Polyconvex Models for Arbitrary Anisotropic Materials

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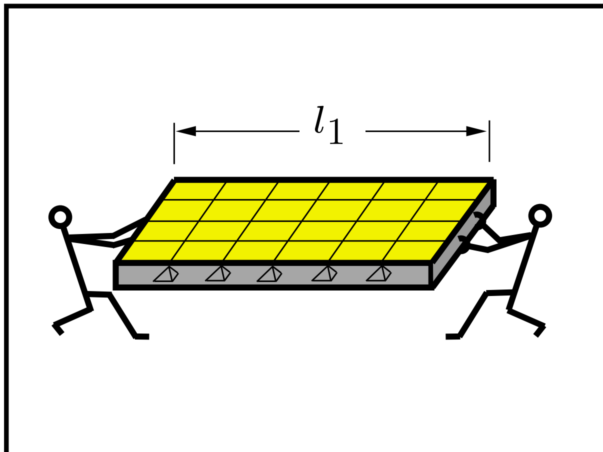
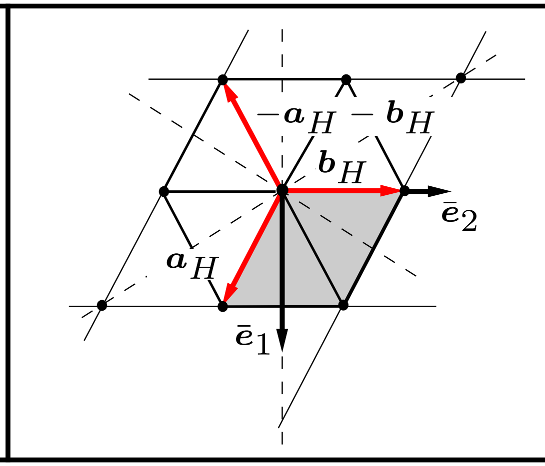
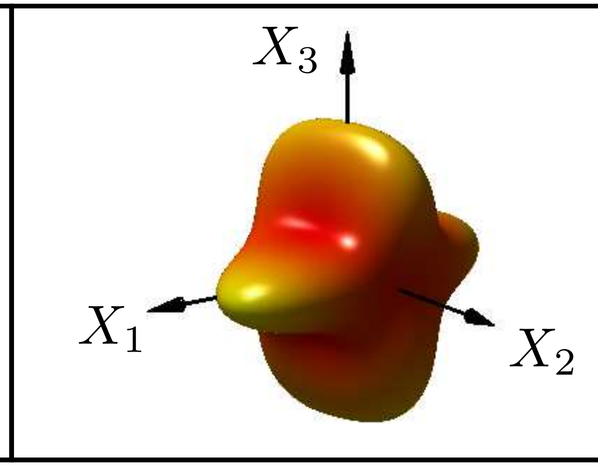






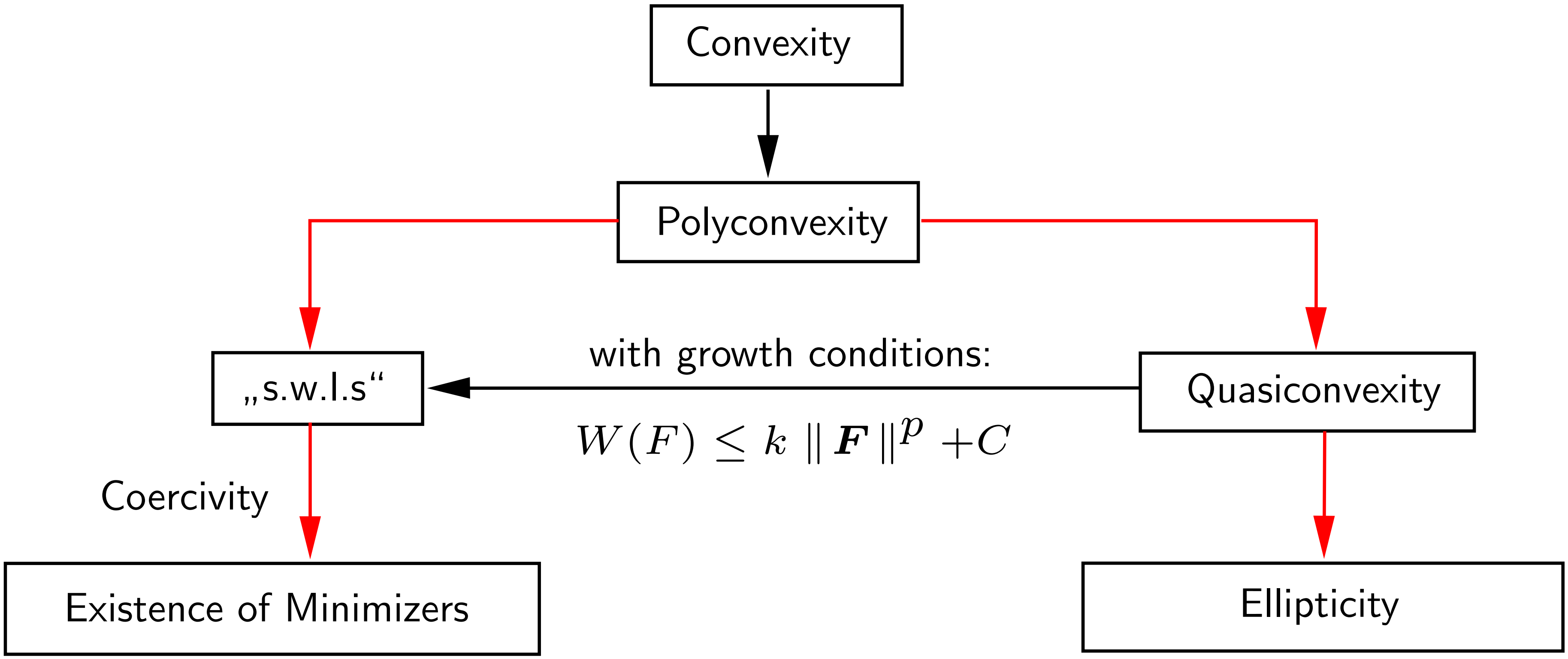
$$\boldsymbol{G} := \boldsymbol{H}\boldsymbol{H}^T \,, \;\; J_4 = \mathrm{tr}[\boldsymbol{C}\boldsymbol{G}], \;\; J_5 = \mathrm{tr}[\mathrm{Cof}[\boldsymbol{C}]\boldsymbol{G}]$$

- Generalized Convexity Conditions
- Crystallographic Motivated Structural Tensors
- Polyconvex Functional Bases
- Polyconvex Anisotropic Free Energy Functions for More General Anisotropy Classes

DFG-Project: NE 902/2-1 SCHR 570/6-1

J. SCHRÖDER, P. NEFF & V. EBBING [2008], “Anisotropic Polyconvex Energies on the Basis of Crystallographic Motivated Structural Tensors”, submitted to *JMPS*

