Conference on Trends in Analysis 2023

University of Duisburg-Essen
LA 013,
Lotharstraße 65, Campus Duisburg
October 04 - 06, 2023

Invited Speakers:

Lars Diening (U Bielefeld)
Franz Gmeineder (U Konstanz)
André Guerra (ETH Zürich)
Jonas Hirsch (U Leipzig)
Filip Rindler (U Warwick)
Bernd Schmidt (U Augsburg)
Thomas Schmidt (U Hamburg)
Anna Siffert (WWU Münster)
Jean van Schaftingen (UCLouvain)
Barbara Zwicknagl (HU Berlin)

Trends in Analysis 2023

J.Hirsch

udue.de/AnaTrends23

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About

Conference on Trends in Analysis 2023

We aim to bring together experienced scientists and researchers and young, up-and-coming scientists around the trending topics in Analysis.

The conference will take place on the Duisburg campus of the University of Duisburg-Essen.

It is organized by Peter Lewintan and completely funded by his PostDoc prize 2022 of the faculty of mathematics of UDE.

The electronic version of this booklet can be found at:
https://www.uni-due.de/mathematik/analysis.trends.2023.php

The open-source LaTeX template, AMCOS_booklet, used to generate this booklet is available at
`https://github.com/maximelucas/AMCOS_booklet`

The colors are based on the corporate design of the University of Duisburg-Essen:
`https://www.uni-due.de/offen-im-denken/farben.php`

The cover poster was designed by Anja Schulte.

Timetable

CT: Contributed Talk, IS: Invited Speaker.

Wednesday, 04 of October

8:30–8:50	**Registration**		
8:55–9:00	**Welcome remarks**		
9:00–09:50	IS	**Franz Gmeineder** U Konstanz	Quasiconvexity, (p, q)-growth and partial regularity
09:50–10:10	**Coffee break**		
10:10–11:00	IS	**Barbara Zwicknagl** HU Berlin	Variational models for pattern formation: from helimagnets to shape memory alloys
11:00–11:50	IS	**Jonas Hirsch** U Leipzig	On the Lawson-Osserman conjecture on the minimal surface system
11:50–14:00	**Lunch break**		
14:00–14:50	IS	**Thomas Schmidt** U Hamburg	Perimeter functionals with measure data
14:50–15:20	CT	**Jule Schütt** U Hamburg	The optimal Hölder exponent in Massari's regularity theorem
15:20–15:50	**Coffee break**		
15:50-16:20	CT	**Eleonora Ficola** U Hamburg	Lower semicontinuity and existence results for total variation functionals with measure data
16:20-16:50	CT	**Paul Stephan** U Konstanz	KMS inequalities for operators of constant rank
16:50-17:20	CT	**Stefan Schiffer** MPI Leipzig	Extensions of divergence-free functions in L^1

Thursday, 05 of October

9:00–09:50	IS	**Lars Diening** U Bielefeld	Regularity of local and non-local non-linear problems
09:50–10:10	**Coffee break**		
10:10–11:00	IS	**Anna Siffert** WWU Münster	p-harmonic maps
11:00–11:50	IS	**Jean van Schaftingen** UCLouvain	$p \nearrow 2$–harmonic maps on three-dimensional domains
11:50–14:00	**Lunch break**		
14:00–14:50	IS	**Bernd Schmidt** U Augsburg	Atomic chains at low temperature: surface energy, boundary layers and crack distribution
14:50–15:20	CT	**Akram Sharif** U Leipzig	Mathematical foundations of contact interactions in continuum mechanics
15:20–15:50	**Coffee break**		
15:50-16:20	CT	**Ferdinand Eitler** U Augsburg	Higher gradient integrability for bounded BD-minimizers of μ-elliptic linear growth functionals
16:20-16:50	CT	**Linus Behn** U Bielefeld	Boundary regularity for nonlinear systems with symmetric gradients
18:00-21:00	**Conference Dinner at Kartoffel-Kiste**		

Kartoffel-Kiste
Schweizer Straße 105
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Tel.: 0203 333 827
`www.kartoffel-kiste.com`

