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ORIGINAL CONTRIBUTION

Roger I. Tanner
Elizabeth Tanner

Heinrich Hencky: a rheological pioneer

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R.I. Tanner (✉)
School of Aerospace,
Mechanical & Mechatronic Engineering,
University of Sydney,
Sydney. N.S.W. 2006, Australia
E-mail: rit@mech.eng.usyd.edu.au

E. Tanner
"Marlowe", Sixth Mile Lane,
Roseville, NSW 2069, Australia

Abstract The literature of continuum mechanics and rheology often mentions the name of Hencky: Hencky strain, Hencky theorems, and many other concepts. Yet there is no coherent appraisal of his contributions to mechanics. Nor is there anywhere any description of his life. This article sets down some of what we have learned so far about this researcher, and appraises his pioneering work on rheology.

Keywords Hencky · Strain · Elasticity · Plasticity · Rheology · Biography

Motivation

The name Hencky is best known to rheologists through the so-called Hencky (or logarithmic) strain, but any student of plasticity theory will also encounter Hencky's equations and theorems associated with slip-line theory, Hencky's interpretation of the von Mises yield criterion, and his deformation theory of plastic flow; see, for example, Chakrabarty (1987). A search of the internet will also show his contribution to the elastic buckling of loaded rings. A study of citations shows that, 50 years after his death, many of his papers are still frequently cited. In spite of this continuing visibility, very little is available about the life of this man (Tanner and Walters 1998), and our curiosity has led us to write down some of what we have been able to find out so far about him. We also discuss his work with a view to evaluating his contributions to solid mechanics and rheology.

Early years

Heinrich Hencky was born in Ansbach in Bavaria, Germany on the 2nd of November 1885. His father was Heinrich Karl Hencky, a Bavarian school administrator whose job meant that he was often moved around so Heinrich Hencky changed schools often. He finished his secondary schooling at Speyer on the Rhine in 1904. He had a brother Karl Georg also born in Ansbach in 1889. Both were mentioned in J.C. Poggendorff's Biographisch-literarisches Handwörterbuch (Poggendorff 1931).

After the early death of his father, his mother – with two sons and a daughter – settled in Munich. Both brothers were students at the Technische Hochschule in Munich; Heinrich 1904–1908 and Karl 1908–1912.

Heinrich Hencky got his Diploma, Civil Engineering from Munich in 1908. In 1909 he was in Military service with the 3rd Pioneer Battalion in Munich. In the program for the student year 1912/13 and 1913/14 for the Technische Hochschule Darmstadt, Heinrich Hencky is listed as Diplom-Ingenieur Konstruktion. In 1913 he received his doctorate of Engineering from the Technical University of Darmstadt. The title of his thesis was "Über den Spannungszustand in rechteckigen ebenen Platten bei gleichmässig verteilter und bei konzentrierter Belastung" (On the stress state in rectangular flat plates under uniformly distributed and concentrated loading). The thesis (Hencky 1913) used a numerical method to study the stresses in flat plates; the thesis has been cited at least 74 times since 1974, and was a substantial contribution to elastic plate theory, which became one of his favourite subjects. He published the findings in the Zeitschrift für Mathematik und Physik (Hencky 1915).

After completing his dissertation he worked on the Alsatian railways from 1 February 1910 until 26 April 1912. In January 1913 he sought a position in the field and got an offer from a company in Kharkov in Ukraine. The war began immediately and he was interned in the Urals region for the period 1915–1918. During this time he met his Russian wife, Alexandra Yuditskaya; they were married in January 1918. She was born in Poltava (now in Ukraine) on 15th June, 1890.

At the end of the war the Henckys were sent back to Germany and went to Munich, where their daughter Lydia was born on 6 December 1918; later, in 1924, they had a son Gerhard Georg, born in Delft on 28 December 1924. Early in the year 1919, after being sent back from Kharkov to Munich, Hencky placed himself at the disposal of the authorities. He was sent to the marine commandos in Warnemünde and was employed as a materials testing engineer on a seaplane project. His departure from there was due to the coming demobilization. In any event, he had found a job. In May 1919, according to papers obtained from the Technical University in Darmstadt, Hencky was a Privatdozent (external lecturer) there, with specialities in statics and building mechanics. In the study-plan for the student years 1919/20 he is listed as Dr. Ing. Heinrich Hencky, Assistant für Ingenieurfach. From Darmstadt he went to the Technical University of Dresden where he felt he would come in contact with more dynamic areas of technical mechanics. There he published (Hencky 1920) his Darmstadt Habilitationsschrift (which gave him a license to teach) on elastic stability, and to this area he contributed substantially during his career.