Generalized Convexity Conditions: Implications



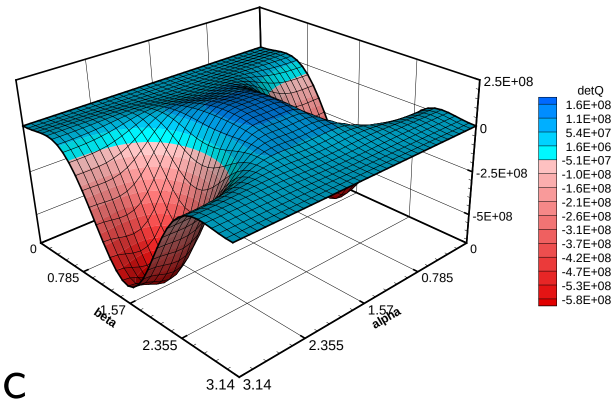
Quasiconv.: $W(\bar{\boldsymbol{F}}) \cdot \mathrm{Vol}(\mathcal{B}) \leq \int_{\mathcal{B}} W(\bar{\boldsymbol{F}} + \nabla \boldsymbol{w}) \, dV \;\rightarrow\; \bar{\boldsymbol{F}}$ is global minimizer.

Ellipticity: Positive definite acoustic tensor $\rightarrow$ Real wave speeds.

Polyconvexity Condition

Polyconvexity ⇒ Quasiconvexity ⇒ Ellipticity

Localization analysis for constrained compression test



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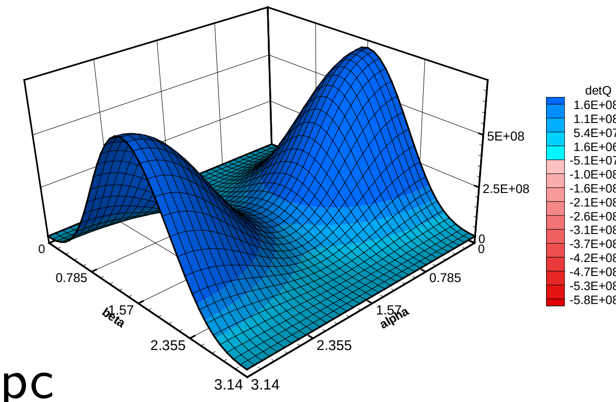

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Polyconvexity, Ball [1977]

The elastic free energy $W(\boldsymbol{F})$ is polyconvex if and only if there exists a function $P : \mathbb{M}^{3\times 3} \times \mathbb{M}^{3\times 3} \times \mathbb{R} \mapsto \mathbb{R}$ (in general non unique) such that

$$W(\boldsymbol{F}) = P(\boldsymbol{F}, \mathrm{Cof}\boldsymbol{F}, \det \boldsymbol{F})$$

and the function $\mathbb{R}^{19} \mapsto \mathbb{R},\ (X, Y, Z) \mapsto P(X, Y, Z)$ is convex.

Common Polyconvex Energy Functions

- **Isotropy**: Polyconvex models, e.g. OGDEN-type models $(\alpha_1, \alpha_2, \delta_1 \geq 0)$

$$\psi^{iso} = \alpha_1 I_1 + \alpha_2 I_2 + \delta_1 I_3^2 - (2\alpha_1 + 4\alpha_2 + 2\delta_1) \ln \sqrt{I_3},$$

formulated in principal invariants $I_1 = \mathrm{tr}\boldsymbol{C},\ I_2 = \mathrm{tr}[\mathrm{Cof}\boldsymbol{C}]$ and $I_3 = \det\boldsymbol{C}$.

> BALL [2002], Some open problems in elasticity, **Problem 2:**
>
> *"Are there ways of verifying polyconvexity and quasiconvexity for a useful class of anisotropic stored-energy functions? "*

- **Anisotropy**: **Transversely isotropic** (a=1) and **orthotropic** (a = 3) polyconvex energy functions first derived in SCHRÖDER & NEFF [2001,2003] formulated in

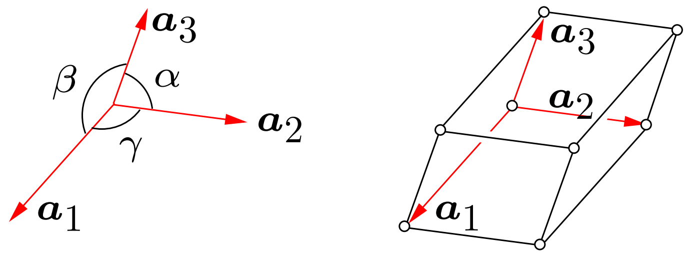
$$\mathrm{tr}[\boldsymbol{C}\boldsymbol{M}^i],\ \mathrm{tr}[\mathrm{Cof}[\boldsymbol{C}]\boldsymbol{M}^i],\ \mathrm{tr}[\boldsymbol{C}(\boldsymbol{1} - \boldsymbol{M}^i)],\ \mathrm{tr}[\mathrm{Cof}[\boldsymbol{C}](\boldsymbol{1} - \boldsymbol{M}^i)], \quad i = 1, ..., a$$

- **Further extensions** and case studies are documented in: STEIGMANN [2003], SNB [2004], ITSKOV & AKSEL [2004], MARKERT, EHLERS & KARAJAN [2005], BALZANI [2006], BNSH [2006], EHRET & ITSKOV [2007].

> *Are there ways of verifying polyconvexity for the further ten classes of anisotropic stored-energy functions?*

Anisotropic Elasticity

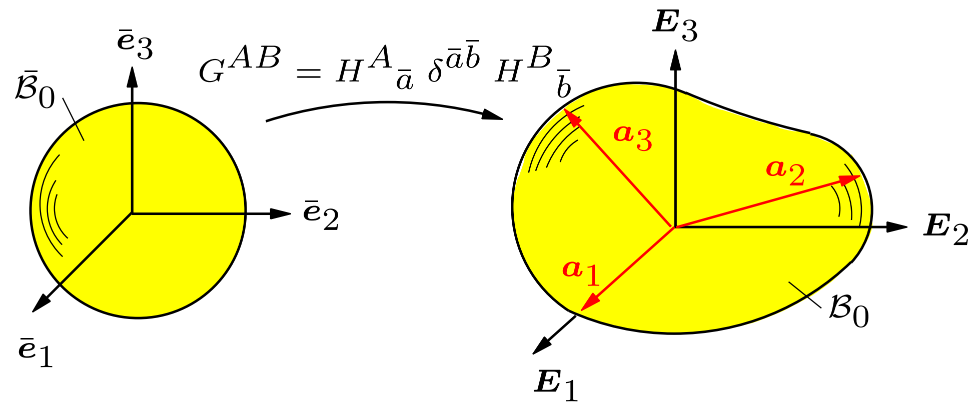
12 anisotropy classes / 32 crystal classes / 7 crystal systems



with $\quad a = ||\boldsymbol{a}_1||, b = ||\boldsymbol{a}_2||, c = ||\boldsymbol{a}_3||$

