

W. Nowacki

THE THEORY OF ASYMMETRICAL ELASTICITY*

Reviewed by G. N. Savin and Yu. N. Nemish

The monograph is a result of studies based on the model of a continuous medium which W. Voigt had proposed in 1887 for analyzing the elastic properties of crystals. The essence of this model is that spin interaction is assumed between adjacent elements of the medium, in addition to the regular center-to-center interaction. This, in turn, gives rise to momental stresses – besides the regular stresses – and, moreover, the regular stress tensor becomes asymmetrical. Hence its name is: The Theory of Elasticity with an Asymmetrical Tensor, or The Theory of Asymmetrical Elasticity.

The further generalization and development of this theory is due to E. Cosserat and F. Cosserat (1909). For this reason, in modern scientific literature is often referred to as the model of a Cosserat medium. For a long time, however, these contributions remained forgotten. Only during the last decade has there been considerably more work on the theory of asymmetrical elasticity, as is evidence in this book.

The monograph consists of four chapters.

Chapter I deals with the general premises and statements concerning elastokinetics. The sources quoted here have served as the basis for developing the theory of asymmetrical elasticity. It is pointed out, specifically, that applying the classical theory of elasticity results in considerable discrepancies with experimental data when the pressure gradients are high, when wave propagation in crystals is considered, etc.

According to the theory of asymmetrical elasticity, every point in the continuum is treated as an infinitesimal element capable, under the influence of external forces, of deforming and orienting itself with respect to a specific system of coordinates. This deformation is generally described in terms of two independent vectors: the displacement vector \vec{u} and the small-rotation vector $\vec{\varphi}$. It is to be noted that several studies had been conducted concerning the Cosserat pseudocontinuum, where the relation $\vec{\varphi} = 1/2 \text{curl } \vec{u}$ was assumed valid. This variant of the theory, apparently due to R. A. Tupin, is in the scientific literature often called the Cosserat theory with restrained rotation.

It seems significant that reference is made in the book to a satisfactory agreement between experiment and theory in the case of discrete media (of the spatial lattice type), where all elastic constants can be determined. This has been investigated by S. Kaliski (1963), W. H. Hoppmann and F. O. Shahwah (1965), and A. Askar and A. S. Cakmak (1968). In the extreme case of transition to a continuous medium one may arrive at the Cosserat model.

Consideration is also given to the restrictions (limits of variation) to which the elastic constants of materials must be subject. We will note, in connection with this, that certain constants of the Cosserat continuum are already known now for some real materials [1].

The equations of equilibrium and of motion are derived, the basic relations and equations of elastokinetics as well as the continuity conditions are analyzed. It has been assumed here that the medium is elastic, homogeneous, isotropic, and has center symmetry (center symmetry is defined as the invariance of the elastic properties of a material with respect to an inversion of the coordinate system).

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The general principles of the asymmetrical elasticity theory are outlined: the principle of energy conservation and the principle of entropy balance, the principle of virtual work, the Hamilton principle, also the reciprocal work theorem and the extended Somigliano theorem.

By some analogy to the classical theory of elasticity, stress potentials and functions are introduced in a certain form and their completeness is proved. Furthermore, the problem concerning the uniqueness of the solution to the equations of asymmetrical elasticity is duly treated here.

Chapter II deals with specific problems in elastokinetics and, in particular, with the theory of elastic wave propagation taking into account the stress tensor asymmetry. Specifically, monochromatic plane waves are considered in an elastic space and half-space, a spin wave and a longitudinal wave in an infinite elastic space, and also problems concerning the generation of waves in an infinite micropolar space.

Consideration is given to the methods by which the fundamental solutions are derived for an infinite elastic space. The problem is reduced here to obtaining the Green functions for the case of micropolar elasticity. Special higher-order solutions are also shown.

The two-dimensional and the axially symmetrical Lamb problem are analyzed for the case of an elastic half-space, as also are the problems of wave propagation in an elastic plate and in an infinitely long beam of circular cross section.

Static problems in the theory of asymmetrical elasticity are outlined in Chapter III. The basic relations and equations are derived from the formula for free energy. The theorems on the uniqueness of solutions, the reciprocal work theorem, the Somigliano theorem, the minimum potential and the complementary energy theorems, and also the extended variational E. Reissner theorem are all stated and proved. Elastostatic equations in displacements and rotations are derived, considering their fundamental solutions in an elastic half-space.

Separate attention is given to such important practical problems as two-dimensional deformation and a two-dimensional stress field.

Finally, the generalized Galerkin and Lyav functions are derived. Particular attention is paid here not only to the vector notation but also to the representation of displacements as well as regular and momental stresses in cylindrical coordinates.