Following the 1920 paper, he published (Hencky 1921b) a famous paper on the stability of closed elastic circular rings with external loads (Fig. 1). He recognized that although a ring might be loaded by inward-pointing in-plane loads, one had to consider the possibility of out-of-plane buckling in order to get a realistic result for the buckling load. This paper continues to be cited, and the results are also shown on the internet. He also continued to work on thin rectangular plates (Hencky 1921a). Unlike the elegant elastic ring buckling problem, this paper contains a lot of quite detailed numerical computations, related in style to his thesis work of 1913. Some criticism (Weber 1921) of the 1921a paper was made, and Hencky (1921c) replied with a graceful clarification.

A further paper, this time on numerical methods for partial differential equations, was written while he was in Dresden (Hencky 1922b). This survey paper contains not only an example on plate theory, but also an early example of the numerical computation of the streamlines of viscous flow past an obstacle (Fig. 2). Although he was busy doing excellent research in Dresden, there is evidence that he was not a successful teacher there. We found, with the help of Dr A.J.Q. Alkemade, a 1924 letter from Professor Trefftz to Professor J.M. Burgers in Delft (Hencky was there in 1924) stating that a search was underway for a Chair of General Mechanics in Dresden. They did not consider that Hencky was in the first rank, but they sought Burgers's views on him, especially about his teaching. Trefftz said that there was some doubt about his teaching abilities following his time in Dresden and not everyone thought he was a competent teacher. Trefftz went on to say "but who among us is a born teacher?" His Dresden difficulties "could have been a beginner's problem on the problem of overcoming his shyness when talking in front of students". (We are greatly indebted to Dr A.J.Q. (Fons) Alkemade for finding this letter in the Burgers archives at the Technical University in Delft).

Soon after, Hencky moved to Delft.

The Delft period 1922–1929

On January 26 1922, Hencky was appointed as Lector in Applied Mechanics at the Technical University in Delft. He was apparently told that the lectureship should

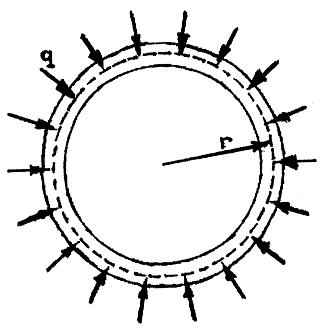

Fig. 1 Buckling of a closed ring. Hencky published the correct solution in 1921b; out-of-plane buckling was considered, in contrast to earlier work

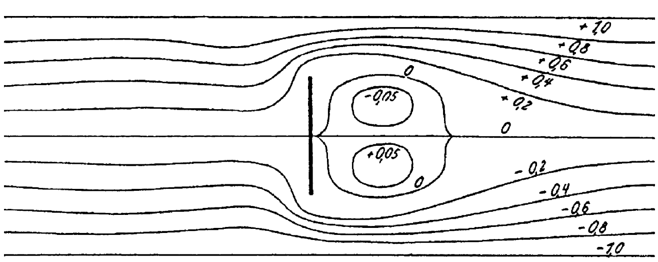

Fig. 2 An example of Hencky's early work (Hencky 1922b) on numerical solutions to field problems. Flow of a viscous fluid around a flat plate at a Reynolds number of 10

become a permanent professorship. Unfortunately this information only related to the University and not to the Technical University and thus his hope for an independent position was not fulfilled. He joined the Department, headed by Professor C.B. Biezeno, situated in the old Mechanical Engineering building which now is an apartment block. J.M. Burgers was also a professor in Delft at the time Hencky was there, and it is from the Burgers Archives that we get some glimpses of Hencky.

Following his move to Delft, Hencky (1922a, 1923a) continued to write on elastic stability theory; the 1922 paper (Hencky 1922a) was a summary of the field. However, in the new environment, possibly under the influence of Burgers and Biezeno, he soon began work on inelastic problems, beginning with plasticity theory and eventually moving on to rheology.

Before continuing with Hencky's contributions, we shall remind ourselves of the state in 1920 of the yet-to-be-named science of rheology, including metal deformation beyond the elastic range (plasticity).

