Reading-up-time

For reviewing purposes of the problem statements, there is a "reading-up-time" of **10 minutes** prior to the official examination time. During this period it is **not** allowed to start solving the problems. This means explicitly that during the entire "reading-up-time" no writing utensils, e.g. pen, pencil, etc. at all are allowed to be kept on the table. Furthermore the use of carried documents, e.g. books, (electronic) translater, (electronic) dictionaries, etc. is strictly forbidden. When the supervisor refers to the end of the "reading-up-time" and thus the beginning of the official examination time, you are allowed to take your utensils and documents. Please **then**, begin with filling in the **complete** information on the titlepage and on page 3.

Good Luck!

LAST NAME	
FIRST NAME	
MATRIKEL-NO.	
TABLE-NO.	

Klausurunterlagen

Ich versichere hiermit, dass ich sämtliche für die Durchführung der Klausur vorgesehenen Unterlagen erhalten, und dass ich meine Arbeit ohne fremde Hilfe und ohne Verwendung unerlaubter Hilfsmittel und sonstiger unlauterer Mittel angefertigt habe. Ich weiß, dass ein Bekanntwerden solcher Umstände auch nachträglich zum Ausschluss von der Prüfung führt. Ich versichere weiter, dass ich sämtliche mir überlassenen Arbeitsunterlagen sowie meine Lösung vollständig zurück gegeben habe. Die Abgabe meiner Arbeit wurde in der Teilnehmerliste von Aufsichtsführenden schriftlich vermerkt.

THE ABOVE REQUIRED STATEMENTS AS WELL AS THE SIGNATURE ARE MANDATORY AT THE BEGINNING OF THE EXAM.

Duisburg, ____________ (Date) ________________________ (Student's signature)

Falls Klausurunterlagen vorzeitig abgegeben: __________ Uhr

Bewertungstabelle

Aufgabe 1	
Aufgabe 2	
Aufgabe 3	
Gesamtpunktzahl	
Angepasste Punktzahl	
%	
Bewertung gem. PO in Ziffern	

____

(Datum und Unterschrift 1. Prüfer, Univ.-Prof. Dr.-Ing. Dirk Söffker)

____

(Datum und Unterschrift 2. Prüfer, Prof. Dr.-Ing. Yan Liu)

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(Datum und Unterschrift des für die Prüfung verantwortlichen Prüfers, Söffker)

Fachnote gemäß Prüfungsordnung:

☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐
1,0	1,3	1,7	2,0	2,3	2,7	3,0	3,3	3,7	4,0	5,0
sehr gut		gut			befriedigend			ausreichend		mangelhaft

Bemerkung: ____

Attention: Give your answers to ALL problems directly below the questions in the exam question sheet.

You are NOT allowed to use a pencil and also NOT red color (red color is used for corrections).

This exam is taken by me as a
☐ mandatory (Pflichtfach)
☐ elective (Wahlfach)
☐ prerequsite (Auflage)
subject (cross ONE option according to your own situation).

Maximum achievable points:	**90 Points**
Minimum points for the grade 1,0:	**95%**
Minimum points for the grade 4,0:	**50%**

General hints:

1) For the multiple-choice and multiple-choice-similar tasks the following rules are effective:
   i) For correct answers of exam task parts the desired number of points will be given.
   ii) For noncorrect answers of exam task parts the desired number of points will be counted negative.
   iii) No answering will neither lead to positive nor to negative points.
   iv) The points of the task will be summarized. The whole number can not be smaller than zero.

2) If in the exam tasks no information is given for the valid range of numbers for time constants or masses etc. : take for time constants (in sec.), for masses (in kg) positive numbers.

3) If in the exam tasks no information is given for applying negative or positive feedback: use the usual negative feedback.

Problem 1 (28 Points)

1a) (3 x 5 Points)
Which of the following statements is 'True' or 'False'?