Friday, 06 of October

9:00–09:50	IS	**André Guerra** ETH Zürich	Stability of the Faber-Krahn inequality for the short-time Fourier transform
09:50–10:10	**Coffee break**		
10:10–11:00	IS	**Filip Rindler** U Warwick	Transport of currents
11:00–11:30	CT	**Olena Atlasiuk** IM NAS Kyiv	The solvability of inhomogeneous boundary-value problems in Sobolev spaces
11:30–12:00	CT	**Richard Schubert** U Bonn	Convergence to the planar interface for the Mullins-Sekerka evolution
12:00–12:10	**Closing**		
12:10–14:00	**Lunch break**		

List of Abstracts – Talks

Wednesday 4th

Quasiconvexity, (p, q)-growth and partial regularity

Franz Gmeineder, U Konstanz

We display new results on the regularity properties of relaxed minimizers of quasiconvex functionals of (p, q)-growth. These results apply to the natural range for which the functionals can be meaningfully extended (or relaxed) and apply to signed integrands as well, thereby extending previous results of Schmidt. Moreover, despite being natural in view of coercivity, signed, quasiconvex integrands allow for different phenomena that are invisible in the convex situation. Specifically, some focus will be put on the non-availability of measure representations à la Fonseca & Maly for the relaxed functionals and, more importantly, why they are not really required for partial regularity. Based on joint work with Jan Kristensen.

Variational models for pattern formation: from helimagnets to shape memory alloys

Barbara Zwicknagl, HU Berlin

In this talk, we follow a variational approach to explain pattern formation in helimagnets. Starting point is a certain two-dimensional frustrated spin system. We discuss a discrete-to-continuum limit (in the sense of Gamma-convergence) at the helimagnetic/ferromagnetic transition point, in the case of incompatible boundary conditions. For the continuum model, we present scaling laws for the minimal energy which indicate the formation of various complex branching-type patterns in certain parameter regimes. We will in particular discuss relations to models for other pattern-forming systems, such as shape memory alloys. This talk is based on joint work with Janusz Ginster and Melanie Koser (both Humboldt-Universität zu Berlin).

On the Lawson-Osserman conjecture on the minimal surface system

Jonas Hirsch, U Leipzig

In the famous Lawson and Osserman paper, non-existence, non-uniqueness and irregularity of solutions to the minimal surface system one finds:

Conjecture 2.1 The system (2.2) and (2.3) are equivalent for any locally Lipschitz function f on Ω.

Where (2.2) is the full minimal surface system

$$\begin{cases} \sum_{i=1}^{n} \frac{\partial}{\partial x^i} \left(\sqrt{g}\, g^{ij} \right) & = 0, \quad j = 1, \dots, n \\ \sum_{i,j=1}^{n} \frac{\partial}{\partial x^i} \left(\sqrt{g}\, g^{ij} \frac{\partial f}{\partial x^j} \right) & = 0, \end{cases}$$

and (2.3) the system involving only the outer variations:

$$\sum_{i,j=1}^{n} \frac{\partial}{\partial x^i} \left(\sqrt{g}\, g^{ij} \frac{\partial f}{\partial x^j} \right) = 0.$$

We are able to answer the conjecture in dimension two in affirmative way. In fact our main result states:

Theorem Let $f : B_1 \subset \mathbb{R}^2 \to \mathbb{R}^n$ be a Lipschitz critical point of the area functional with respect to outer variations, then f is smooth.

Having stated the conjecture and our result I will use the remainder of the time to present ideas to the proof and how 2 dimensions enter.
This is a joint work with Connor Mooney and Riccardo Tione.

Perimeter functionals with measure data

Thomas Schmidt, U Hamburg

IS

The talk is concerned with the parametric variational approach to prescribed-mean-curvature boundaries in $\mathbb{R}^n$, based on the minimization of Massari's functional among sets of finite perimeter. It will be explained that this approach generalizes, to some extent, from L^1 mean curvature to mean curvature given by a possibly lower-dimensional signed measure and then yields various existence results under a new and optimal assumption on the measure, called the small-volume isoperimetric condition. This assumption in turn will be illustrated at hand of examples, will be shown to admit a wide class of $(n-1)$-dimensional measures and will be decisive for lower semicontinuity with cancellation-compensation effects between perimeter and measure terms.