The sciences of classical small-deformation elasticity, Newtonian fluid mechanics and linear viscoelasticity (Boltzmann 1874) were already essentially complete by 1900, and with the rise of polymer-based industries interest was being shown in going beyond the linear regimes, which were already known to be inadequate in many practical situations. Eugene Bingham's book "Fluidity and plasticity", published in 1922 (Bingham 1922), gives us an important reference point. Although the book does not discuss viscosity that varies with shear rate very much, it does contain extensive discussion on slip at solid walls (he concluded that slip did not occur) and on systems with a yield stress. Variable viscosity was already well-known since Schwedoff's (Schwedoff 1890) experimental work.

On the theoretical side, Zaremba (1903) had discovered the need to treat the rate of change of stress carefully, and his work was rediscovered by Jaumann (1905, 1911). Jaumann's works influenced Hencky later on.

In the science of plasticity, which was also to become an important field of activity for Hencky, we note that early work concentrated on predicting how a material would yield under a general stress state. In the familiar tensile test for a metal, the only non-zero stress component is σ (≡ Load W/area A). What happens if a shear stress, or more normal stresses, are also imposed on the sample? For many metals it was found that hydrostatic pressure had no effect on yielding (Hill 1950). H. Tresca (Tresca 1864) had suggested that the maximum shear stress caused yielding. R. von Mises (von Mises 1913) had suggested another criterion, involving the six stress components. The Tresca and so-called von Mises criteria give quite similar results for many metals (Chakrabarty 1987). M. Levy (Levy 1871) and R. von Mises (von Mises 1913) had used total strain (that is, elastic plus plastic strain) in constructing constitutive relations connecting stress and strain but an acceptable set of such relations for plastically deforming metals was not at hand in 1920, and was clearly an active area of research, especially in Germany.

Hencky (1923b) is perhaps his most famous paper. In it he studied statically determined cases of a rigid-plastic body, in the plane case, using the Tresca (maximum shear stress) yield criterion. Several theorems on slip-lines, now named after Hencky, were produced. In these problems Hencky showed how to satisfy the equations of equilibrium and the yield criterion; there is no mention of deformation in such problems, and so no discussion of a constitutive equation was made. Richard von Mises (von Mises 1925) referred to this "beautiful discovery" of Hencky. This paper is discussed in every book on plasticity theory (e.g. Chakrabarty 1987) and is still cited; we found 80 citations in the Science Citation Index between 1974 and 2000. Several examples of applications of the theory were given in this paper.

In 1924 the First International Congress for Applied Mechanics was held in Delft and Hencky (1924a) read a paper on his deformation theory of plasticity. It provoked considerable discussion, some of a sceptical nature, from Ludwig Prandtl, T. von Karman and others. The full text of this work (Hencky 1924b) was published in the Zeitschrift für angewandte Mathematik und Mechanik (ZaMM) and is still frequently cited (77 citations since 1974) and is found in all texts. In this paper he proposed his energy criterion for yielding, and the result is sometimes called the Hencky-von Mises criterion. For some time Hencky's deformation theory of plasticity found favour for practical applications (Nadai 1950) but one has to recognize that it does not reflect the physics of plasticity accurately, and may only be used in restricted circumstances. At the time it was published it was the only work which tried to incorporate both elastic and plastic general responses, but Prandtl (1924) (two-dimensional case) and finally Reuss (1930) produced the now accepted Prandtl-Reuss constitutive relations soon after. The limitations of Hencky's total deformation theory are spelt out by Chakrabarty (1987). Basically, provided loads and stresses increase monotonically, so that the deviatoric stresses[1] are always in the same ratio to one another, then the Hencky and Prandtl-Reuss theories give the same result. When loading-unloading-reloading cycles are considered, with reloading to a different stress state, the results of the Hencky theory will often be erroneous.

Hencky (1925b) continued to write in ZaMM, (then the leading journal on mechanics) and this 1925 paper set up a general tensorial formulation of his deformation theory of plasticity. However, for rheology it is more significant for a paragraph near the end of the paper

[1]If the stress tensor is denoted by σ, then the deviatoric stress tensor $\mathbf{s} \equiv \sigma - 1/3\ \mathrm{tr}\sigma\ \mathbf{I}$.

(paragraph 4, page 146), in which the idea of a coordinate system embedded in the material and deforming with the material is introduced. Later, Oldroyd (1950) and Lodge (1964, 1974) were much influenced by this suggestion in their formulation of fundamental rheological constitutive relations, and so this paper is a milestone. By contrast, the third 1925 paper (Hencky 1925c) recapitulates his current ideas on plasticity theory, and appears to contain nothing new. Hencky (1925a) looked at rolling, pressing and drawing problems using his deformation theory, and here he also began to think about the relation of viscous fluid theory to plasticity.