Chapter IV is devoted to problems in thermoelasticity. The basic equations and relations of dynamic thermoelasticity are derived for an isotropic center-symmetrical medium with momental stresses. Also the basic theorems of thermoelasticity (variational theorems, the reciprocal work theorem) are extended to a medium describable by two vectors: the displacement vector and the small-rotation vector.

A solution is given to the problem of thermoelastic stress potentials and functions, based on the asymmetrical elasticity vector equations in displacements and rotations with volume forces and moments taken into account. Then, the displacement and the rotation vectors as well as the volume forces and moments are represented in terms of potential and solenoidal components, and a system of equations which the said functions satisfy is set up. The equation of thermoelasticity is then added to this system, which makes it possible now to analyze the propagation of elastic waves.

In the same chapter there can be found an extension of the V. M. Meisel formulas, by which the displacements and the small rotations of a body can be determined, if displacements, rotations, and the temperature change are given on one part of its surface while surface forces, moments, and the temperature are given on the remaining part. The fundamental solutions to the thermoelasticity problem and certain types of solutions applicable to micropolar thermoelasticity are examined here.

The two-dimensional problems in the theory of asymmetrical elasticity are formulated for the case of constrained thermoelasticity. The elasticity relations, the equations of motion, the stress functions, and the conditions of compatibility are established here. Thermal stresses produced by a rupturing temperature field as well as composite boundary conditions applicable to thermoelasticity are dealt with.

Thus, the essential purpose of this book is to discuss the general problems concerning the mathematical theory of asymmetrical elasticity: to formulate the basic assumptions, principles, and theorems, then to derive the determining equations and relations. However, there are only a few specific examples given and little is said about methods of handling them. No attention is paid, as should be, to specific procedures

and recommendations for analyzing the mechanical phenomena in real bodies and structures, and the anticipated effects of these phenomena are not analyzed. The problems that have been just touched upon would, together with experimental data, define the limits of applicability of the theory and would bring it closer to engineering practice.

These comments apply generally to the overall present status of the research work done in asymmetrical elasticity and are not meant to detract from the merits of this book. The extensive bibliography (over 200 entries) referred to throughout the text and listed separately at the end gives the reader a good idea of how the theory of elasticity with an asymmetrical stress tensor evolved historically and of what results it has produced.

On the whole, W. Nowacki's monograph, *The Theory of Asymmetrical Elasticity*, is written on a mathematical level which is quite up to date and it deserves high praise.

LITERATURE CITED

1. G. N. Savin, A. A. Lukashov, E. M. Lysko, S. V. Veremeenko, and G. G. Agas'ev, "Propagation of elastic waves in the Cosserat continuum with restrained rotation of particles," *Prikl. Mekhan.*, 6, No. 6 (1970).

A. D. Kovalenko

FUNDAMENTALS OF THERMOELASTICITY*

Reviewed by A. Yu. Ishlinskii

A common feature of modern research on the mechanics of continuous media is the concern about mechanical phenomena in relation to their external effects. This trend is of considerable scientific interest and practical significance in such areas as thermoelasticity, magnetohydrodynamics, and electro- and magnetoelasticity. Thermoelasticity, as an extension of the elasticity theory to nonisothermal deformation, has in recent years attained a high level of recognition in connection with important engineering problems.

In this country and abroad a great deal has been published on the subject of thermoelasticity. At the same time, in the Soviet literature there has been no work providing a much-needed correlation of all the many individual contributions to this trend of research in the mechanics of deformable bodies.

The appearance of A. D. Kovalenko's monograph, *Fundamentals of Thermoelasticity*† fills this gap and represents an important milestone in the development of the theory and the methods of thermoelasticity in this country.

The rather small volume contains a creatively conceived generalization of the results of thermoelasticity studies reported in the world's technical literature and explains the basic conclusions arrived at by Soviet scientists, particularly by the author's co-workers.

Following the modern trend in the development of phenomenological theories on the mechanics of continuous media, the author presents the theory of thermoelasticity based on the laws of the thermodynamics of irreversible processes and on the laws of classical mechanics. This has allowed him to take a unified point of view in the analysis of the mechanical and the thermal processes occurring and interacting in deformable solid bodies. In deriving the basic equations of constrained dynamic thermoelasticity, it is not assumed here as is usual that the temperature rise is small relative to the initial temperature level. Consequently, the theory of thermoelasticity, for small deformations, becomes more general and comprises the linear theory of constrained thermoelasticity as well as the theory of unconstrained thermoelasticity for large thermal perturbations, using the linear equations of motion and the nonlinear equations of heat conduction. Within this framework, the basic problems of thermoelasticity with small and with large thermal perturbations are defined in this monograph more precisely.

