Participants presentations: talks

1 **Leandro del Pezzo, Buenos Aires**:
Title: Discontinuous Galerkin method for the $p(x)$- Laplacian

In this talk, we construct an "Interior Penalty"Discontinuous Galerkin method to approximate the minimizer of a variational problem related to the $p(x)-$Laplacian. The function $p : \Omega \to [p_1, p_2]$ is log- Hölder continuous and $1 < p_1 \leq p_2 < \infty$. We prove the weak convergence of the sequence of minimizers of the discrete functional to the minimizer. We also make some numerical experiments in dimension one to compare this method with the Conform Galerkin method, in the case where p_1 is next to one. This example is motivated by its applications to image processing.

2 **Afaf Bouharguane, Montpellier**:
Title: A nonlinear and nonlocal model for dune morphodynamics

Abstract: We will introduce a PDE describing the morphodynamics of sand dunes sheared by a fluid flow, recently proposed by Andrew C. Fowler. This model involves a nonlocal term which can be seen as an anti-diffusive fractional differential operator. We propose numerical schemes based on the finite difference method and the splitting method to approximate the solutions of this equation. Numerical stability and convergence of these schemes are investigated. To finish with, we will see that this model can be also applied to signal processing.

3 **Faustino Maestre, Sevilla**:
Title: Homogenization for the wave equation

Abstract: In this talk we present a recent result about the homogenization of the wave equation. We can find in the literature some classical results for the homogenization for hyperbolic system with coefficients which do not depend on the time variable or assuming Lipschitz-continuity in time. We analize the case for discontinuous coefficients, we generalize the classical results assuming BV coefficients in the time variable.

4 **Marcin Malogrosz, Warsaw**:
Title: Models of morphogen transport: global solutions and asymptotics

Abstract: Transport of morphogens is a process occurring in the tissue, affecting cell differentiation. In 1) authors proposed several mathematical models (systems of PDEs of reaction-diffusion type) of this process. In 2) a detailed analysis of two of those models was made in 1D setting. I will present my recent results concerning global in time existence and asymptotic behavior for the 3D setting.

1) Lander, A. D., Nie, Q., Wan, Y. M. *Do Morphogen Gradients Arise by Diffusion?* Dev. Cell, Vol. 2, pp. 785-796.

2) Krzyżanowski, P., Laurençot, P., Wrzosek, D. *Well-posedness and convergence to the steady state for a model of morphogen transport*, SIAM J.MATH. ANAL. Vol. 40, No. 5, pp. 1725-1749.

5 **Farid Bozorgnia, Lissabon**
Title: Numerical approximations for variational problems of spatial segregation of reaction-diffusion system

Abstract: In this talk, numerical approximations for a class of stationary states for reaction-diffusion system for m densities having disjoint support, which are governed by a minimization problem, are presented. We use quantitative properties of the solution and free boundaries to derive our scheme. Furthermore, the proof for the convergence of method is given. Also we applied our numerical scheme to the spatial segregation limit of diffusive Lotka-Volterra models in presence of high competition and inhomogeneous Dirichlet boundary conditions. Moreover, we discuss the numerical implementation of the resulting approach and present computational tests.

6 **Kaushik Bal, Pau**
Title: On a Singular Evolution Equation

Abstract: We investigate the following quasilinear parabolic and singular equation,

$$\begin{cases} u_t - \Delta_p u = \dfrac{1}{u^\delta} + f(x,u) \text{ in } Q_T = (0,T] \times \Omega \\ u = 0 \text{ on } \Gamma = [0,T] \times \partial\Omega, \quad u > 0 \text{ in } Q_T \\ u(0,x) = u_0(x) \text{ in } \Omega \end{cases} \tag{P$_t$}$$

where Ω is an open bounded domain with smooth boundary in $\mathbb{R}^N$, $1 < p < \infty$, $0 < \delta$ and $T > 0$. We assume that $(x,s) \in \Omega \times \mathbb{R}^+ \to f(x,s)$ is a non negative Caratheodory function, locally Lipschitz in respect to s uniformly in $x \in \Omega$ and sublinear at ∞, i.e.

$$0 \leq \lim_{t \to +\infty} \frac{f(x,t)}{t^{p-1}} = \alpha_f < \lambda_1(\Omega), \tag{1}$$

(where $\lambda_1(\Omega)$ is the first eigenvalue of $-\Delta_p$ in Ω with homogeneous Dirichlet boundary conditions) and $u_0 \in L^\infty(\Omega) \cap W_0^{1,p}(\Omega)$, satsfying a cone condition defined below. Then, for any $\delta \in (0, 2 + \frac{1}{p-1})$, we prove the existence and the uniqueness of a weak solution $u \in \mathbf{V}(Q_T)$ to (P$_t$). The proof involves a semi-discretization in time approach and the study of the stationnary problem associated to (P$_t$). The key points in the proof is to show that $u \in \mathcal{C}$ (defined below) and by the weak comparison principle that $\frac{1}{u^\delta} \in L^\infty(0,T; W^{-1,p'}(\Omega))$ and $u^{1-\delta} \in L^\infty(0,T; L^1(\Omega))$. When $t \to \frac{f(x,t)}{t^{p-1}}$ is non-increasing for a.e. $x \in \Omega$, we show that $u(t) \to u_\infty$ in $L^\infty(\Omega)$ as $t \to \infty$, where u_∞ is the unique solution to the stationnary problem.

Finally, in the last section we analyse the case of $p = 2$. Using the interpolation spaces theory and the semi-group theory, we prove existence of weak solutions to (P$_t$) for any $\delta > 0$ in $C([0,T], L^\infty(\Omega))$ and give sharp condition on δ to get solutions in $C([0,T], H_0^1(\Omega))$ and describe their asymptotic behaviour in $L^\infty(\Omega) \cap H_0^1(\Omega)$.

7 **Richard Norton, Oxford** Title: Finite element approximation error for an H^1 gradient flow

Abstract: An H^1 gradient flow of an energy integral where the energy density is the sum of a double-well potential and a bending energy can be rewritten as a semilinear parabolic equation. However, existing finite element approximation theory does not completely cover our situation because our problem is posed in H^1 (instead of the more usual L^2), our nonlinear term is a nonlocal operator, and we want to know how the error bounds depend on the relative size of the bending energy. We successfully adapt existing theory for all of these difficulties.

8 **Martin Lukarevski, Hannover** Title: Evolution equation for the Stefan Problem

Abstract: The Stefan Problem is a well-known phase transition free boundary problem which can be modelled and solved in many different ways. One of them is to reduce it to a single evolution equation. We will apply maximal regularity on this equation to show existence of local solutions in Sobolev spaces.