The Second International Congress for Applied Mechanics was held in Zürich in September 1926 (Hencky 1927). Hencky's contribution was a strange one – he considered using general tensor calculus, no doubt influenced by Schouten's book (Schouten 1924), to study the elastica, which was a several-hundred-year-old problem. This paper has not been recently cited, as far as we can tell.

In 1928 Hencky began work in a new research area – finite elastic deformations. His paper (Hencky 1928) looks for the form of the finite deformation equations for elastic bodies. Curiously, he did not assume incompressibility, and he also stuck with the classical definition of strain. We recall that it was not until 1948 that Rivlin (Tanner and Walters 1998) managed to solve a significant number of finite strain elasticity problems. Nevertheless, Hencky's paper has been cited 37 times since 1974.

Hencky's first paper in English was published with Professor Biezeno in the Dutch Academy of Sciences (Biezeno and Hencky 1928, 1929). It is a long paper in two parts entitled "On the general theory of elastic stability", of no great significance now perhaps. However, 1929 was a year of great interest in Hencky's life.

Hencky (1929a) introduced into finite deformation elasticity the logarithmic, or so-called Hencky strain ϵ:

$$\varepsilon = \ell n \text{ (final length/original length)} \tag{1}$$

and continued his attempts to define plastic flow models of the deformation type, and models for rubberlike elasticity. The motivation for defining the Hencky strain was the following (Ludwik 1909). Instead of computing a strain increment $d\epsilon$ by using the increment of length dx divided by the original length (x_o), where integration provides the linear result $\epsilon=(x-x_o)/x_o$, one can refer the increase $d\epsilon$ to the current length (x), so that we get $d\epsilon=dx/x$, and hence by integration $\epsilon=\ln(x/x_o)$.

His final paper (Hencky 1929b) from Delft is of great interest to rheologists. A translation of the title is "The superposition law of a finitely deforming elastic continuum capable of relaxation and its importance for an exact derivation of the equations for a viscous fluid in the Euler form". In a long preamble he discusses the need for clarity in forming equations for continua. It seems to have been inspired by reading some work by Reiger (1919) and in fact the paper contains a not-too-easy to follow set of arguments beginning with finite strains (logarithmic strain is mentioned, of course) and proceeding to the formulation of a Maxwell-type model, but including finite elastic strains.

If we take Hencky's Maxwell-type superposition, we have, in his notation, a co-rotational type of stress derivative (p. 627 of his paper):

$$\frac{\delta\sigma_{mn}}{\delta t} = \frac{\partial\sigma_{mn}}{\partial t} + v_i\frac{\partial\sigma_{mn}}{\partial x_i} + \sigma_{ni}w_{im} + \sigma_{mk}w_{kn} \tag{2}$$

where σ_{mn} are the stress components, v_i are the velocity components and the vorticity tensor w_{mn} is defined here as (p. 624)

$$w_{mn} = \frac{1}{2}\left(\frac{\partial v_n}{\partial x_m} - \frac{\partial v_m}{\partial x_n}\right). \tag{3}$$

The Maxwell-type model is then (p. 628)

$$\frac{\delta\sigma_{mn}}{\delta t} = \phi_{mn} - \frac{\sigma_{mn}}{T} \tag{4}$$

where T is the relaxation time and ϕ_{mn} is a function of various quantities:

$$\phi_{mn} = \phi_{mn}\left(v_i, \frac{\partial v_i}{\partial x_k},\ \sigma_{mn}, \frac{\partial\sigma_{mn}}{\partial x_i}\right) \tag{5}$$

If we let

$$\phi_{mn} = G\left(\frac{\partial v_m}{\partial x_n} + \frac{\partial v_n}{\partial x_m}\right) \tag{6}$$

be a viscous term, where GT is the viscosity (η) then Eq. (4) becomes

$$T\frac{\delta\sigma_{mn}}{\delta t} + \sigma_{mn} = GT\left(\frac{\partial v_m}{\partial x_n} + \frac{\partial v_n}{\partial x_m}\right) \tag{7}$$

which is a co-rotational Maxwell model. Clearly, as $T\to 0$ but $GT\to\eta$, we return to viscous fluid flow. No questions of objectivity of ϕ_{mn} were explicitly discussed.