No.	crystal system	edge lengths	axial angle
1	triclinic	$a \neq b \neq c$	$\alpha \neq \beta \neq \gamma \neq 90^\circ$
2	monoclinic	$a \neq b \neq c$	$\alpha = \beta = 90^\circ;\ \gamma \neq 90^\circ$
3	trigonal	$a = b = c$	$\alpha = \beta = \gamma \neq 90^\circ$
4	hexagonal	$a = b \neq c$	$\alpha = \beta = 90^\circ;\ \gamma = 120^\circ$
5	rhombic	$a \neq b \neq c$	$\alpha = \beta = \gamma = 90^\circ$
6	tetragonal	$a = b \neq c$	$\alpha = \beta = \gamma = 90^\circ$
7	cubic	$a = b = c$	$\alpha = \beta = \gamma = 90^\circ$

General Anisotropy in Polyconvex Framework



Main idea: Introduction of an **anisotropic metric tensor** $\boldsymbol{G}$:

- $\boldsymbol{G}$ is a **second-order**, **symmetric** and **positive definite** structural tensor.
- $\boldsymbol{G}$ is the **push-forward** of a cartesian metric of a fictitious reference configuration $\bar{\mathcal{B}}_0$ onto the real reference configuration $\mathcal{B}_0$:

$$\rightarrow \quad \boldsymbol{G} = \boldsymbol{H}\boldsymbol{H}^T \quad \rightarrow \quad \boldsymbol{G} = \boldsymbol{Q}\boldsymbol{G}\boldsymbol{Q}^T \quad \forall\, \boldsymbol{Q} \in \mathcal{G} \subset \mathrm{O}(3)\,.$$

$\Longrightarrow$ Principle of Material Symmetry is automatically satisfied :

$$\boldsymbol{C} \cdot \boldsymbol{G} = \boldsymbol{Q}\boldsymbol{C}\boldsymbol{Q}^T \cdot \boldsymbol{G} = \boldsymbol{C} \cdot \boldsymbol{Q}^T\boldsymbol{G}\boldsymbol{Q} \quad \forall\, \boldsymbol{Q} \in \mathcal{G} \subset \mathrm{O}(3)\,.$$

Literature: Zheng & Spencer [1993], Zheng [1994], Xiao [1996], Apel [2004], Xiao, Bruhns & Meyer [2007]

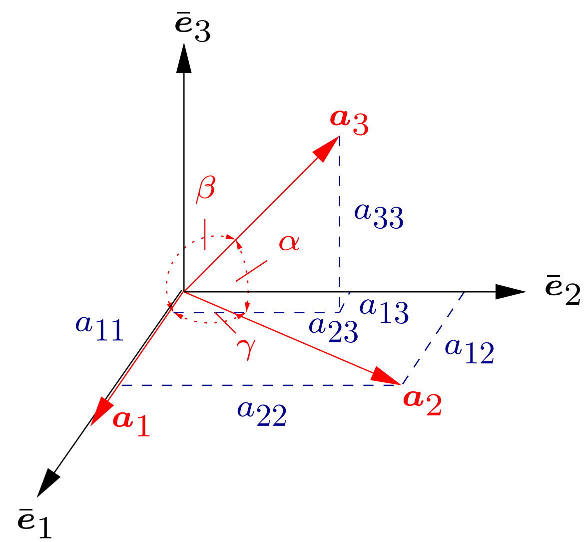
Metric Tensors for the Seven Crystal Systems

Linear Mapping of the cartesian base vectors onto crystallographic motivated base vectors:

$$\boldsymbol{H} : \bar{\boldsymbol{e}}_i \mapsto \boldsymbol{a}_i \rightarrow \boldsymbol{H} = [\boldsymbol{a}_1, \boldsymbol{a}_2, \boldsymbol{a}_3] \quad \text{with} \quad \boldsymbol{a}_i = \boldsymbol{H}\, \bar{\boldsymbol{e}}_i$$

Special choice: $\boldsymbol{a}_1 \parallel \boldsymbol{e}_1, \quad \boldsymbol{a}_2 \perp \boldsymbol{e}_3 :$

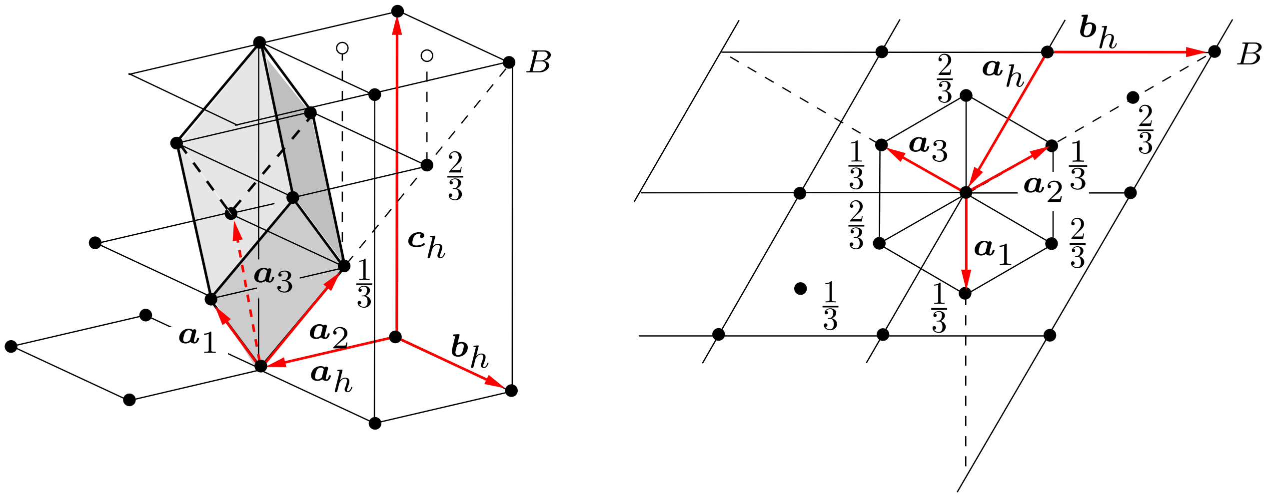
$$\boldsymbol{H} = \begin{bmatrix} a & b\cos\gamma & c\cos\beta \\ 0 & b\sin\gamma & c(\cos\alpha - \cos\beta\cos\gamma)/\sin\gamma \\ 0 & 0 & c\,[1 + 2\cos\alpha\cos\beta\cos\gamma - (\cos^2\alpha + \cos^2\beta + \cos^2\gamma)]^{1/2}/\sin\gamma \end{bmatrix}$$



