Universität Duisburg-Essen
Computational Mechanics
Fakultät für Mathematik

Summer Term 2012
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Advanced Numerical Methods. Exercise 10.

Exercise 1: (8 + 4 = 12 Points)
Compute the regions of absolute stability for the schemes

1. Explicit Euler
2. Implicit Euler
3. Modified Euler
4. Heun scheme

as follows. Use the `rand` function in MATLAB to sample the square $[-2, 2] + i[-2, 2] \subset \mathbb{C}$ uniformly. If the random point is inside the set of absolute stability keep it, otherwise reject it. Generate enough random samples to guess the shape of the sets of absolute stability and plot the points inside.

Exercise 2: (7 Points)
The solution to the *test equation*

$$z'(t) = \lambda z(t), \quad z(0) = 1, \quad t \in [0, \infty)$$

with $\lambda \in \mathbb{C}$ is given by $z(t) = \exp(t\lambda)$.

1. How does $|z(t)|$ depend on the real and imaginary parts of λ?
2. Determine the sets of values for $\lambda \in \mathbb{C}$ such that
   (a) $\lim_{t\to\infty} |z(t)| = 0$
   (b) $\lim_{t\to\infty} |z(t)| = \infty$
   (c) $\lim_{t\to\infty} |z(t)| = 1$

   and draw a picture of the corresponding regions in the complex plane.
3. What is the connection between the test equation with the asymptotic study of a given linear system of ODEs $y'(t) = Ky(t)$, assuming $K \in \mathbb{R}^{n\times n}$ can be diagonalized?

Due Date: 07/05/2012 $12:30h$