^l}{2}\right) V_{js}V_{js}^* - \boldsymbol{Y}_{ij}^* \frac{V_{js}V_{is}^*}{T_{ij}} \quad (i,j) \in E_s$$
$$|S_{ijs}| \le \overline{|\boldsymbol{S}_{ijs}|} \quad \forall (i,j) \in E \cup E^R$$

Scenario Replicates

Solving the Competition OPF

Variables:

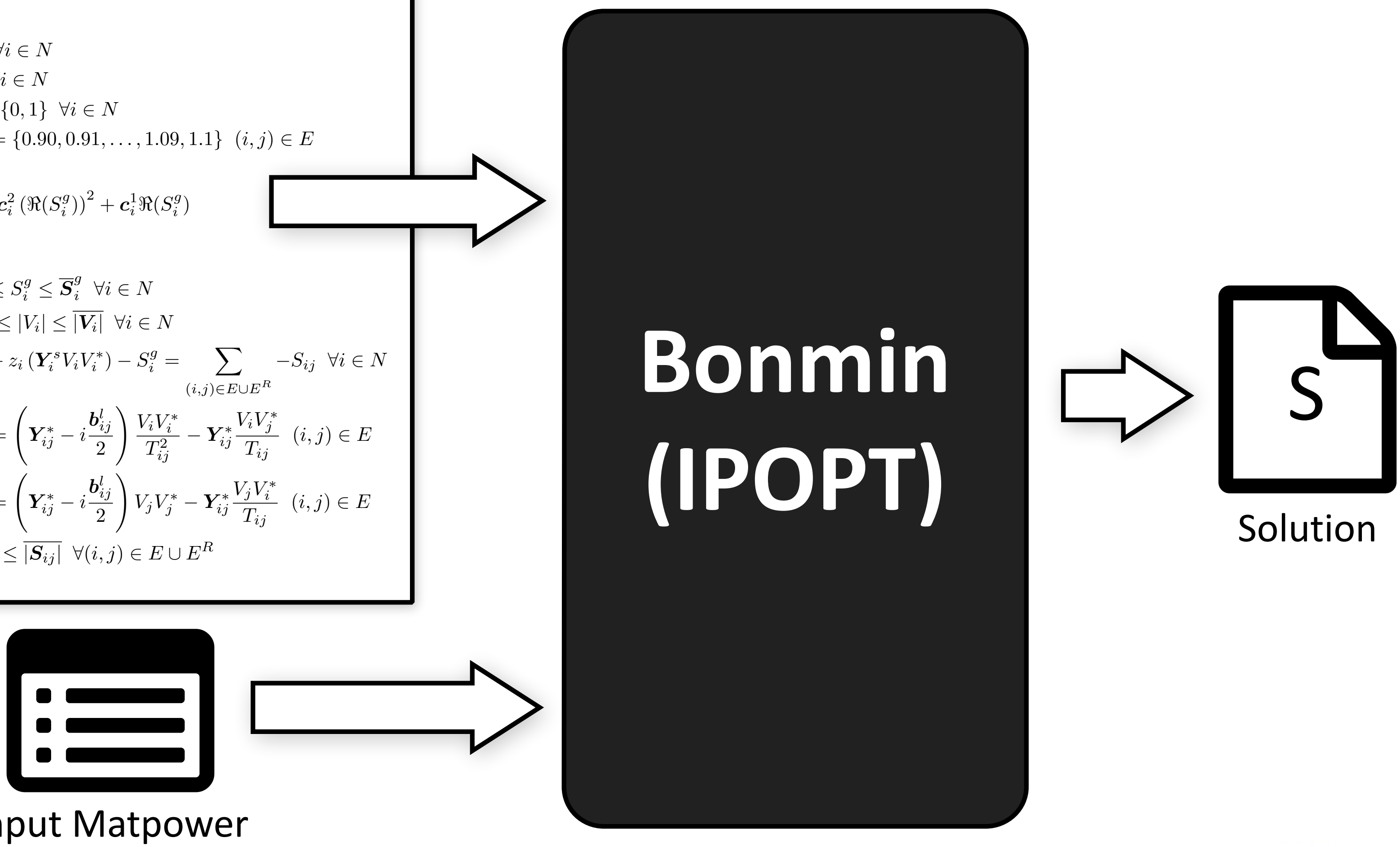
$$S_i^g \ \ \forall i \in N$$
$$V_i \ \ \forall i \in N$$
$$z_i \in \{0,1\} \ \ \forall i \in N$$
$$T_{ij} = \{0.90, 0.91, \ldots, 1.09, 1.1\} \ \ (i,j) \in E$$

Minimize:

$$\sum_{i \in N} \boldsymbol{c}_i^2 \left(\Re(S_i^g)\right)^2 + \boldsymbol{c}_i^1 \Re(S_i^g)$$

Subject to:

$$\underline{\boldsymbol{S}}_i^g \leq S_i^g \leq \overline{\boldsymbol{S}}_i^g \ \ \forall i \in N$$
$$|\underline{\boldsymbol{V}}_i| \leq |V_i| \leq |\overline{\boldsymbol{V}}_i| \ \ \forall i \in N$$
$$\boldsymbol{S}_i^d + z_i \left(\boldsymbol{Y}_i^s V_i V_i^*\right) - S_i^g = \sum_{(i,j) \in E \cup E^R} -S_{ij} \ \ \forall i \in N$$
$$S_{ij} = \left(\boldsymbol{Y}_{ij}^* - i\frac{\boldsymbol{b}_{ij}^l}{2}\right)\frac{V_i V_i^*}{T_{ij}^2} - \boldsymbol{Y}_{ij}^* \frac{V_i V_j^*}{T_{ij}} \ \ (i,j) \in E$$
$$S_{ji} = \left(\boldsymbol{Y}_{ij}^* - i\frac{\boldsymbol{b}_{ij}^l}{2}\right) V_j V_j^* - \boldsymbol{Y}_{ij}^* \frac{V_j V_i^*}{T_{ij}} \ \ (i,j) \in E$$
$$|S_{ij}| \leq |\overline{\boldsymbol{S}_{ij}}| \ \ \forall (i,j) \in E \cup E^R$$



Global Optimality?