E.g. Monoclinic Metric Tensor ($a \neq b \neq c, \alpha = \beta = 90°, \gamma \neq 90°$)**:**

$$\boldsymbol{G}^m = \boldsymbol{H}^m \boldsymbol{H}^{mT} = \begin{bmatrix} a^2 + b^2\cos^2\gamma & b^2\cos\gamma\sin\gamma & 0 \\ b^2\cos\gamma\sin\gamma & b^2\sin^2\gamma & 0 \\ 0 & 0 & c^2 \end{bmatrix} \rightarrow \boldsymbol{G}^m = \begin{bmatrix} \tilde{a} & \tilde{d} & 0 \\ \tilde{d} & \tilde{b} & 0 \\ 0 & 0 & \tilde{c} \end{bmatrix}$$

Crystallographic Motivated Trigonal Metric Tensor



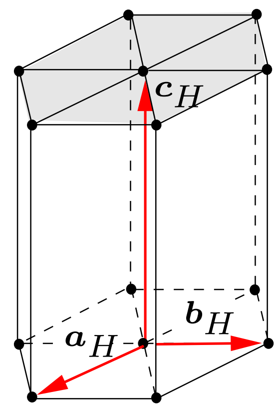
Considering **rhombohedral base vectors** in **hexagonal centered cell**

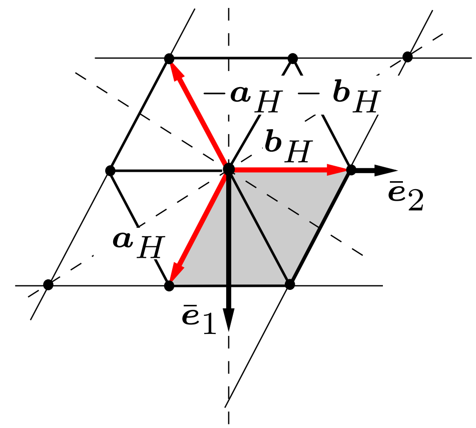
$$\boldsymbol{a}_1 = \frac{1}{3}(2\boldsymbol{a}_h + \boldsymbol{b}_h + \boldsymbol{c}_h)\,, \quad \boldsymbol{a}_2 = \frac{1}{3}(-\boldsymbol{a}_h + \boldsymbol{b}_h + \boldsymbol{c}_h)\,, \quad \boldsymbol{a}_3 = \frac{1}{3}(-\boldsymbol{a}_h - 2\boldsymbol{b}_h + \boldsymbol{c}_h)\,,$$

with threefold axis $\boldsymbol{c}_h = (\boldsymbol{a}_1 + \boldsymbol{a}_2 + \boldsymbol{a}_3) \;\|\; \boldsymbol{e}_3$, leads to trigonal metric tensor

$$\boldsymbol{G}^{ht} = \boldsymbol{H}^{ht}\boldsymbol{H}^{htT} = \mathrm{diag}(a^2, a^2, c^2) \quad \rightarrow \quad \boldsymbol{G}^{ht} = \boldsymbol{Q}\boldsymbol{G}^{ht}\boldsymbol{Q}^T \;\; \forall\, \boldsymbol{Q} \in \mathcal{G}^{ht}$$

Crystallographic Motivated Hexagonal Metric Tensor





Bravais [1866]: Inherent sixfold symmetry of **primitive hexagonal cell** is captured by **three hexagonal base vectors** in the $\bar{\boldsymbol{e}}_1 - \bar{\boldsymbol{e}}_2-$plane

$$\boldsymbol{a}_H, \quad \boldsymbol{b}_H, \quad (-\boldsymbol{a}_H - \boldsymbol{b}_H)\,.$$

This symmetry indicates that $\bar{\boldsymbol{e}}_1 - \bar{\boldsymbol{e}}_2-$plane acts as **isotropy plane**, Love [1907]. Therefore the fictitious deformation has to be of the type

$$\boldsymbol{H}^h = \mathrm{diag}(a, a, c) \;\rightarrow\; \boldsymbol{G}^h = \mathrm{diag}(a^2, a^2, c^2) \;\rightarrow\; \boldsymbol{G}^h = \boldsymbol{Q}\boldsymbol{G}^h\boldsymbol{Q}^T \;\;\forall\, \boldsymbol{Q} \in \mathcal{G}^h$$

Proof of Polyconvexity of $\mathrm{tr}[\boldsymbol{C}\boldsymbol{G}]$ and $\mathrm{tr}[\mathrm{Cof}[\boldsymbol{C}]\boldsymbol{G}]$

Generic **polyconvex** anisotropic functions

$$[\mathrm{tr}(\boldsymbol{F}^T\boldsymbol{F}\boldsymbol{G})]^k \quad \text{and} \quad [\mathrm{tr}(\mathrm{Cof}(\boldsymbol{F}^T)\mathrm{Cof}(\boldsymbol{F})\boldsymbol{G})]^k$$

with $k \geq 1$ and $\boldsymbol{G} \in \mathrm{P}_{\mathrm{sym}}$.

Proof of Convexity. With identity $[\mathrm{tr}(\boldsymbol{F}^T\boldsymbol{F}\boldsymbol{G})]^k = \| \boldsymbol{F}\boldsymbol{H} \|^{2k} = \langle \boldsymbol{F}\boldsymbol{H}, \boldsymbol{F}\boldsymbol{H} \rangle^k$ we obtain

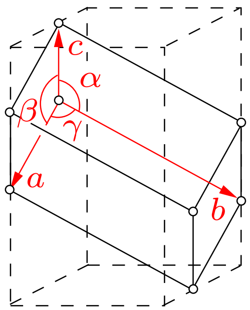
$$\begin{aligned}
D_F(\langle \boldsymbol{F}\boldsymbol{H}, \boldsymbol{F}\boldsymbol{H} \rangle^k) \,.\boldsymbol{\xi} = \;& 2k\langle \boldsymbol{F}\boldsymbol{H}, \boldsymbol{F}\boldsymbol{H} \rangle^{k-1} \langle \boldsymbol{F}\boldsymbol{H}, \boldsymbol{\xi}\boldsymbol{H} \rangle \\
D_F^2(\langle \boldsymbol{F}\boldsymbol{H}, \boldsymbol{F}\boldsymbol{H} \rangle^k) \,.(\boldsymbol{\xi}, \boldsymbol{\xi}) = \;& 2k\langle \boldsymbol{F}\boldsymbol{H}, \boldsymbol{F}\boldsymbol{H} \rangle^{k-1} \langle \boldsymbol{\xi}\boldsymbol{H}, \boldsymbol{\xi}\boldsymbol{H} \rangle \\
& +4k(k-1)\langle \boldsymbol{F}\boldsymbol{H}, \boldsymbol{F}\boldsymbol{H} \rangle^{k-2} \langle \boldsymbol{F}\boldsymbol{H}, \boldsymbol{\xi}\boldsymbol{H} \rangle^2 \\
= \;& 2k \, \| \boldsymbol{F}\boldsymbol{H} \|^{2k-2} \, \| \boldsymbol{\xi}\boldsymbol{H} \|^2 \\
& +4k(k-1) \, \| \boldsymbol{F}\boldsymbol{H} \|^{2k-4} \; \langle \boldsymbol{F}\boldsymbol{H}, \boldsymbol{\xi}\boldsymbol{H} \rangle^2 \geq 0.
\end{aligned}$$

Detailed proofs are given in Schröder, Neff & Ebbing [2008].

