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Advanced Numerical Methods. Exercise 12.

Exercise 1: (10 Bonus Points)

Solve the following heat equation

$$\begin{aligned}u_t(x, t) &= u_{xx}(x, t), & (x, t) \in (0, \pi) \times (0, \infty) \\u(x, 0) &= \varphi(x), & x \in [0, \pi] \\u(0, t) &= u(\pi, t) = 0, & t \in [0, \infty)\end{aligned}$$

Use the method of separation of variables, i.e., use a product ansatz

$$u(x, t) = f(x)g(t).$$

Hint:

1. Derive independent ordinary differential equations for $f(x)$ and $g(t)$ (separation of variables).
2. Solve these ordinary differential equations taking into account the boundary values of the PDE. This gives solutions for certain initial values.
3. Sum up these possible solutions (i.e., use Fourier sine series) to construct a solution u to the initial value problem which matches the initial data φ at time $t = 0$.

Exercise 2: (10 Points)

Apply the method of lines to the initial value problem from the previous exercise. Explain how it works for that particular case.