

True NURBS in Approximation Theory: An Introduction

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It is commonly accepted that, more than 60 years ago, Schoenberg [4] was the first to give a mathematical reference to *splines*. This is probably the first place where the term is used in connection with smooth, piecewise polynomial functions. Years later, Schoenberg [5] also introduced a variation diminishing spline method as an extension of the classical Bernstein polynomial approximation. Since then, splines have developed a lot in use and importance, and have known an increasing applicability, not only in approximation theory, but also in Computer Aided Geometric Design (CAGD). It was in CAGD that NURBS («non-uniform rational B-spline») curves and surfaces were introduced, the thesis of Versprille [6] being one of the first papers on the subject. They provide a common framework, not only for treating free-form curves and surfaces, but also for the precise representation of conic sections.

It is very important, however, to distinguish the parametric case from the functional one.

The aim of this presentation is to clarify the concept of *true NURBS functions* in approximation theory, the interest being in the truly rational case. The main question is: how can the degrees of freedom introduced by the weights be exploited in order to get a better approximation?

Attention is drawn to this controversial but challenging subject by discussing the unsolved problems that arise in the existing sources [1–3].

References

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