The optimal Hölder exponent in Massari's regularity theorem

Jule Schütt, U Hamburg

CT

The regularity theorem of Massari provides a strong connection between the existence of variational mean curvatures H with good integrability, i.e. $H \in \mathrm{L}^p$ for large p, and the $\mathrm{C}^{1,\alpha}$-regularity of the corresponding hypersurface. This, in fact, is far away from being trivial in view of the lack of continuity of H. The optimal Hölder exponent α_{opt} was yet unknown but Massari conjectured it to converge to 1 as $p \to \infty$. We confirm this conjecture by presenting the exact shape of α_{opt}.

Lower semicontinuity and existence results for total variation functionals with measure data

Eleonora Ficola, U Hamburg

CT

We deal with functionals of the kind

$$\mathcal{F}_\mu[u] := \int_\Omega f(\,\cdot\,, \nabla w)\,\mathrm{d}x + \int_\Omega w^*\,\mathrm{d}\mu \qquad \text{for } w \in u_0|_\Omega + \mathrm{W}^{1,1}_0(\Omega)\,, \tag{F.1}$$

where $u_0 \in \mathrm{W}^{1,1}_0(\mathbb{R}^N)$ determines a Dirichlet condition on bounded Lipschitz domains $\Omega \subseteq \mathbb{R}^N$. The leading term in (F.1) is a functional with linear growth, whereas μ denotes a prescribed finite (signed) Radon measure on Ω, possibly singular with respect to L^N or even lower-dimensional. More specifically, we focus on the anisotropic total variation problem - primarily imposing 1-homogeneity on the function f - and in search of minimizers it comes natural to suitably reformulate (F.1) in the broader frame of BV-functions. We discuss and characterize appropriate measure conditions which balance the interplay between the two terms and enable us to establish lower semicontinuity and hence the existence of minima in $\mathrm{BV}(\Omega)$. A quick overview of some prominent examples will clarify the setting and emphasize the sharpness of our assumptions.

KMS inequalities for operators of constant rank

Paul Stephan, U Konstanz

CT

This talk focuses on the study of Korn-Maxwell-Sobolev (KMS) inequalities for constant-rank operators. Such inequalities generalise inequalities of Korn-type to incompatible matrix fields, and previous work by Gmeineder, Lewintan & Neff essentially has been confined to the framework of elliptic operators. As we discuss in this talk, the introduction of a correction term then makes it possible to establish inequalities of this kind for operators of constant rank too. Joint work with Peter Lewintan.

Extensions of divergence-free functions in L^1

Stefan Schiffer, MPI Leipzig

CT

In this talk, we discuss extension theorems for divergence-free functions. In more detail, the central question may be posed as follows: For which domains can we find a linear and bounded extension operator that maps L^p into L^p and preserves the constraint $divu = 0$? Even for quite simple domains such an extension naturally is challenging in the end-point cases $p = 1$ and $p = \infty$. We discuss the case of Lipschitz bounded sets and the exponent p belonging to those endpoint cases where we can answer above question in the affirmative. This talk is based on joint work with F. Gmeineder (U Konstanz).

Thursday 5th

Regularity of local and non-local non-linear problems

Lars Diening, U Bielefeld IS

We discuss the regularity of the solutions to certain local and non-local versions of the p-Laplacian. We will also talk about the weighted case and counterexamples.

p-harmonic maps

Anna Siffert, WWU Münster IS

We will discuss recent results on p-harmonic maps. Among others, we proved the existence of infinite families of p-harmonic maps between certain spheres. Some of the results presented are joint work with Volker Branding.

$p \nearrow 2$–harmonic maps on three-dimensional domains

Jean van Schaftingen, UCLouvain, Louvain-la-Neuve, Belgium IS

The problem of finding a harmonic map on a three-dimensional domain with a fixed boundary data prescribed as a map from the boundary to a given compact manifold has no solution in general, because topological obstructions force the natural Sobolev space of maps satisfying the constraint on the boundary to be empty. On the other hand when $p < 2$, minimising p–harmonic maps satisfying the boundary condition exist. We study the asymptotics when $p \nearrow 2$ of these maps. When the fundamental group of the target manifold is finite or, equivalently, its universal covering is compact, we prove that the minimisers converge to a singular harmonic map and that the singular set is a union of segment of minimal total weighted length, with weights given by the singular energy of the topological charges of the segments.

