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## Advanced Numerical Methods. Exercise 8.

### Exercise 1: (4 + 8 = 12 Points)

Consider again the motion of a space probe exposed to the gravity of the moon and earth. We shall assume that the orbit can be described by two component functions  $(u(t), v(t))^T$ , i.e., that it is essentially planar. The coordinates are chosen such that the earth resides at  $(0, 0)^T$  and the moon is located at  $(1, 0)^T$  and obtain:

$$\begin{aligned}\ddot{u} &= u + 2\dot{v} - (1 - \mu) \frac{u + \mu}{[(u + \mu)^2 + v^2]^{3/2}} - \mu \frac{u - 1 + \mu}{[(u - 1 + \mu)^2 + v^2]^{3/2}}, \\ \ddot{v} &= v - 2\dot{u} - (1 - \mu) \frac{v}{[(u + \mu)^2 + v^2]^{3/2}} - \mu \frac{v}{[(u - 1 + \mu)^2 + v^2]^{3/2}}.\end{aligned}$$

Here, the parameter  $\mu$  denotes the mass of the moon relative to the earth. Note that we used the dot notation to indicate time derivatives, i.e.,  $\dot{u} := \frac{du}{dt}$ . In Homework 5 you transformed the system of ordinary differential equations into a first order system in  $y_1, y_2, y_3, y_4$ .

Consider the *implicit* Euler method for the corresponding reduced first order system, which is given by

$$y_{i+1} = y_i + hf(t_{i+1}, y_{i+1}).$$

1. Write down a fixed point iteration for  $y_{i+1}$ . Give necessary conditions for the convergence of the fixed point iteration.

(**Note:** You do not have to prove the convergence of the fixed point iteration).

2. Rewrite the implicit Euler scheme as a root finding problem  $G(y_{i+1}) = 0$  where  $G$  is some appropriate auxiliary function. Work out Newtons method to compute a zero of  $G$ , i.e., write down the Newton iteration and the system to be solved in each Newton iteration (at a fixed timestep  $t_i$ ).

What might be a good starting value for the Newton iteration?

### Exercise 2: (Programming) (12 Points)

Use the MATLAB functions `ode23` and `ode45` to solve the initial value problem described in Exercise 2 above using  $\mu = \frac{1}{82.45}$  and initial values:

$$u_0 = 1.2, v_0 = 0, \dot{u}_0 = 0, \dot{v}_0 = -1.049357509830350.$$

Plot the trajectory of the space probe during the interval  $[0, 10]$  with both schemes (e.g., using `odephas2`). Turn on the statistics for the solver using `odeset('stats', 'on')` and summarize the differences between the two sets of solver statistics. Try to explain the statistics.

**Note:** Please do not forget to turn in a printout of your program codes for correction and send the MATLAB code to `anm.uebung@uni-due.de`.

**Due Date:** 06/21/2012 12 : 30h