Universität Duisburg-Essen
Computational Mechanics
Campus Essen

Winter Term 2011/12
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Dipl.-Math. Andreas Fischle
Dipl.-Math. Alexander Heinlein

Introduction to Numerical Methods
Homework 5

Exercise 1: (4 + 4 Points)
Determine the smallest number of intervals, which are needed to approximate

$$I(f) := \int_0^1 e^x \, dx$$

up to an accuracy of $5 \cdot 10^{-4}$ when using

(i) the composite Trapezoidal rule,

(ii) the composite Simpson's rule.

Exercise 2: (6 + 6 Points)

(i) Use the composite Trapezoidal rule with $n = 6$ intervals to approximate the integral

$$I(f) := \int_2^{2.3} \sqrt{x} \, dx.$$

Compute the exact error

$$E := |I(f) - \tilde{I}(f)|$$

between the exact integral $I(f)$ and the approximation $\tilde{I}(f)$ with the composite Trapezoidal rule and an estimate for this error.

(ii) Do the same calculations as in **(i)** for $I(f) := \int_0^{\frac{\pi}{2}} \sin(x)\cos(x) \, dx$ with the composite Simpson's rule and $m = 3$ intervals.

Exercise 3: (20 Points / Programming)
Consider the functions

$$
\begin{aligned}
f(x) &:= 4x - 1, \\
g(x) &:= x^2 + 1, \\
h(x) &:= \sin(x)^2.
\end{aligned}
$$

(i) Write a MATLAB program, that computes approximate values of $I(f) := \int_0^5 f(x)\, dx$, $I(g) := \int_0^3 g(x)\, dx$ and $I(h) := \int_0^{\frac{3\pi}{2}} h(x)\, dx$ using the composite Trapezoidal and Simpson's rule respectively. Allow for a variable number of intervals.

(ii) Compute, either by solving the integrals by hand **or** by using your program from **(i)** (and the formulae for the error estimates), the values of $I(f)$, $I(g)$ and $I(h)$ up to an accuracy of 10^{-10}.

(iii) Use the values for $I(f)$, $I(g)$ and $I(h)$ computed in **(ii)** as reference values to plot the approximation errors for both composite integration rules in one diagram per function, for $n = 2, 4, 8, 16, 32, 64, 128$. What connections can be drawn between your results and the theory from the lecture?

Bonus Exercise 4*: (4 Points)
Show how the composite Simpson's rule, see equation (3.5) in the lecture notes, can be deduced from Simpson's rule, equation (3.3).
Hint: Look at the deduction of the composite Trapezoidal rule in the lecture.

Delivery:
17. November 2011 (Theory)
24. November 2011 (Programming)