Let us compute the response in a steady simple shearing flow. Hencky did not do this, perhaps because he was concerned with the compressibility of the material, in which he set the pressure equal to $-1/3\ \sigma_{ii}$.

We set $v_1=\dot\gamma x_2, v_2= v_3= 0$, a simple shearing flow.

The components of the vorticity tensor w_{mn} are all zero except $w_{12}=-\dot\gamma/2$ and $w_{rm21}=\dot\gamma\big/2$. Solving, we find

$$\sigma_{13} = \sigma_{23} = \sigma_{33} = 0$$
$$\sigma_{12} = GT\dot\gamma/\left(1 + T^2\dot\gamma^2\right)$$
$$\sigma_{11} = -\sigma_{12}\dot\gamma T, \sigma_{22} = -\sigma_{11}.$$

Except for the minus signs, this is the response expected for the co-rotational Maxwell model. We believe Hencky's equations are in error, since this is intended to be a co-rotational model (see for example Chakrabarty 1987, p. 33, Eq. (59) and p. 42, Eq. (81)). A change in the definition of w_{mn} (to $-w_{mn}$) would leave the results unchanged except that now we would get the familiar result

$$\sigma_{11} = \sigma_{12}\dot{\gamma}T = G(T\dot{\gamma})^2/\left(1 + T^2\dot{\gamma}^2\right)$$

$$\sigma_{22} = -\sigma_{11}$$

and

$$N_1 = \sigma_{11} - \sigma_{22} = 2G(T\dot{\gamma})^2\Big/(1 + T^2\dot{\gamma}^2) \quad (8)$$

$$N_2 = \sigma_{22} - \sigma_{33} = -N_1/2. \quad (9)$$

Without this correction the Hencky model does not fall into the general Oldroyd model category (see Tanner 2000, p. 157) and is not objective. Subsequent work by Fromm (1933, 1947), which built on Hencky's results, gave the correct corotational results (Eqs. 8 and 9); see Tanner and Walters (1998).

Hencky in this important paper was influenced by his 1925 work on deformation and by his work on buckling (Biezeno and Hencky 1928, 1929), leading to the co-rotational ideas, but he was also preoccupied, as was Fromm (1933, 1947), with the volumetric response. Hencky, followed by Fromm, set the pressure $p = -1/3\ \sigma_{ii}$. Researchers do not seem to have realized, in these early days, that the pressure is determined, in an incompressible medium, by the momentum balance, not by the constitutive equation.

In spite of the lack of examples and the errors detected above, this paper is a milestone in rheology, leading ultimately to Oldroyd's work (Oldroyd 1950) on convected derivatives. It is not very widely known (12 citations since 1974). Hencky sent this paper to the Annalen der Physik on 7 July 1929, and then departed for the United States. Clearly his research career was going well at Delft, but he was 44 years old and not yet a professor, and one needs to understand his situation to see the motives for leaving Delft. By 1929 it was apparent that relations between Hencky and Biezeno were not good and that Hencky was not happy with his position in the Department at Delft.

Biezeno was told that Hencky wanted leave for a year but later he found out that Hencky intended to stay in the U.S.A. In a letter to Burgers he states that he "has heard lately that Hencky can stay in the U.S.A. Let us hope so!"

On the 26th July 1929 Hencky left Delft for the USA where his family joined him. In June 1930 he took up the position of Associate Professor at the Massachusetts Institute of Technology (MIT) in Cambridge, Mass.

According to the administrative archives Hencky resigned from his position at Delft on the 1st of July 1931.

Life in America

From June 1st 1930 to June 1933 Hencky was Associate Professor of Mechanics in the Department of Mechanical Engineering at MIT. A photograph of him (courtesy of MIT Museum) at that time is shown in Fig. 3. The MIT President's report 1929–1930 states: "Professor Hencky, who came to the Institute a year ago has spent the greater part of his time on an investigation of the plastic flow of material under stress and on lectures in Rheology to a class of graduate students". The 1930–1931 MIT Directory of Officers and Students states: The following additions have been made to the faculty – Dr Heinrich Hencky: Associate Professor of Mechanics. While at MIT Hencky taught what was, we believe, the first ever course in rheology (at that time a year-long graduate elective course in Course II (Mechanical Engineering) numbered 2.341). The course description, taken from the MIT course catalogue of 1931, is as follows:

2.341, 2.342. Rheology (A).