Polyconvex Functional Bases $\mathcal{P}$

– Evaluation of Polyconvex Invariants $\boldsymbol{C} \cdot \boldsymbol{G}$ and $\mathrm{Cof}\boldsymbol{C} \cdot \boldsymbol{G}$ –

Triclinic system: 2nd-order SST



$$\boldsymbol{G}^a = \begin{bmatrix} \tilde{a} & \tilde{d} & \tilde{e} \\ \tilde{d} & \tilde{b} & \tilde{f} \\ \tilde{e} & \tilde{f} & \tilde{c} \end{bmatrix}$$

Ordering with respect to individual entries of $\boldsymbol{G}$ yields

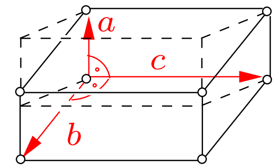
$$\mathcal{P}^a := \{ C_{11}, C_{22}, C_{33}, C_{12}, C_{13}, C_{23} \}.$$

Classical Basis:

$$\mathcal{I}^a := \{ C_{11}, C_{22}, C_{33}, C_{12}, C_{13}, C_{23} \}.$$

→ complete description

Monoclinic system: 2nd-order SST



$$\boldsymbol{G}^m = \begin{bmatrix} \tilde{a} & \tilde{d} & 0 \\ \tilde{d} & \tilde{b} & 0 \\ 0 & 0 & \tilde{c} \end{bmatrix}$$

$$\begin{aligned} \mathcal{P}^m := \{ & C_{11}, C_{22}, C_{33}, C_{12}, \\ & \mathrm{Cof}C_{11} = C_{22}C_{33} - C_{23}^2, \\ & \mathrm{Cof}C_{22} = C_{11}C_{33} - C_{13}^2, \\ & \mathrm{Cof}C_{33} = C_{11}C_{22} - C_{12}^2, \\ & \mathrm{Cof}C_{12} = C_{13}C_{23} - C_{12}C_{33} \}. \end{aligned}$$

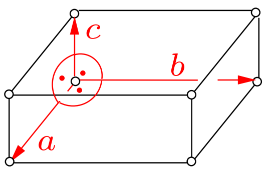
Classical Basis:

$$\mathcal{I}^m := \{ C_{11}, C_{22}, C_{33}, C_{12}, C_{13}^2, C_{23}^2, C_{13}C_{23} \}.$$

→ complete description

Polyconvex Functional Bases $\mathcal{P}$

Rhombic system: 2nd-order SST

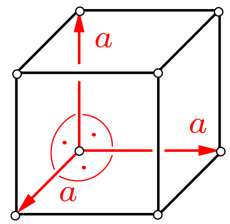


$$\boldsymbol{G}^o = \begin{bmatrix} \tilde{a} & 0 & 0 \\ 0 & \tilde{b} & 0 \\ 0 & 0 & \tilde{c} \end{bmatrix}$$

$$\mathcal{P}^o := \{\, C_{11}\,,\ C_{22}\,,\ C_{33}\,, C_{22}C_{33} - C_{23}^2\,,\ C_{11}C_{33} - C_{13}^2\,,\ C_{11}C_{22} - {C_{12}}^2\,\}.$$

$\longrightarrow$ complete description

Cubic system: 4th-order SST

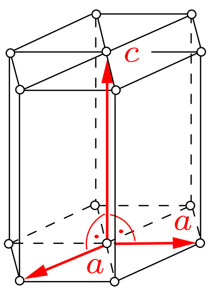


$$\boldsymbol{G}^c = \begin{bmatrix} \tilde{a} & 0 & 0 \\ 0 & \tilde{a} & 0 \\ 0 & 0 & \tilde{a} \end{bmatrix}$$

$$\mathcal{P}^c := \{I_1,\ I_2\}.$$

$\longrightarrow$ incomplete description

Hexagonal system: 6th-order SST

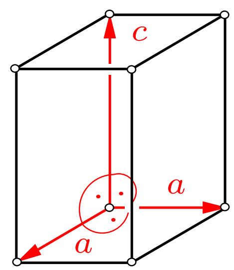


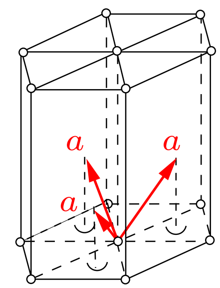
$$\boldsymbol{G}^h = \begin{bmatrix} \tilde{a} & 0 & 0 \\ 0 & \tilde{a} & 0 \\ 0 & 0 & \tilde{c} \end{bmatrix}$$

$$\mathcal{P}^h := \{\, C_{11} + C_{22}\,,\ C_{33}\,,\ C_{22}C_{33} + C_{11}C_{33} - (C_{23}^2 + C_{13}^2)\,,\ C_{11}C_{22} - C_{12}^2\}\,.$$

$\longrightarrow$ complete description

Tetragonal, Trigonal system: 4th-order SSTs





$$\boldsymbol{G}^t = \boldsymbol{G}^{ht} = \boldsymbol{G}^h$$
$$\mathcal{P}^t = \mathcal{P}^{ht} = \mathcal{P}^h$$

$\longrightarrow$ incomplete description

Generic Polyconvex Anisotropic Energy Functions

Additive decomposition of the free energy in isotropic and anisotropic terms, i.e.,

$$\psi = \psi^{iso}(I_1, I_2, I_3) + \psi^{aniso}(I_3, J_{4j}, J_{5j}),$$

with $J_{4j} = \mathrm{tr}[\boldsymbol{C}\boldsymbol{G}_j]$, $J_{5j} = \mathrm{tr}[\mathrm{Cof}[\boldsymbol{C}]\boldsymbol{G}_j]$ and the j-th metric tensor $\boldsymbol{G}_j$.