NO!

Why?

Continuous non-linear relaxation (**IPOPT**) is not a Lower Bound (non-convex)!

Bonmin's Branch and Bound not guaranteed to find OPT

Still a Heuristic,

Heuristic MINLP

Good Enough?

Results

Wind Power Plant Case (96 scenarios)

Obj. - Time - Violations

IEEE 118 - ORPD

Node Obj. Node Iter. - time

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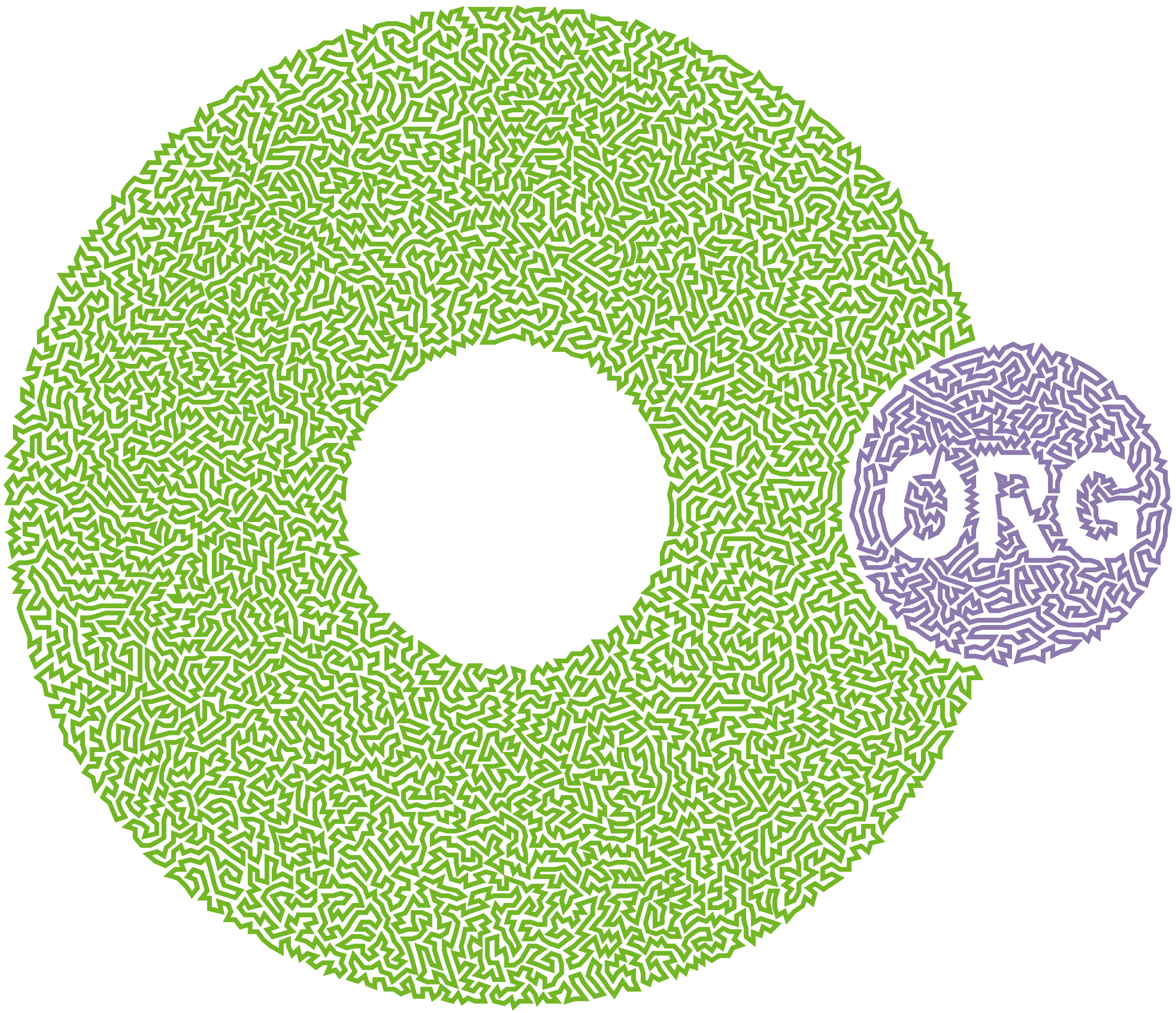
IEEE 118 Case - ORPD - Comparisons

Model	Cont.	Objective	Time
NLP	Basecase	114.68	0.085s
MINLP	Basecase	114.72	2.4s
NLP	N-1	115.48	0.6s
MINLP	N-1	115.54	15s

Conclusions

- **Shocked** by how effective **Bonmin/IPOPT** are at solving these problems.
- Would love to see **phase shifters** added to the decision variables.
- Great exercise, **BIG** thanks the organizers!
  - (hope this becomes an annual event)



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