A study of the science of the flow of matter, especial attention being given to the relations between experimental results and theory. The theory is developed as far as possible to meet the needs of the research engineer. Examples taken from the theories of hydrodynamics, elasticity and plasticity are given to illustrate the general principles underlying the laws of the flow of matter. A special study is made of the behavior of semi-elastic and semi-plastic fluids of metals at high temperatures used in forging and in welding and in the rolling mill, as well as the behavior of materials under forced vibrations, the fatigue of metals and frictional resistances in such bodies.

The course taught by Hencky was listed in the 1930, 1931 and 1932 catalogues at MIT.

Hencky's MIT office was Room 1–321, now used as the Mechanical Engineering student office for graduate

Fig. 3 Professor Dr. Heinrich Hencky about 1930, at MIT. (Courtesy MIT Museum)

students in Mechanics and Materials. Information about the theses he supervised and co-supervised was found in the catalogue and on microfiche in the MIT library. Hyman Friedman's (1931 B.S.) thesis entitled, "The law of elasticity of rubber" was supervised by H. Hencky and H.W. Hayward. H.W. Hayward the co-supervisor was Professor of Materials and Engineering and Assistant Director, Division of Industrial Cooperation and Research. Theodore Pian's 1931 thesis "Yield conditioning of plates and shells by Mises-Hencky Criterion" and Robert Conrad's 1932 "Stress field of a plate reinforced by a longitudinal guide and subject to tension" were also supervised by Hencky.

Hencky had a difficult position as a basic scientist at MIT in a department mainly interested in very practical problems. President Stratton, who had supported his candidature, died and in 1932 Hencky was made redundant during a reorganization. He started a business as a consultant engineer, living in Lisbon, New Hampshire. As a consultant, Hencky travelled a lot. On a temporary basis and only for a short time he found employment at Lafayette College in Easton, Penn., courtesy of E.C. Bingham.

We now consider his American-based work.

By 1929 the word "rheology" had been invented by Bingham and his colleagues at Lafayette College, and Hencky began to study finite deformations in rubber elasticity and publish in the Journal of Rheology, beginning in April 1931. His paper (Hencky 1931) deals with finite deformation, and, naturally, explains the logarithmic strain. He continued to assume a constant shear modulus for high deformations, but the compression (bulk) modulus was allowed to be more complex. He was in contact with Percy Bridgman at Harvard who was doing experiments under high pressures and Hencky fitted Bridgman's experimental data with a bulk behaviour rule. In the Summary of his 1931 paper, Hencky says that he has developed a law of elasticity which "is theoretically incontestible". A lively discussion ensued. Dr Karrer asked about logarithmic strain, and Hencky replied that "the logarithmic measure of deformation was already used by the technologist, Ludwid (sic). I think everybody comes to the same idea automatically". The Ludwid reference is to Ludwik (1909). Mr Peek asked about treating shear, but Hencky never discussed what happens to the shear strain in the logarithmic case, and he perhaps wisely avoided the question (or it avoided him).

Corresponding to the logarithmic principal strains ϵ_1, ϵ_2, ϵ_3, one can rotate to non-principal axes and observe what the shear components then become – clearly the result is not simple, and this represents one of the weaknesses of using the logarithmic formulation for general mechanics problems. This paper has had 10 citations since 1974.

Another paper in the Journal of Rheology (Hencky 1932b) on hardening in polycrystalline metals followed; the log strain is again in evidence. Hencky (1932a) was published in the prestigious Philosophical Magazine and, inspired by Bridgman's work, it discusses wave propagation in materials under variable pressure, with particular concern for the increase of pressure as one descends into the Earth's interior. The log strain again crops up.

Returning to rubber mechanics, a paper (Hencky 1933c) was published in the Journal of Applied Mechanics (an American Society of Mechanical Engineers journal, begun in 1933). Again we see the log strain, but it is really unnecessary since it mainly appears as e^{ϵ}, which simply gives the stretches. Some experiments were given, data were fitted using a Mooney-like equation, and some discussion is reported. One of his MIT colleagues (Professor William Hovgaard) was sceptical about using rubber for steam locomotive tyres, and asked whether this had actually been done (it had not, but eventually the Paris Metro began to use some rubber-typed stock, but not for steam locomotives). He also said that Hencky had "a (mathematical) development which is somewhat difficult to follow ...". We believe Hencky waffled somewhat in reply. He was somewhat sharp to other discussers, saying that "... in reading their discussion I could not only not detect a proof of this statement, but also no relation to my own work". This was in reply to a long discussion by Karpov and Templin (from the Aluminum Company of America). In another ASME paper (Hencky 1933a) he returned to plasticity and creep of metals, and a Maxwell-type model is introduced, plus the unavoidable log strain. No discussion was reported. Hencky (1933b) discusses the behaviour of vulcanized rubber. A semi-popular article on stresses in rubber tyres (Hencky 1935) in the ASME magazine Mechanical Engineering, which naturally contains the log-strain again, and which is, in our view, inconclusive, completes the scientific publications of Hencky in the U.S.A. Note that, by 1935, he was living on a farm at Lisbon, New Hampshire, and was no longer with MIT.