NO.	Task/Question/Judgement	True	False
A1	State space description is a description only for linear time invariant systems.	○	⊗
A2	The Rosenbrock matrix describes the system behavior in time domain.	○	⊗
A3	The transfer function matrix includes less information than the Rosenbrock matrix.	⊗	○
A4	The solution of the state equation can be get only from the mathematical series $e^{At} = \sum_{i=0}^{\infty} \frac{A^i t^i}{i!}$.	○	⊗
A5	The free motion does not depend on the input $u(t)$.	⊗	○

NO.	Task/Question/Judgement	True	False
B1	Eigenvectors of a system are always linear independent.	○	⊗
B2	Observability of a system can be checked directly with C matrix, if the system matrix is a diagonal matrix.	⊗	○
B3	Decoupling zeros are equal to some eigenvalues.	⊗	○
B4	An input decoupling zero is always observable.	○	⊗
B5	Poles can not be transmission zeros for the same system.	⊗	○

NO.	Task/Question/Judgement	True	False
C1	Lyapunov stability criterion is suitable for checking state stability of a system.	⊗	○
C2	Hurwitz criterion is applied to check the Lyapunov stability of a system.	○	⊗
C3	Optimal state feedback control is designed by given desired poles.	○	⊗
C4	Observer can be designed independently from controller design of a system.	⊗	○
C5	Output control is also a feedback technique.	⊗	○

For the following tasks, a system description in controllable canonical form is given with

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ a_1 & a_2 & a_3 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \quad C = \begin{bmatrix} c_1 & 0 & 0 \\ 0 & c_2 & c_3 \end{bmatrix}, \quad \text{and} \quad D = \begin{bmatrix} 0 \\ 0 \end{bmatrix},$$

with $a_{1,2,3} \neq 0$ and $c_{1,2,3} \neq 0$.

1b) (1 Point)
State the Rosenbrock matrix of the system depending on the parameters $a_{1,2,3}$ and $c_{1,2,3}$.

$$P(s) = \begin{bmatrix} s & -1 & 0 & 0 \\ 0 & s & -1 & 0 \\ -a_1 & -a_2 & s-a_3 & -1 \\ c_1 & 0 & 0 & 0 \\ 0 & c_2 & c_3 & 0 \end{bmatrix}$$

☐

1c) (3 Points)
Calculate the decoupling zeros. Indicate if they are input or output decoupling zeros.

If $\text{Rank}\,[s_0 I - A \quad -B] < n \Rightarrow s_0$: input decoupling zero.

$$\text{Rank} \begin{bmatrix} s_0 & -1 & 0 & 0 \\ 0 & s_0 & -1 & 0 \\ -a_1 & -a_2 & s_0-a_3 & -1 \end{bmatrix} = 3$$

$\Downarrow$

$\det |\cdots| = -1 \neq 0$ for all values of s_0

$\Rightarrow$ no input decoupling zero.

If $\operatorname{Rank}\begin{bmatrix} s_0 I - A \\ C \end{bmatrix} < n \Rightarrow s_0$ is output decoupling zero.

$$\operatorname{Rank}\begin{bmatrix} s_0 & -1 & 0 \\ 0 & s_0 & -1 \\ -a_1 & -a_2 & s_0 - a_3 \\ c_1 & 0 & 0 \\ 0 & c_2 & c_3 \end{bmatrix} = 3 = n \quad \text{for all values of } s_0.$$

(see Hint)

$\Rightarrow$ no output decoupling zeros

$*$ Hint:

The rank of the $5 \times n$ Matrix can be directly determined from its $n \times n$ Sub matrix:

$$\det\begin{bmatrix} c_1 & 0 & 0 \\ 0 & c_2 & c_3 \\ s_0 & -1 & 0 \end{bmatrix} = c_1 c_3 \neq 0$$

(see assumption)

$$\Rightarrow \operatorname{Rank}\begin{bmatrix} s_0 & -1 & 0 \\ 0 & s_0 & -1 \\ -a_1 & -a_2 & s_0 - a_3 \\ c_1 & 0 & 0 \\ 0 & c_2 & c_3 \end{bmatrix} = 3$$

1d) (4 Points)
Calculate the characteristic polynomial of the system matrix A depending on the parameters $a_{1,2,3}$. Is the system asymptotically stable?

$$\det(\lambda I - A) = \lambda^3 - a_3\lambda^2 - a_2\lambda - a_1$$

Hurwitz – criterion:

① all coefficients should have same sign:

$\Rightarrow a_1, a_2, a_3 < 0$

② Hurwitz determinants:

$$H = \begin{vmatrix} -a_3 & -a_1 & 0 \\ 1 & -a_2 & 0 \\ 0 & -a_3 & -a_1 \end{vmatrix}$$

$$H_1 = -a_3 \overset{!}{>} 0$$

$$H_2 = \begin{vmatrix} -a_3 & -a_1 \\ 1 & -a_2 \end{vmatrix} = a_2a_3 + a_1 \overset{!}{>} 0$$

$$H_3 = -a_1 \cdot H_2 \overset{!}{>} 0 \Rightarrow a_1 \overset{!}{<} 0$$

$\Rightarrow$ The system is asymptotically stable.

if $a_1, a_2, a_3 < 0$ and $a_2a_3 + a_1 > 0$.

1e) (2 Point)
How many outputs has the system? Give the equation(s) for the output(s).

2 Outputs.

$y_1 = C_1 x_1$

$y_2 = C_2 x_2 + C_3 x_3$

1f) (3 Points)
Is the system fully controllable? State reason.

Yes. Because there is no input decoupling zero.

$\sum$

Problem 2 (31 Points)

2a) (4 Points)
A system is described by A, B, and C. The dimension of the state vector x is 35. The invertible modal matrix of A is V. What can be concluded from the given facts?

NO.	Task/Question/Judgement	True	False
1	The observable eigenvalues can be calculated numerically by $\tilde{B} = V^{-1}B$.	○	⊗
2	The transformed system matrix $\tilde{A} = V^{-1}AV$ has new and different eigenvalues as in A.	○	⊗
3	The modal matrix describes the structure of the solution by $x(t) = V\tilde{x}(t)$.	⊗	○
4	From $\tilde{C} = CV$ the observability of some eigenvalues can not be seen.	○	⊗

☐

2b) (4 Points)
From $\det(\lambda_i I - A) = 0$ the left and right eigenvectors $\tilde{\tilde{x}}$ and $\tilde{x}$ can be calculated by $\tilde{\tilde{x}}_i^T(\lambda_i I - A) = 0$ and $(\lambda_i I - A)\tilde{x}_i = 0$ for all λ_i of A. In the detailed case, $\tilde{x}_1$, $\tilde{\tilde{x}}_1$, $\tilde{x}_2$, $\tilde{\tilde{x}}_2$ are calculated as $\tilde{\tilde{x}}_1 = \begin{bmatrix} 8 \\ 5 \\ 7 \\ 2 \\ 8 \end{bmatrix}$, $\tilde{\tilde{x}}_2 = \begin{bmatrix} 8 \\ 5 \\ 1 \\ 1 \\ 1 \end{bmatrix}$, $\tilde{x}_1 = \begin{bmatrix} 8 \\ 5 \\ 6 \\ 2 \\ 9 \end{bmatrix}$, and $\tilde{x}_2 = \begin{bmatrix} 8 \\ 3 \\ 3 \\ 1 \\ 8 \end{bmatrix}$ for a system with $B = \begin{bmatrix} -2 \\ 1 \\ 4 \\ 2 \\ 5 \end{bmatrix}$,
$C = \begin{bmatrix} c & 9 & -3 & 5 & -5 \end{bmatrix}$.

What can be concluded from the given facts?

NO.	Task/Question/Judgement	True	False
1	The system has 5 eigenvalues.	⊗	○
2	For $c \neq 1$ the second eigenvalue is controllable.	○	⊗
3	The system is fully controllable, because there exists no zero in B.	○	⊗
4	For $c = 1$ the first eigenvalue λ_1 is a pole.	○	⊗

☐

2c) (4 Points)
Which of the following statements is 'True' or 'False'?

NO.	Task/Question/Judgement	True	False
1	An observer can be used for diagnostic purposes.	⊗	○
2	A residuum is defined by the difference between output and reference value $r = y - y_{ref}$.	○	⊗
3	The eigenvalues of the controlled system should definitely lie left to the eigenvalues of the plant.	○	⊗
4	The sampling rate of the calculation for simulation of observer-based state feedback control should be larger than or equal to $2\omega_{max}$ (ω_{max}: the maximum frequency of the eigenvalues of the closed-loop system and the observer).	⊗	○

☐

2d) (7 Points)
For the system with

$$\lambda_{1,2} = -5 \pm 2j$$
$$\lambda_{3,4} = -1 \pm 3j$$
$$\lambda_5 = -2$$
$$\lambda_{6,7} = 3 \pm j$$
$$\lambda_{8,9} = 0 \pm 3j$$

a controller design has to be realized. It has to be stated that the non-observable eigenvalues are $\lambda_{3,4} = -1 \pm 3j$, as well as non-controllable eigenvalues with $\lambda_{3,4} = -1 \pm 3j$ and $\lambda_{6,7} = 3 \pm j$.