For the isotropic part ψ^{iso} we choose a compressible Mooney-Rivlin model,i.e.,

$$\psi^{iso} = \alpha_1\; I_1 + \alpha_2\; I_2 + \delta_1\; I_3 - \delta_2 \ln(\sqrt{I_3}), \quad \forall\; \alpha_1,\; \alpha_2,\; \delta_1,\; \delta_2 \geq 0.$$

Suitable polyconvex anisotropic energies in terms of $f_{3rj}, f_{4rj}, f_{5rj}, f_{6rj}, f_{7rj}$

$$\begin{aligned}
\psi_1^{aniso} &= \sum_{r=1}^{n} \sum_{j=1}^{m} \left[f_{3rj}(I_3) + f_{4rj}(J_{4j}) + f_{5rj}(J_{5j}) \right], \\
\psi_2^{aniso} &= \sum_{r=1}^{n} \sum_{j=1}^{m} \left[f_{3rj}(I_3) + f_{6rj}(I_3, J_{4j}) + f_{7rj}(I_3, J_{5j}) \right], \\
\psi_3^{aniso} &= \sum_{r=1}^{n} \sum_{j=1}^{m} \left[f_{3rj}(I_3) + f_{4rj}(J_{4j}) + f_{5rj}(J_{5j}) \right. \\
&\qquad \left. + f_{6rj}(I_3, J_{4j}) + f_{7rj}(I_3, J_{5j}) \right].
\end{aligned}$$

Further details: Schröder, Neff & Ebbing [2008].

Generic Polyconvex Anisotropic Functions

Some specific functions which **automatically** satisfy the condition $\boldsymbol{S}|_{C=1} = \boldsymbol{0}$

$$\psi_I^{aniso} = \sum_{r=1}^{n}\sum_{j=1}^{m} \xi_{rj} \left\{ \left[\frac{1}{\alpha_{rj}+1}\frac{1}{(g_j)^{\alpha_{rj}}}(J_{4j})^{\alpha_{rj}+1} + \frac{1}{\beta_{rj}+1}\frac{1}{(g_j)^{\beta_{rj}}}(J_{5j})^{\beta_{rj}+1} + \frac{g_j}{\gamma_{rj}}(I_3)^{-\gamma_{rj}} \right] \right\},$$

with $\xi_{rj}, \alpha_{rj}, \beta_{rj} \geq 0$, $\gamma_{rj} \geq -1/2$. $\longrightarrow$ coercive

$$\psi_{II}^{aniso} = \sum_{r=1}^{n}\sum_{j=1}^{m} \xi_{rj} \left\{ \left[\frac{1}{\alpha_{rj}+1}\frac{1}{(g_j)^{\alpha_{rj}}}(J_{4j})^{\alpha_{rj}+1} + \frac{1}{\beta_{rj}+1}\frac{1}{(g_j)^{\beta_{rj}}}(J_{5j})^{\beta_{rj}+1} - \ln(I_3^{g_j}) \right] \right\},$$

with $\xi_{rj},\ \alpha_{rj},\ \beta_{rj} \geq 0$.

$$\psi_{III}^{aniso} = \sum_{r=1}^{n}\sum_{j=1}^{m} \left[\frac{J_{4j}^{\alpha_{rj}}}{I_3^{1/3}} + \frac{J_{5j}^{\alpha_{rj}}}{I_3^{1/3}} + \frac{g_j^{\alpha_{rj}}}{\beta_{rj}} I_3^{-\beta_{rj}} \right],$$

with $\alpha_{rj} = 5/3$, $\beta_{rj} \geq -1/2$.

$$\psi_{IV}^{aniso} = \sum_{r=1}^{n}\sum_{j=1}^{m} \left[\frac{J_{4j}^{\alpha_{rj}}}{I_3^{1/3}} + \frac{J_{5j}^{\alpha_{rj}}}{I_3^{1/3}} - g_j^{\alpha_{rj}}\ \beta_{rj} \ln(I_3) \right],$$

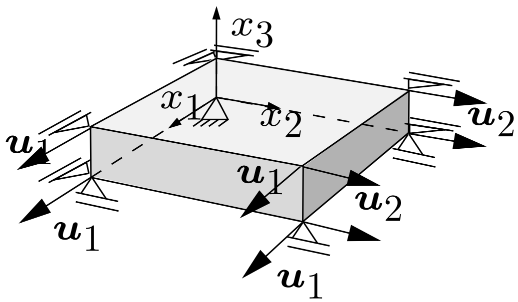
with $\alpha_{rj} \geq 1$, $\beta_{rj} = \alpha_{rj} - 2/3$.

A priori stress free reference configuration: ITSKOV & AKSEL [2004].

Proof of coercivity: SCHRÖDER, NEFF & EBBING [2008].

A First Applications to Transverse Isotropy

Biaxial Homogeneous Tension Test

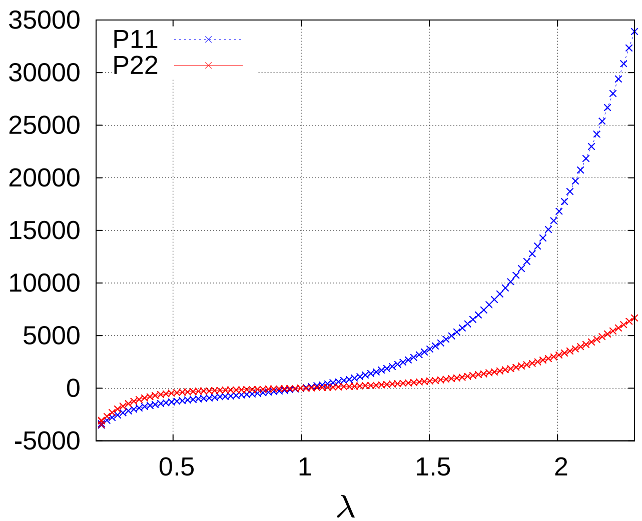


Unimodular metric tensor:

$$\boldsymbol{G}^{ti} = \mathrm{diag}(\gamma, \frac{1}{\sqrt{\gamma}}, \frac{1}{\sqrt{\gamma}})$$

Additive construction of energy function $\psi = \psi^{iso} + \psi^{ti}$, with

$$\begin{aligned} \psi^{iso} &= \alpha_1 I_1 + \alpha_2 I_2 + \delta_1 I_3 - \delta_2 \ln\sqrt{I_3} \\ \psi^{ti} &= \eta_1 (J_4^{\alpha_4} + \beta_1 J_5^{\alpha_5}) \end{aligned}$$



$J_4 = \mathrm{tr}[\boldsymbol{C}\boldsymbol{G}^{ti}]$, $J_5 = \mathrm{tr}[\mathrm{Cof}[\boldsymbol{C}]\boldsymbol{G}^{ti}]$

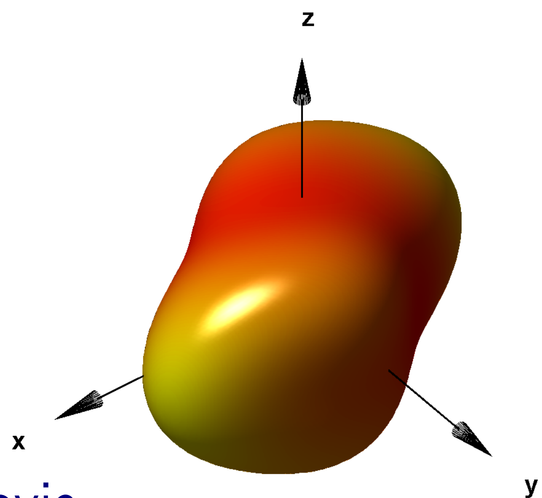
$\gamma = 4.0,\ \alpha_1 = 2.0,\ \alpha_2 = 0.0,$