Hencky, formerly of MIT, was listed as a member of the advisory board of the Philosophy of Science quarterly published by the Philosophy of Science Association. The first issue appeared in January 1934.

The Russian years

We have some information about the years 1936–1937. Before leaving the United States, Hencky was in a difficult position as he had no fixed job. He was unable to find work in Germany and then an invitation came in 1935 from Galerkin of the Russian Scientific Academy in Moscow. They had become acquainted at the International Conference on Mechanics held in Delft in 1924 and Galerkin had shown an interest in Hencky's work.

As he had no other possibilities, considering the political climate, he stated in his 1938 CV that he was forced to take up the proposal. The proposition was made for a professorship in Technical Mechanics at the Kharkov (Charkow) Chemical Technical Institute and also a leadership role in various research institutions. He led the scientific teaching and activities of the researchers in the Institute for Civil Engineering and was in two other institutions as well. He was advisor to the active material testing then under the Scientific Institute for Rubber Testing in the Mechanics Institute of Moscow University, with A.A. Ilyushin.

Hencky's scientific work made a successful takeoff in Russia but his ordinary life was too difficult to bear and he stated that he soon felt a prisoner in his location. In 1938 he and his family were told, with 24 h notice, to leave Russia.

The Mainz years

After the Henckys left Russia they were met in Germany by his brother Karl, who had a high position in Leverkusen at IG Farben. The Henckys stayed two months with brother Karl who helped obtain a position for him at the MAN (Maschinenfabrik-Augsburg-Nürnburg) Company in the Gustavsburg district near Mainz, under the direction of Dr. Richard Reinhardt (Superintendent of MAN Gustavsburg). In the Foreword of his 1951 book (Hencky 1951a) Hencky thanked Reinhardt and the leaders of the MAN company for the welcome support of his research.

After his American and Russian residencies he was clearly regarded as potentially politically unsound by the Nazi party in Germany and a document in the MAN archives was annotated by the local SS unit to say that he was not to be given access to secret data. He was critical of the Nazi regime and joined no political party. He seems to have survived because his supervisor at MAN (Dr. Richard Reinhardt) was able to argue that Hencky was a very valuable technical person.

Hencky began work with MAN on 1st January 1938. His children left that year for the U.S. where they settled. Hencky was given a position with employment in various MAN departments working on specialist problems in the area of statics, dynamics and materials engineering. He was also asked to lecture in the continuing education and training program of the company and to work on specialist articles for publication in journals. In 1941 he was promoted to the position of Chief Engineer. In 1943 he received an Honorarium for a publication published in Der Stahlbau (Hencky 1943).

Hencky worked at MAN for 13 years retiring on 31st December 1950 but he continued consulting for the company in the following year.

On July 6th 1951 Hencky died in a mountain sports accident; he had been an enthusiastic mountaineer. This information was found in Z Angew Math Mech 31(10):332 (Oct 1951) in the news section. He was 65 years old.

Hencky's scientific output naturally diminished when he was with MAN in Gustavsburg-Mainz, but in June 1941 he submitted a paper on plates and shells to an ASME Applied Mechanics Division meeting in Philadelphia (Hencky 1942). At this time the Second World War was in full swing although the USA was not yet directly involved until 1942. He gives his position only as "Mechanical Engineer, Mainz". The paper is a useful contribution to the energy method for studying critical loads in plates and shells under initial stress. It has been cited several times, and does not refer to logarithmic strain.

His next paper (Hencky 1943) is a short summary of plasticity and contains no new work. At the end of the war (1944) he wrote on shear stresses in flat plates using a virtual work method and he gives an estimate of shear stresses in plates; the work was published about three years after it was received by the editor, (Hencky 1947) and has been highly cited; we found 69 citations since 1974.