This is a joint work with Bohdan Bulanyi (UCLouvain) and Benoît Van Vaerenbergh (UCLouvain).

Atomic chains at low temperature: surface energy, boundary layers and crack distribution

Bernd Schmidt, U Augsburg

IS

We study the low-temperature behavior of a one-dimensional chain of atoms that interact via a Lennard-Jones type potential. In particular: (1) We provide a detailed analysis of the boundary layers and surface energy at temperature $T = 0$. (2) At $T > 0$ and positive pressure $p > 0$ we show that at the Gibbs measures for infinite chains and semi-infinite chains satisfy path large deviations principles and find that the surface correction to the Gibbs free energy converges to the zero temperature surface energy. (3) Deriving sufficiently strong bounds on the decay of correlations, we see that at $T > 0$ and $p = 0$ one obtains clusters whose diverging number and diverging size can be quantified asymptotically.

Mathematical foundations of contact interactions in continuum mechanics

Akram Sharif, U Leipzig

CT

In continuum mechanics the underlying balance laws have to be satisfied not only for an entire body but also for all of its subbodies. Classically these balance laws account for contact interactions between contiguous subbodies exerted across their common boundary. A well known result by Cauchy constitutes that contact interactions with a continuous density function can be represented as the surface integral over the common boundary of the subbodies of the product of the normal vector field times a stress field, called Cauchy's stress tensor. In the past 50 years many extensions of this result have been discovered and recent ones cover the occurrence of special concentrated contact interactions with a suitable collection of subparts of sets of finite perimeter as common boundaries of subbodies. However, at the same time these approaches have not been able to cover the case when concentrations occur directly on the boundaries of subbodies. The aim of the talk is to give a short introduction to the field and present an approach by Schuricht, which retains the advantages of the classical theory and allows a better description of concentrations, resolving some problems of earlier approaches. This is joined work with Friedemann Schuricht.

Higher gradient integrability for bounded BD-minimizers of μ-elliptic linear growth functionals

Ferdinand Eitler, U Augsburg

CT

This talks focuses on regularity theory for bounded generalised minimisers of μ-elliptic linear growth functionals in the space BD of functions of bounded deformation. For this kind of variational problems the latter space is natural, but a priori the symmetric gradient exists only as a matrix valued Radon measure. For generalised minimisers which are locally bounded we establish a higher gradient integrability result for the full range of $\mu \in (1, 3]$. In order to use the available a priori bounds on the symmetric gradients, we must simultanously allow for algebraic manipulations and keep track of the L^∞-constraint. The potential non-uniqueness of generalised minimisers moreover requires these tasks to be compatible with a suitable application of Ekeland's variational principle. This is joint work with Lisa Beck (Augsburg) and Franz Gmeineder (Konstanz).

Boundary regularity for nonlinear systems with symmetric gradients

Linus Behn, U Bielefeld

CT

We study boundary regularity for nonlinear systems depending on the symmetric gradient $\varepsilon u := \frac{1}{2}(\nabla u + \nabla^T u)$. An important example is given by the symmetric p-Laplace system $-\operatorname{div}(|\varepsilon u|^{p-2}\varepsilon u) = f$ for $1 < p < \infty$, where our result is new in the degenerate case $p > 2$. Furthermore, we consider more general systems of Orlicz growth.
Joint work with Lars Diening (Bielefeld University).

Friday 6th

Stability of the Faber-Krahn inequality for the short-time Fourier transform

André Guerra, ETH Zürich, Switzerland

IS

For a given signal, its short-time Fourier transform (STFT) is a measure of its "instantaneous frequency". For generic signals, however, the uncertainty principle says that the concept of instantaneous frequency is not well defined and thus the STFT can only be so much concentrated on a given set of finite, positive measure. The Faber-Krahn inequality for the STFT asserts that the STFT is optimally localized if the localization domain is a ball and the signal is a suitable Gaussian. In this talk we will discuss a recent proof of an optimal, quantitative version of this result, joint with J. Gómez, J. P. G. Ramos and P. Tilli.

