Universität Duisburg-Essen
Computational Mechanics
Campus Essen

Winter Term 2011/12
22. December 2011

Dipl.-Math. Andreas Fischle
Dipl.-Math. Alexander Heinlein

Introduction to Numerical Methods
Homework 11

Bonus-Exercise 1*: ($4 \times 4 = 16$ points)
Test the 4 methods introduced in the lecture for the one dimensional case to solve nonlinear equations. Therefore always make the following steps

- Formulate the problem as root finding or fixed point problem.
- Plot the problem, i.e., plot the function(s) to see where the root or fixed point is approximately located.
- Choose start iterates and compute 5 steps with an accuracy of at least 5 digits, i.e., try to work with exact arithmetic as long as possible round only if the number seems to have an infinite number of digits. Don't forget to have a look how good your solution already is.

Use all 4 methods, i.e., choose a different method for each problem. You can choose them as you like the methods suggested in the brackets are only suggestions.

(i) $x^2 - 2 = 0$ for $x \in \mathbb{R}_+$, i.e., only positive values. (Newton method)

(ii) Find the first positive $x \neq 0$ such that $\tan(x) = \tanh(x)$, with $\tanh(x) = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$. (Bisection method)

(iii) $\arctan(x) = x$ for $x \in (\frac{\pi}{2}, \frac{3\pi}{2})$ (Fixed point iteration)
Hint: Take care which values your pocket calculator or computer gives for arctan you may have to change the functions a little bit.

(iv) $x^3 = 2$ (Secant method)

Bonus-Programming 2*: ((5 + 5) + (2 + 2) = 14 points)

(i) Write two programs:

(1) for the Bisection method to solve $x^2 - 2 = 0$ for $x \in \mathbb{R}_+$ and with variable initial values a and b,

(2) for the Fixed point iteration to solve $\arctan(x) = x$ for $x \in (\frac{\pi}{2}, \frac{3\pi}{2})$ with variable initial guess $x^{(0)}$, do not forget what you did in Exercise 1.

(ii) Add a graphical output to the programs in part (i) to illustrate the computing process.

You don't have to turn in this programming exercise!

Delivery:
12. January 2012

We wish you nice Christmas holidays
and a happy New Year!