The next paper (Hencky 1949) is in English and its title is "Mathematical Principles of Rheology". It is, regrettably, poor in terms of organization, explanation and content, and we simply quote Clifford Truesdell's review (Truesdell 1950): "Author obtains "universal equations of rheology". His claim that paper "demands from reader some concentrated penetrating thinking" is substantiated by scarcity of logical definitions and reasoning, and by superfluity of confusing misprints. The notation changes in middle of paper; reviewer uses author's second system here. After an incorrect statement of history of finite strain theory, author introduces "projective strain and rotation" e_{ij} and w_{ij}, which are respectively the symmetric and skew parts of $v_{ij}+c_i v_j$, where v_i is velocity vector and c_i is an unknown vector. If parameter t used in definition of v_j is time, e_{ij} and w_{ij} have the dimension of rates. Author's general dynamical equations are of incremental or rate type, although in treating case of simple extension he unexpectedly interprets e_{ij} as his earlier logarithmic finite strain measure. The vague considerations motivating these proposals are not understood by reviewer."

Hencky says, in summary "Elasticity and hydrodynamics are now connected by the same set of equations and form a whole, as was anticipated by MAXWELL and corroborated by the founders of the Society of Rheology".

He also states his indebtedness to R. Reinhardt, the Superintendent of MAN-Gustavsburg, for sponsoring his research. A German summary of the ideas in this paper was published posthumously (Hencky 1951b).

He sent a book manuscript (title Neuere Verfahren in der Festigkeitslehre – New Pathways in Solid Mechanics) to the publisher in 1943. Unfortunately, most of the manuscript was burnt in a fire in that year, and only the remaining fragment was published in 1951 (Hencky 1951a. Again he thanks the direction of MAN-Gustavsburg for their support. The book was produced under great difficulties (especially a paper shortage) and it mainly deals with elastic problems. Daniel Drucker (Drucker 1951) reviewed it for Applied Mechanics Reviews as follows: "Many typographical errors occur, and at times rather sweeping claims are made and difficult points passed over. These difficulties are undoubtedly due to the circumstances under which the book was published".

The legacy of Heinrich Hencky

As a researcher in mechanics, Hencky's career was disrupted by two world wars, the Russian revolution and the 1930 economic depression. His academic career (Delft, MIT) spanned only about 14 years, mostly at Delft, which was undoubtedly his most creative period. He worked (alone) in many branches of mechanics, but the ones we are most concerned with may be summarized as follows:

1. Elasticity. This section includes his initial work on plates, and subsequent work on buckling of rings and shells. As mentioned, he was an expert in this area and citations of his work continue to be made. The circular ring buckling load problem and the estimation of shear in plates are prime examples of his work.
2. Plasticity. His 1923 paper on slip line theorems continues to be heavily cited. It is not clear why he did no further work on this new area, which ultimately led to practical solutions to many rigid-plastic flow problems (see, for example, Hill 1950). His deformation theory of plasticity also continues to be cited, even though it is not a model that can be used in complex loading paths. It was, when invented and soon after, used in practice to solve some problems.
3. Finite deformations in materials. He discovered the virtues of the logarithmic strain measure in 1929 and championed it thereafter. He says, in the discussion of his 1931 Journal of Rheology paper, that P. Ludwik invented this measure in about 1909. While Hencky never discussed how to deal with shear in the logarithmic strain (wisely!) he may claim to have been the foremost user and exploiter of the concept, and so perhaps the present embedded nomenclature "Hencky strain" is not inappropriate.
4. Rheology. Aside from his work on elasticity and plasticity, three of his papers (and one posthumous abstract) are explicitly concerned with relaxing materials. His 1925 paper on convected coordinates and his 1929 paper, although not highly cited now, have influenced rheologists via the work of Oldroyd (1950) and Lodge (1974) following Zaremba (1903) and Fromm (1933, 1947).

One must admit that his life was not easy – the early Russian internment and later problems with the Soviets, his teaching problems in Dresden, his difficulties with Biezeno in Delft, and ultimately his loss of the MIT position, need to be borne in mind. Despite these troubles, his scientific achievement was outstanding. His sponsorship by Dr Reinhardt at MAN kept him going for more than ten years, but again luck was not with him and most of his book manuscript was destroyed by fire in 1943. His death in a mountain-sports accident in July 1951 was an unexpected end to a notable career in mechanics and rheology.

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