According to the author's classification, the problem most important in practice is that of quasistatic thermoelasticity, which describes the deformation of a solid body without the inertia terms in the equation of motion and without taking into consideration the coupling between deformation and the temperature field. A sizable portion of the monograph deals with quasistatic problems of thermoelasticity. Known results based on the theory of two-dimensional isothermal thermoelasticity, on the theory of plates and shells, and on the three-dimensional axially symmetrical thermoelasticity problem are all extended here to include thermal effects. Much space is devoted to formulating the problems of heat conduction, to the methods and to the analysis of their solutions – as the first step in the analysis of thermal stresses.

Of distinct originality is the presentation of the now most highly developed aspect in the theory of quasistatic thermoelasticity – the two-dimensional problem. The uniqueness conditions for displacements

*Naukova Dumka, Kiev, 1970.

†The monograph is a revised and enlarged edition of the book published in Russian earlier (A. D. Kovalenko, *Introduction to Thermoelasticity*, Naukova Dumka, Kiev, 1965) and translated into English in 1969.

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are more precisely analyzed here in the case of multiply constrained regions, taking into account the characteristic aspects of the two-dimensional thermoelasticity problem, the conditions for a steady-state two-dimensional field producing no stresses in a multiply constrained body are derived in a simpler form, and the dislocation analogy is stated more rigorously.

The analysis of quasistatic thermoelasticity problems in plates and shells is preceded by a discussion and formulation, in terms of the thermoelasticity problems, of the basic premise in the theory of thin plates and shells concerning the invariability of a normal element. It is shown that introducing generalized purely thermal deformations makes it possible to reduce the problem of thermoelasticity in a thin-walled structure to the corresponding problem in isothermal theory. In this way, the author is able fully to utilize his earlier results for developing the theory of stress calculations in circular plates and in shells of revolution.

One of the chapters in this monograph deals with axially symmetrical thermoelasticity problems. The conciseness of the presentation and the rigorous selection of material make for a clear definition of specific problems in axially symmetrical thermoelasticity and for an adequately complete characterization of methods applicable to their solution.

The analysis of thermoelastic stress fields in elastic bodies becomes quite complicated by the very nature of the temperature effects. The essentially three-dimensional character of temperature fields in modern structure components limits the feasibility of using simplified approaches in the analysis of their thermal stress intensity.

Of interest within this framework are the specific examples considered in the book, namely a cylinder of finite length and a thick-walled spherical shell. The quantitative estimates of stress intensity in these problems do quite comprehensively illustrate the particular nature of axially symmetrical thermoelasticity problems mentioned earlier and the feasibility of applying the proposed methods to their solution.

Much attention is given in this monograph to the dynamic thermoelasticity problem. Based on the theory of unconstrained dynamic thermoelasticity, the effects of inertia forces during thermal pulse loading are examined for two typical cases: thermal shock at the surface of an elastic half-space and at the surface of a circular plate. The latter case, which is a problem in the transverse vibration of plates, has great practical importance on account of the considerable dynamic effects here. The analysis of dynamic problems is accompanied by an estimate of the effect which a finite rate of change of thermal forces has on the magnitude of the dynamic parameters. This provides a useful basis for evaluating the practical necessity of including the dynamic effects in calculations, depending on the geometry of a structural component and on its rate of heating.

The concluding chapter of the monograph deals with the coupling between thermal and mechanical processes in elastic bodies. Analyses are presented of the problem of thermal perturbations of an elastic layer due to a momentary application of surface forces, and the problem concerning the propagation of harmonic thermoelastic expansion waves in an infinite medium and thermoelastic Rayleigh and longitudinal waves in an infinitely long cylinder.

The results shown here present a good qualitative and quantitative idea about the effect of coupling between the processes and they are meaningful when the accuracy of respective simplified theories of thermoelasticity is to be evaluated.

The presentation of general solutions in the Galerkin–Papkovitch form and also the application of the Lagrange variational principles extended to constrained dynamic thermoelasticity are quite useful for solutions of specific problems.

The examples where the thermal stress distribution in elastic bodies and the thermomechanical effects are considered in relation to the nature of the thermal forces are markedly oriented toward power-apparatus design practice.

In summarizing, we note that the book adds a significant contribution to the theory of thermoelasticity, it is concise but, nevertheless, its content is lucid and purpose-oriented, and it should assist in the development of effective methods for thermoelasticity analysis necessary for the design of property and reliably functioning modern engineering structures.