Universität Duisburg-Essen
Computational Mechanics
Fakultät für Mathematik

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Advanced Numerical Methods. Exercise 3.

Exercise 1 (10 Points) (Programming)
Write a program to calculate the approximation to $y(0.3)$ for the IVP

$$y' = x^2 + y^2, \quad y(0) = 0;$$

Implement the methods of Euler and Heun. Use the approximation using a step length of $h = 10^{-4}$ as a reference. What do you observe if you choose a coarser discretization, i.e. $h = 0.1, 0.05, 0.025 \ldots$.

Exercise 2 (6 Points)
Derive the explicit Euler's, the modified Euler method and Heun's method as approximations of an integral.

Exercise 3 (optional, 8 + 2 + 4 = 14 Bonus Points)*
The following is a so-called linear system of ODEs with constant coefficients

$$\begin{pmatrix} u'(t) \\ v'(t) \end{pmatrix} = M \begin{pmatrix} u(t) \\ v(t) \end{pmatrix},$$

with $M = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \mathbb{R}^{2\times 2}$ diagonalizable, that means $M = TDT^{-1}$ with a diagonal matrix D and invertible T.

1. Find out, by studying the literature if necessary, how to solve such a system. Write down the procedure.
2. Solve the system you obtain for
$$M = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$
3. Solve the system you obtain for
$$M = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$
Hint: The matrix can be diagonalized over the complex numbers.

Due Date: 05/03/2012 .