$\delta_1 = 10.0,\ \delta_2 = 786,\ \eta_1 = 1.0,\ \beta_1 = 7.5,$

$\alpha_4 = 2.0,\ \alpha_5 = 3.0$ [MPa]

Fitting of Monoclinic Moduli $\mathbb{C} = 4\partial^2_{CC}\psi$: Augite

Characteristic surfaces



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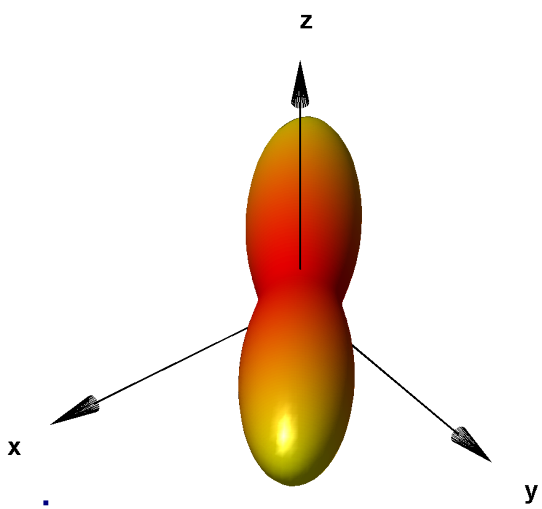
Young's modulus



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Bulk modulus

Elasticities in [GPa], SIMMONS [1971]:

$$\mathbb{C}^{(V)exp} = \begin{bmatrix} 217.8 & 72.4 & 33.9 & 24.6 & 0 & 0 \\ & 181.6 & 73.4 & 19.9 & 0 & 0 \\ & & 150.7 & 16.6 & 0 & 0 \\ & & & 51.1 & 0 & 0 \\ & sym. & & & 55.8 & 4.3 \\ & & & & & 69.7 \end{bmatrix}$$

Error for $\psi = \psi_I^{aniso}$ and $n = m = 3$:

$$e = \frac{\| \mathbb{C}^{(V)comp} - \mathbb{C}^{(V)exp} \|}{\| \mathbb{C}^{(V)exp} \|} \approx 3.48[\%]$$

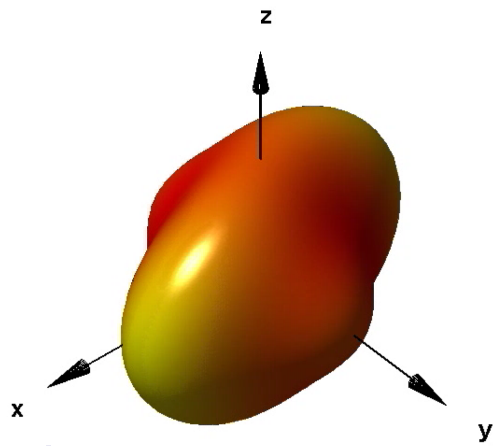
Metric Tensors:

$$\boldsymbol{G}_1^{ti} = \text{diag}(0.419, 0.419, 1.953),$$

$$\boldsymbol{G}_2^{m} = \begin{bmatrix} 1.503 & -0.513 & 0 \\ -0.513 & 0.934 & 0 \\ 0 & 0 & 1.572 \end{bmatrix}, \quad \boldsymbol{G}_3^{m} = \begin{bmatrix} 2.719 & 0.496 & 0 \\ 0.496 & 0.547 & 0 \\ 0 & 0 & 1.008 \end{bmatrix}$$

