Universität Duisburg-Essen
Computational Mechanics
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Advanced Numerical Methods. Exercise 5.

Exercise 1: (6 Points)
Reduce the following system of ODEs

$$\begin{aligned} u'' &= e^{tv} + u^3 - t^2 u' + 3v'u \\ v'' &= \cos(u') - t^3 u'v' + uv^2 \end{aligned}$$

to an equivalent system of ordinary differential equations of first order, i.e., to a system of the form

$$x'(t) = f(t, x(t))$$

which only contains derivatives up to first order.

Exercise 2: (6 + 2 = 8 Points)
Consider the motion of a space probe exposed to the gravity of the moon and earth. We shall assume that the orbit can be described by two component functions $(u(t), v(t))^T$, i.e., that it is essentially planar. The coordinates are chosen such that the earth resides at $(0,0)^T$ and the moon is located at $(1,0)^T$ and obtain:

$$\begin{aligned} \ddot{u} &= u + 2\dot{v} - (1-\mu)\frac{u+\mu}{[(u+\mu)^2+v^2]^{3/2}} - \mu\frac{u-1+\mu}{[(u-1+\mu)^2+v^2]^{3/2}}, \\ \ddot{v} &= v - 2\dot{u} - (1-\mu)\frac{v}{[(u+\mu)^2+v^2]^{3/2}} - \mu\frac{v}{[(u-1+\mu)^2+v^2]^{3/2}}. \end{aligned}$$

Here, the parameter μ denotes the mass of the moon relative to the earth. Note that we used the dot notation to indicate time derivatives, i.e., $\dot{u} := \frac{\mathrm{d}u}{\mathrm{d}t}$.

1. Transform the system of ordinary differential equations into a four dimensonal system of first order.
2. Which initial values do you need to prescribe in order to obtain a solvable inital value problem, i.e., to actually compute a trajectory of the space probe? Give an example.

Exercise 3: (4 + 6 = 10 Points)
Verify that:

1. The explicit Euler's scheme has consistency order 1.
2. The modified Euler's scheme has consistency order 2.

Hint: Use Taylor expansions of the solution (in terms of f) and the increment function Φ.

Due Date: 05/17/2012 . 12:30h