Transport of currents

Filip Rindler, U Warwick, United Kingdom

IS

The transport of singular structures, such as vortex lines/sheets in fluids, topological singularities in magnetism, or dislocation lines in plastic solids, can all be seen as fundamentally governed by the geometric (Lie) transport equation

$$\frac{d}{dt}T_t + \mathcal{L}_{b_t}T_t = 0$$

for a time-indexed family of integral or normal k-currents $t \mapsto T_t$ in the ambient space $\mathbb{R}^d$. Here, $\mathcal{L}_{b_t}$ denotes the Lie derivative with respect to the vector field b_t, defined by duality. Written in coordinates, this PDE encompasses the classical transport equation ($k = d$), the continuity equation ($k = 0$), the equation for the transport of lines ($k = 1$), and the advection of membranes ($k = d - 1$). This talk will report on recent progress on the analysis of this equation for arbitrary k, covering in particular existence and uniqueness of solutions, structure theorems, rectifiability, and Rademacher-type differentiability results. This is joint work with Paolo Bonicatto and Giacomo Del Nin.

The solvability of inhomogeneous boundary-value problems in Sobolev spaces

Olena Atlasiuk, IM NAS Kyiv, Ukraine

CT

We study linear systems of ordinary differential equations on a finite interval with the most general (generic) inhomogeneous boundary conditions in Sobolev spaces. These boundary problems include all known types of classical and numerous nonclassical conditions. The latter may contain derivatives of integer and fractional order, which may exceed the order of the differential equation.We investigate the character of solvability of inhomogeneous boundary-value problems, prove their Fredholm properties, and find the indices, the dimensions of the kernel, and the cokernel of these problems. These results are illustrated by examples.

Convergence to the planar interface for the Mullins-Sekerka evolution

Richard Schubert, U Bonn

CT

We consider the evolution of fairly wild perturbations of the plane in three space dimensions by the Mullins-Sekerka (MS) law. MS is a non-local third-order geometric evolution that is characterized by preservation of mass and reduction of surface area. Only assuming initial finiteness of the excess mass and the excess surface energy, we prove that the surface eventually becomes a graph, and that the energy converges with an optimal algebraic rate towards the flat ground state. I will discuss how the energy dissipation structure and the L^1-method for conservation laws enter the proof. Based on joint work with Felix Otto (Leipzig) and Maria G. Westdickenberg (Aachen).

