Universität Duisburg-Essen
Computational Mechanics
Fakultät für Mathematik

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Advanced Numerical Methods. Exercise 13.

Exercise 1:
Solve the initial value problems

1. $y'(t) = (t-1)^2\, y(t), \quad y(0) = 1$
2. $y'(t)\, y(t) = t, \quad y(0) = 1$

Exercise 2:
Reduce the second order system of ODEs

$$y''(t) = Ay(t), \quad A \in \mathbb{R}^{n\times n}, \quad y(0) = y_0 \in \mathbb{R}^n, \quad y'(0) = 0 \in \mathbb{R}^n$$

to a first order system in the set of variables $z = (z_1, z_2)^T \in \mathbb{R}^{2n}$.
Write this in block matrix form and compute one step of the explicit Euler method for the reduced system.

Exercise 3:
Compute the set $\mathcal{A} \subset \mathbb{C}$ of absolute stability for the explicit and implicit Euler methods and draw them.

Exercise 4:
Write down the Runge-Kutta method corresponding to the following Butcher tableau:

$$\begin{array}{c|ccc} 0 & 0 & 0 & 0 \\ 1/2 & 1/2 & 0 & 0 \\ 1 & -1 & 2 & 0 \\ \hline & 1/6 & 2/3 & 1/6 \end{array}$$

Apply this scheme to the ODE

$$y'(t) = y(t), y(0) = 1$$

and compute the first step.

Exercise 5:
Use the 'method of lines' to discretize the following initial boundary value problem in space:

$$\begin{aligned} u_t(x,t) + c\,u_x(x,t) &= 0, \quad (x,t) \in (0,2\pi) \times (0,\infty), c > 0 \\ u(x,0) &= \sin(x), x \in [0,2\pi] \\ u(0,t) &= u(2\pi,t) = \sin(-ct), \quad t \in [0,\infty). \end{aligned}$$

Use the finite difference approximation

$$u'(x) \approx \frac{u(x+h) - u(x-h)}{2h}$$

Use a grid $0 = x_0 < x_1 < \ldots < x_n < x_{n+1} = 1$ with step size $h = \frac{1}{n+1}$.

Exercise 6:
Derive the modified Euler method from a suitable quadrature rule and prove that its order of consistency is 2.

Due Date: none