NO.	Task/Question/Judgement	True	False
1	$\lambda_{3,4} = -1 \pm 3j$ are both input and output decoupling zeros.	⊗	○
2	The system has 5 transmission zeros.	○	⊗
3	The system is asymptotically stable.	○	⊗
4	The poles are $s_{p1,2} = -5 \pm 2j$, $s_{p3} = -2$, and $s_{p4,5} = 0 \pm 3j$.	⊗	○
5	$\lambda_{3,4} = -1 \pm 3j$ are not eigenvalues.	○	⊗
6	The system can be stabilized using a state feedback controller.	○	⊗
7	The system is detectable.	⊗	○

☐

2e) (12 Points)

A system is given by

$$A = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -2 & 1 \\ 0 & 1 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} b_1 \\ 0 \\ b_2 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & c & 0 \end{bmatrix}, \quad \text{and} \quad D = [0].$$

i) (3 Points)
The transfer function matrix $G(s)$ is

- ○ $\frac{2b_2c}{s^2+4s+3}$
- ☒ $\frac{b_2c}{s^2+4s+3}$
- ○ $\frac{b_1c}{s^2+4s+3}$
- ○ None

☐

ii) (2 Points)
Calculate the eigenvalues of A. The result is:

- ○ $\lambda_1 = -1$, $\lambda_{2,3} = 0 \pm j$
- ○ $\lambda_{1,2} = 1$ and $\lambda_3 = 3$
- ☒ $\lambda_{1,2} = -1$ and $\lambda_3 = -3$
- ○ None

☐

iii) (2 Points)
Is the system fully controllable?

- ○ Yes, for $b_1 \neq 0$ and $b_2 \neq 0$.
- ○ No.
- ☒ Yes, for $b_2 \neq 0$.
- ○ None

☐

iv) (2 Points)
Is the system fully observable?

- ○ Yes.
- ☒ No.
- ○ Yes, for $c \neq 0$.
- ○ None

☐

v) (3 Points)
Calculate the feedback gains for state feedback control of the given system using pole placement. The desired eigenvalues of the controlled system should be $\lambda_1 = -1$, $\lambda_2 = -4$ and $\lambda_3 = -6$.

- [x] $k_1 = 0$, $k_2 = 9/b_2$, $k_3 = 6/b_2$
- [ ] $k_1 = 1/b_2$, $k_2 = 9/b_2$, $k_3 = 6/b_2$
- [ ] $k_1 = 1/b_2$, $k_2 = b2$, $k_3 = b2/3$
- [ ] None

Σ ☐

Problem 3 (31 Points)
A system description is given with

$$A = \begin{bmatrix} 7 & 0 & -8 \\ 10 & -2 & -10 \\ 4 & 0 & -5 \end{bmatrix},\ B = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & b \end{bmatrix},\ C = \begin{bmatrix} c & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix},\ \text{and } D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}\ (b \neq 0,\ c \neq 0).$$

The eigenvalues of the system matrix A are $\lambda_1 = -1$, $\lambda_2 = -2$, and $\lambda_3 = 3$.

