Universität Duisburg-Essen
Computational Mechanics
Fakultät für Mathematik

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05/31/2012

Advanced Numerical Methods. Exercise 7.

Exercise 1: (8 Points)
The Crank-Nicholson rule is given by the implicit scheme

$$y_{k+1} = y_k + \frac{h}{2}\left(f(x_k, y_k) + f(x_{k+1}, y_{k+1})\right).$$

Derive it from the trapezoidal quadrature rule and show that its order of consistency is 2.

Exercise 2: (10 Points)
Consider the following Butcher arrays:

$$\begin{array}{c|c} 0 & \\ \hline 1 & 1 \end{array} \qquad \begin{array}{c|cc} 0 & & \\ 1 & 1 & \\ \hline 1 & \frac{1}{2} & \frac{1}{2} \end{array} \qquad \begin{array}{c|ccc} 0 & & & \\ \frac{1}{2} & \frac{1}{2} & & \\ \frac{3}{4} & 0 & \frac{3}{4} & \\ \hline 1 & \frac{2}{9} & \frac{1}{3} & \frac{4}{9} \end{array} \qquad \begin{array}{c|cccc} 0 & & & & \\ \frac{1}{2} & \frac{1}{2} & & & \\ \frac{1}{2} & 0 & \frac{1}{2} & & \\ 1 & 0 & 0 & 1 & \\ \hline 1 & \frac{1}{6} & \frac{1}{3} & \frac{1}{3} & \frac{1}{6} \end{array}$$

Write down the corresponding Runge-Kutta schemes from left to right. If you recognize a scheme from the lecture write down its name.

Due Date: 06/08/2012 . 12:30h