Universität Duisburg-Essen
Computational Mechanics
Campus Essen

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Introduction to Numerical Methods
Homework 7

Exercise 1: (10 Points)
Compute manually the solution of the linear equation system $Ax = b$ with

$$A = \begin{pmatrix} 25 & 10 & 25 & 15 \\ 10 & 13 & 16 & -3 \\ 25 & 16 & 33 & -15 \\ 15 & -3 & -15 & 12 \end{pmatrix} \quad \text{and} \quad b = \begin{pmatrix} -5 \\ 10 \\ 23 \\ 15 \end{pmatrix}$$

using LU-decomposition with column pivot search.
Make sure that your result is correct by multiplying L with U.
Hint: Extend A by a 4×4-identity matrix I as follows:

$$\left(\begin{array}{cccc|cccc} a_{11} & a_{12} & a_{13} & a_{14} & 1 & 0 & 0 & 0 \\ a_{21} & a_{22} & a_{23} & a_{24} & 0 & 1 & 0 & 0 \\ a_{31} & a_{32} & a_{33} & a_{34} & 0 & 0 & 1 & 0 \\ a_{41} & a_{42} & a_{43} & a_{44} & 0 & 0 & 0 & 1 \end{array}\right)$$

Permute the rows of this identity matrix when you permute the rows of A.

Exercise 2: (5 + 5 = 10 Points)
The linear equation system

$$\begin{pmatrix} 0.001 & -2.3 \\ -1.35 & 0.03 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -2.295 \\ -6.72 \end{pmatrix}$$

has the solution $x_1 = 5$ and $x_2 = 1$.
To see the influence of errors occuring when a system is solved without column pivot search you should now solve the system yourself. But you are only allowed to use 3 digits, i.e., each number can be displayed in exponential form by

$$\pm 0.y_1 y_2 y_3 \cdot 10^E \quad (y_1, y_2, y_2 \in \{0, 1, \ldots, 9\}, y_1 \neq 0, E \in \mathbb{Z}).$$

If you would need more digits for a number it is cut without rounding, e.g., 12.45 becomes $0.124 \cdot 10^2$.
Compute the solution of the system with this representation of numbers using Gaussian elimination

(i) without using column pivot search,

(ii) using column pivot search.

Do you recognize a difference in the solutions?

Exercise 3: (20 Points / Programming)
Write a program which computes for a given matrix $A \in \mathbb{R}^{n\times n}$ a LU-decomposition with column pivot search and solves the linear equation system $Ax = b$ when an additional right hand side $b \in \mathbb{R}^n$ is given.
Use your program to solve the system $Ax = b$ given by

(i) the example in exercise 1,

(ii) the matrix $A = (a_{ij})_{i,j=1..n}$ with $a_{ij} = \dfrac{1}{(i+j-1)}$

and the vector $b = (b_i)_{i=1,..,n}$ with $b_i = \displaystyle\sum_{j=1}^{n} \frac{1}{(i+j-1)}$,

(iii) the matrix $A = (a_{ij})_{i,j=1..n}$ with

$$a_{ij} = \begin{cases} \dfrac{1}{(i+j-1)} & i \neq j \\ 1 & i = j \end{cases}$$

and the vector $b = (b_i)_{i=1,..,n}$ with $b_i = 1 + \displaystyle\sum_{\substack{j=1 \\ i\neq j}}^{n} \frac{1}{(i+j-1)}$.

Show using Linear Algebra that the solutions you obtain are correct.
Test your program for different $n \in \mathbb{N}$, e.g., $n = 5, 10, 25, 100$.
Program the forward- and backward-substitution by yourself and do not use the matlab routine to solve the linear system.

Hint: Start with an LU-decomposition without pivot-search and then implement the pivot search in a second step.
Don't implement the pivot search and the elimination as matrix-matrix-products.

You can store L and U in A if you like, cf., the lecture notes concerning the advantages of this procedure, but you can also store them separately and then test your result by computing $LU - PA$.

Delivery:
1. December 2011 (Theory)
8. December 2011 (Programming)