Fitting of Anisotropic Moduli

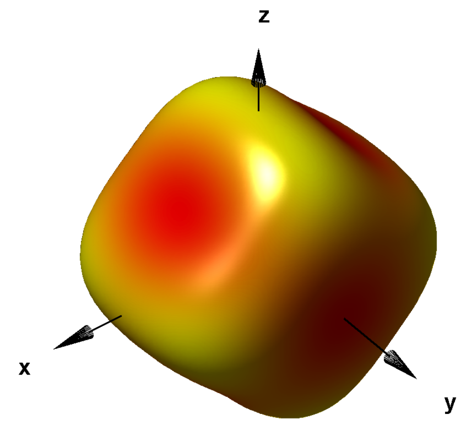
Monoclinic Materials



movie

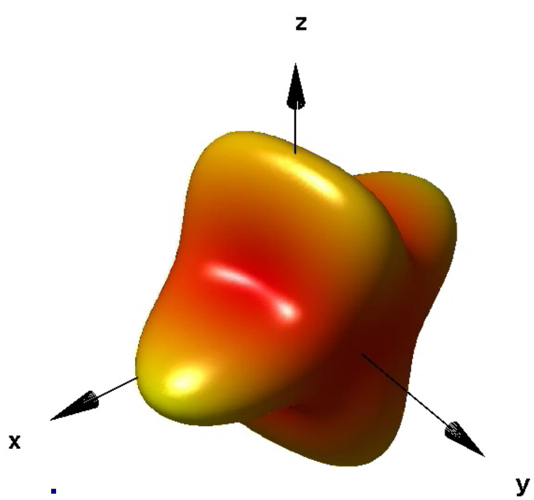
Aegirite: $e = 1.65\%$

Rhombic Materials



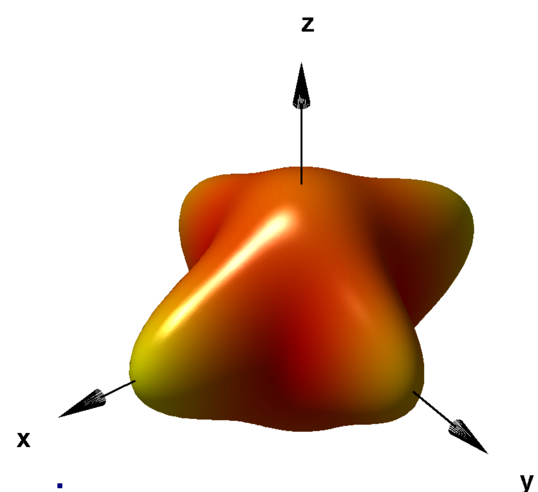
movie

Ammonium Sulfate: $e = 3.46\%$



movie

Feldspar(Labradorite): $e = 5.28\%$

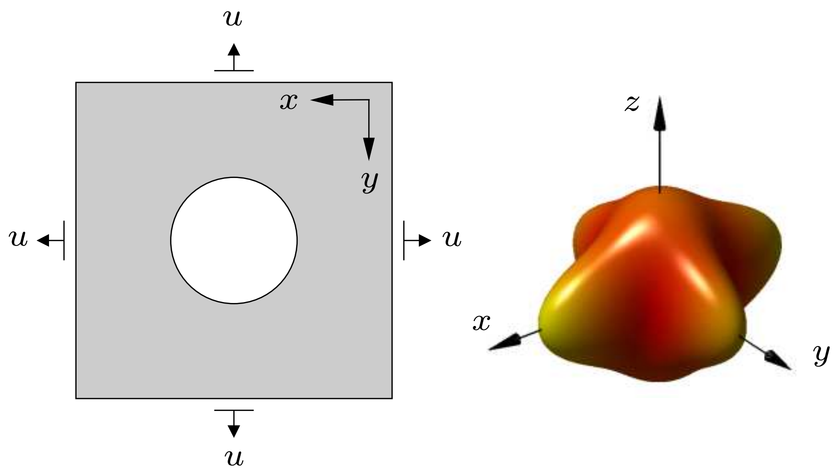


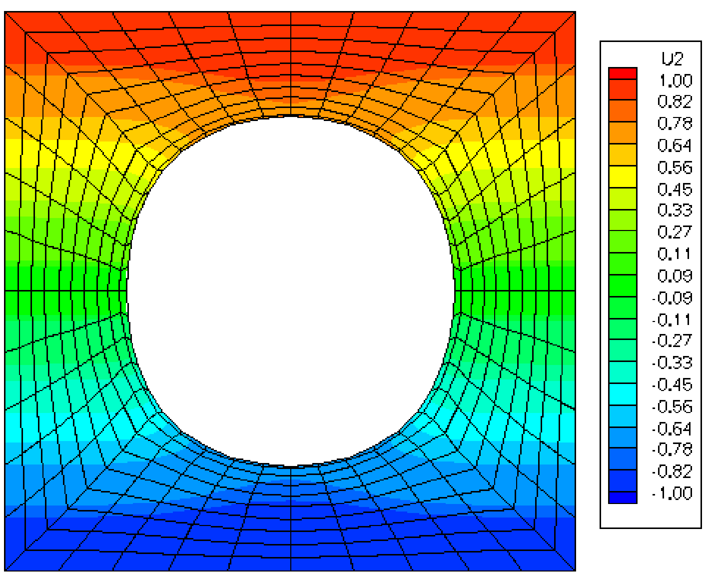
movie

Acenaphthene: $e = 2.28\%$

Biaxial Tension Test of Perforated Plate with Centered Hole

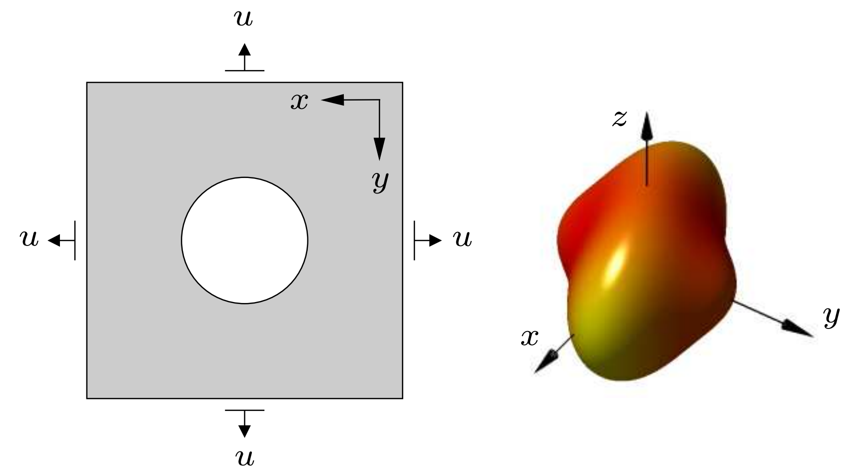
Rhombic Acenaphthene

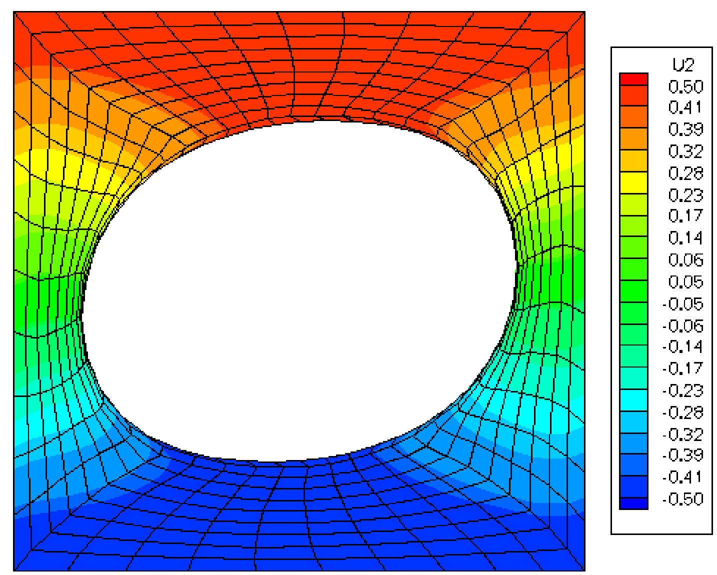




movie

Monoclinic Aegirite

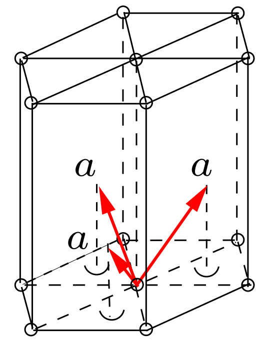




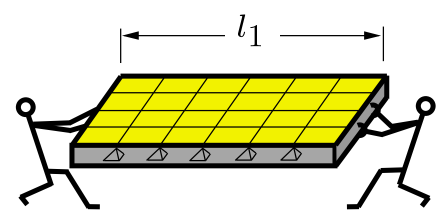
movie

Polyconvex Models for Arbitrary Anisotropic Materials

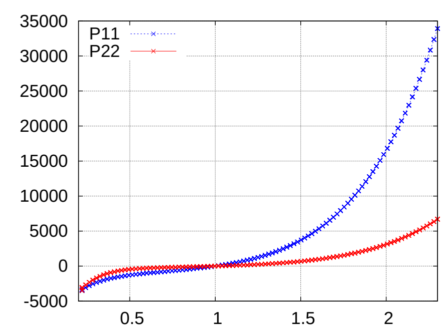
- Triclinic, monoclinic, rhombic, hexagonal as well as transversely isotropic symmetries can be *"completely described"* by generic invariant functions in terms of **single, second-order, positive definite anisotropic metric tensors**.



- These invariant functions **automatically fulfill** the **polyconvexity condition**.



- The requirement of a **stress-free reference configuration** is also **automatically satisfied** .



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