Universität Duisburg-Essen
Computational Mechanics
Fakultät für Mathematik

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Advanced Numerical Methods. Exercise 9.

Note: Both programming exercises on this sheet are optional. Bonus points of the first exercise will count for the first half of the semester to give you an additional chance to reach the criteria. Bonus points for the second exercise will count for the second half.

Exercise 1: (20 Bonus points / Programming)
In this exercise we want to simulate a harmonic oscillator with and without damping. The equations of movement are governed by Hooke's Law $F(x) = -kx$, where $x(t)$ denotes the displacement at time t and $k > 0$ is a spring constant. In the undamped case, we thus consider the initial value problem

$$\begin{aligned} x(0) &= 0 \\ x'(0) &= 1 \\ mx''(t) &= -kx(t). \end{aligned}$$

A *damped* harmonic oscillator is subject to an additional friction force proportional to the velocity x'. This gives a total force $F(x) = -kx - bx'$, where $b > 0$ is the friction coefficient leading us to:

$$\begin{aligned} x(0) &= 0 \\ x'(0) &= 1 \\ mx''(t) &= -kx(t) - bx'(t). \end{aligned}$$

1. Write a MATLAB code which solves the *undamped* oscillator IVP with $m = 1$ and $k = 1$ numerically using the following schemes:

   (a) Explicit Euler

   (b) Heun's scheme

   (c) Classical Runge Kutta (4th order)

   Use the stepsizes $h = 10^{-k}$, $k = 1, 2, 3, 4$ and plot the computed approximate solutions for the time interval $[0, 8\pi]$ into one common plot per step size. Guess the exact solution and plot it into the same plot.

2. Do the same for the *damped* harmonic oscillator, setting the friction coefficient to $b = 1/2$. You do not need to plot the exact solution.

Hint: Reduce the initial value problems to first order systems of differential equations first.

Exercise 2: (Programming) (12 Bonus points / Programming)
Consider a point mass $p \in \mathbb{R}^2$ with unit mass $m = 1$ which is connected to the origin $(0, 0)$ by a spring with spring constant $k = 1$. Let $p(0) = (1, 0)^T$ and $p'(0) = (0, 1)^T$. The spring is initially at rest, i.e., it does not generate any forces in the initial configuration at $t = 0$. We turn off gravity and neglect friction.

1. Write down the equations of movement for the point mass p using Hooke's Law (without damping).
2. Reduce the equations of movement to first order.
3. Use the matlab solvers `ode23` and `ode45` to solve and plot the trajectory of $p(t)$ during the time interval $[0, 100]$. Turn on the statistics by using `odeset('Stats', 'on')`.

Please: Do not forget to turn in a printout of your program codes and plots. Send the code to `anm.uebung@uni-due.de`.

Due Date: 07/05/2012 $12:30h$