Universität Duisburg-Essen
Computational Mechanics
Fakultät für Mathematik

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04/19/2012

Advanced Numerical Methods. Exercise 2.

Exercise 1 (7+7 Points) (Theory/Programming)
Consider the initial value problem: Find $y : [0,1] \to \mathbb{R}$,

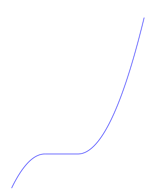
$$y' = 1 + y^2, \qquad y(0) = 0.$$

Why does the theorem of Picard-Lindelöf apply? Solve the IVP numerically by using Euler's and Heun's methods. Compare with the exact solution and compute the global approximation error. Which method converges faster (in what sense?) Which are the advantages and disadvantages of the methods?

Exercise 2 (5 Points)
Give several solutions of

$$y' = 2\sqrt{|y|}, \qquad y(0) = 0. \quad (*)$$

and prove that they solve $(*)$. Draw the solutions and the direction field. Can the following function be a solution to the IVP (*)?



Why? Why does the uniqueness theorem from the lecture not apply here?

Exercise 3 (3 + 6 Points)

1. Lookup Taylors theorem in 1D and state it. You don't have to prove it, just recall it and write it down.

2. Compute the following Taylorexpansions:

   (a) $f(x) = x^7 + 4x^2 - x + 3$ at $x_0 = 0$, up to terms of order 7

   (b) $g(x) = 1/x$ at $x_0 = 1$, up to terms of order 4

   (c) $h(x) = \sin(x)$ at $x_0 = \frac{\pi}{2}$, up to terms of order 6

Exercise 4 (3 Points)
Consider the ODE $y' = f(x, y)$. Assuming $f(x, y)$ to be differentiable, derive the identity

$$y''(x) = f_x(x, y) + f_y(x, y)f(x, y).$$

Note:
You can turn in your MATLAB programs via email to anm.uebung@uni-due.de with a subject along the lines of
Homework 1 Lisa Muster ES8484859201
Homework 1 Karl Muster ES8484859202

Please do not forget to print out your MATLAB programs and to turn them in for correction together with your written solutions.

Due Date: 04/26/2012 .