List of Participants

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Linus Behn (U Bielefeld), CS
Dominic Breit (TU Clausthal)
David Buchberger (U Augsburg)
Lars Diening (U Bielefeld), IS
Ferdinand Eitler (U Augsburg), CS
Eleonora Ficola (U Hamburg), CS
Tobias Friesel (UDE)
Lukas Fußangel (UDE)
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André Guerra (ETH Zürich), IS
Jonas Hirsch (U Leipzig), IS
Vanessa Hüsken (UDE)
Christopher Irving (TU Dortmund)
Mark Jagalski (UDE)
Peter Lewintan (KIT), Organizer
Robert Martin (UDE)
Kristian Moring (UDE)
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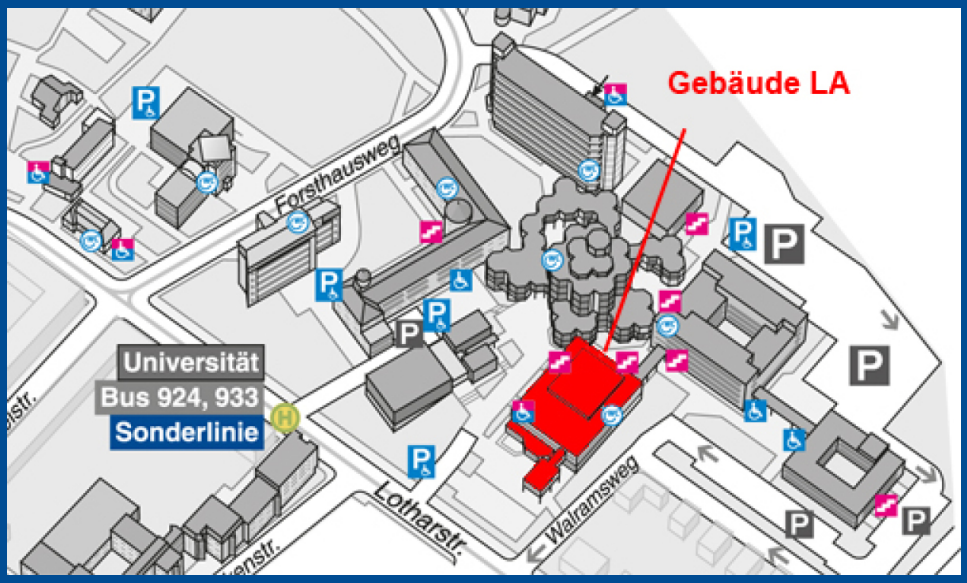
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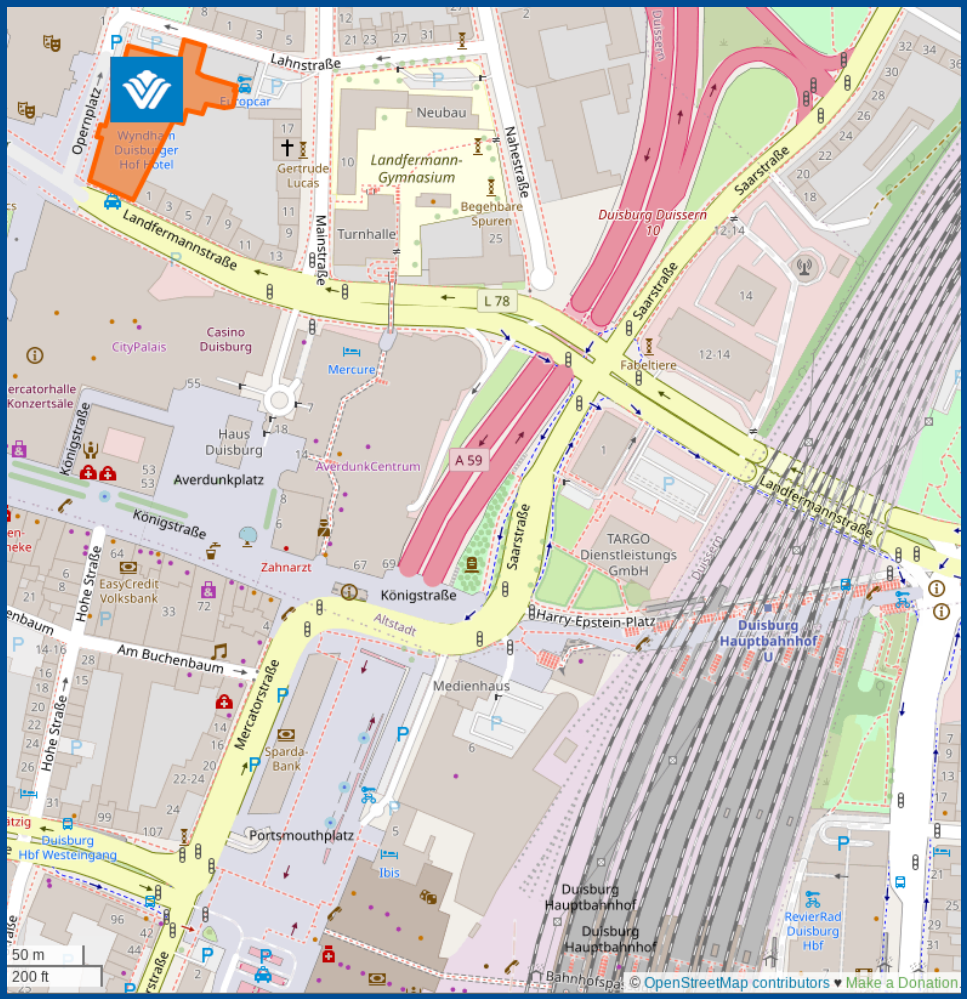
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