

5 Gruppenübungen

A. 1: (a)

$$\int e^x \sin(e^x) dx = \left\{ \begin{array}{l} u = e^x \\ du = e^x dx \end{array} \right\} = \int \sin u du = -\cos u + c = -\cos(e^x) + c$$

(b)

$$\int \frac{\log x}{x} dx = \left\{ \begin{array}{l} u = \log x \\ du = \frac{dx}{x} \end{array} \right\} = \int u du = \frac{u^2}{2} + c = \frac{(\log x)^2}{2} + c$$

(c)

$$\begin{aligned} \int_1^{e^2} \frac{e^x}{e^{2x} + 2e^x + 1} dx &= \left\{ \begin{array}{l} u = e^x \\ du = e^x dx \end{array} \right\} = \int_e^{e^2} \frac{du}{(u+1)^2} = \\ &= \int_e^{e^2} (u+1)^{-2} du = -(u+1)^{-1} \Big|_e^{e^2} = -\frac{1}{e^2+1} + \frac{1}{e+1} \end{aligned}$$

A. 2: (a)

$$\int \frac{x}{f} \cdot \frac{e^x}{g'} dx = \frac{x}{f} \cdot \frac{e^x}{g} - \int \frac{1}{f'} \cdot \frac{e^x}{g} dx = xe^x - e^x + c.$$

(b)

$$\begin{aligned} \int \frac{\sqrt{x}}{f'} \cdot \frac{\log x}{g} dx &= \frac{2}{3} \cdot \frac{x^{\frac{3}{2}}}{f} \cdot \frac{\log x}{g} - \int \frac{2}{3} \cdot \frac{x^{\frac{3}{2}}}{f} \cdot \frac{1}{x} dx = \\ &= \frac{2}{3} x^{\frac{3}{2}} \cdot \log x - \frac{2}{3} \int x^{\frac{1}{2}} dx = \\ &= \frac{2}{3} x^{\frac{3}{2}} \cdot \log x - \frac{2}{3} \cdot \frac{2}{3} x^{\frac{3}{2}} + c \end{aligned}$$

(c)

$$\begin{aligned} \int_1^4 \log x dx &= \int_1^4 \frac{1}{f'} \cdot \frac{\log x}{g} dx = \frac{x}{f} \cdot \log x \Big|_1^4 - \int_1^4 \frac{x}{f} \cdot \frac{1}{x} dx \\ &= 4 \cdot \log 4 - 4 \cdot 0 - (4 - 1) = 4 \log 4 - 3. \end{aligned}$$

A. 3:

$$\begin{aligned} \int_0^x t^2 e^{-t^2} dt &= \int_0^x \frac{t}{f} \cdot \frac{te^{-t^2}}{g'} dt = \frac{t}{f} \cdot \left(-\frac{1}{2} \cdot e^{-t^2} \right) \Big|_0^x - \int_0^x \frac{1}{f'} \cdot \left(-\frac{1}{2} \cdot e^{-t^2} \right) dt = \\ &= -\frac{x}{2} \cdot e^{-x^2} + \frac{1}{2} \int_0^x e^{-t^2} dt = -\frac{x}{2} e^{-x^2} + \frac{1}{2} \psi(x). \end{aligned}$$