3a) (7 Points)
- Calculate the transfer function matrix for the system.
- State the poles.
- Is the system BIBO-stable (state reason)?
- Is the system state-stable (state reason)?

$$G(s) = C(sI-A)^{-1}B + D$$

$$= \begin{bmatrix} c & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} s-7 & 0 & 8 \\ -10 & s+2 & 10 \\ -4 & 0 & s+5 \end{bmatrix}^{-1} \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & b \end{bmatrix}$$

$$= \frac{1}{(s+1)(s+2)(s-3)} \cdot \begin{bmatrix} c(s+2)(s+5) & -8cb(s+2) \\ 4(s+2) & b(s-7)(s+2) \end{bmatrix}$$

$$= \begin{bmatrix} \frac{c(s+5)}{(s+1)(s-3)} & \frac{-8cb}{(s+1)(s-3)} \\ \frac{4}{(s+1)(s-3)} & \frac{b(s-7)}{(s+1)(s-3)} \end{bmatrix}$$

poles : $s_{p_1} = -1 \quad s_{p_2} = 3$

$\Rightarrow$ not BIBO-stable $(s_{p_2} = 3 > 0)$

$\Rightarrow$ not state-stable $(s_{p_2} = \lambda_3 = 3 > 0)$

3b) (4 Points)

Give the modal matrix of the system $A = \begin{bmatrix} 7 & 0 & -8 \\ 10 & -2 & -10 \\ 4 & 0 & -5 \end{bmatrix}$.

Model matrix : $V = [V_1 \quad V_2 \quad V_3]$

for $\lambda_1 = -1$:

$$\begin{bmatrix} 8 & 0 & -8 \\ 10 & -1 & -10 \\ 4 & 0 & -4 \end{bmatrix} \cdot \begin{bmatrix} V_{11} \\ V_{12} \\ V_{13} \end{bmatrix} = 0 \Rightarrow V_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

for $\lambda_2 = -2$:

$$\begin{bmatrix} 9 & 0 & -8 \\ 10 & 0 & -10 \\ 4 & 0 & -3 \end{bmatrix} \cdot \begin{bmatrix} V_{21} \\ V_{22} \\ V_{23} \end{bmatrix} = 0 \Rightarrow V_2 = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$$

for $\lambda_3 = 3$:

$$\begin{bmatrix} 4 & 0 & -8 \\ 10 & -5 & -10 \\ 4 & 0 & -8 \end{bmatrix} \cdot \begin{bmatrix} V_{31} \\ V_{32} \\ V_{33} \end{bmatrix} = 0 \Rightarrow V_3 = \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$$

$$\Rightarrow V = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$$

3c) (4 Points)
Give the decoupling zeros for the system and classify the decoupling zeros.

$\lambda_2 = -2$ is an eigenvalue but not a pole.

$\Rightarrow \lambda_2 = -2$ is a decoupling zero

Using Gilbert:

$$\tilde{B} = V^{-1} \cdot B = \begin{bmatrix} -1 & 0 & 2 \\ -2 & 1 & 2 \\ 1 & 0 & -1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & b \end{bmatrix}$$

$$= \begin{bmatrix} -1 & 2b \\ -2 & 2b \\ 1 & -b \end{bmatrix}$$

no zero row $\Rightarrow$ no input decoupling zero.

$$\tilde{C} = C \cdot V = \begin{bmatrix} c & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c & 0 & 2c \\ 1 & 0 & 1 \end{bmatrix}$$

$\hookrightarrow$ second column is a column of zeros

$\Rightarrow \lambda_2 = -2$ is an output decoupling zero.

3d) (4 Points)
Is the system stabilizable (state reason)? Is it possible to realize full state feedback (state reason)?

Yes.
No input decoupling zero ⇒ The system is fully controllable.

No; not possible.

i) using observer: no, because 2nd mode is not observable.

ii) using no observer: no, because the full state is not measured completely.

3e) (4 Points)
A third measurement, which is $y_3 = 2x_1 + cx_2 + 3x_3$, is realized. Determine the new output matrix C_{new}. Calculate the condition for full observability with respect to parameter c ($c \neq 0$) for the new system (A, C_{new}).

$$C_{new} = \begin{bmatrix} c & 0 & 0 \\ 0 & 0 & 1 \\ 2 & c & 3 \end{bmatrix}$$

$$\tilde{C}_{new} = C_{new} \cdot V = \begin{bmatrix} c & 0 & 0 \\ 0 & 0 & 1 \\ 2 & c & 3 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 1 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} c & 0 & 2c \\ 1 & 0 & 1 \\ 5 & c & 7+2c \end{bmatrix}$$

due to $c \neq 0$

$\Rightarrow$ no column zero in $\tilde{C}_{new}$

$\Rightarrow$ fully observable.

3f) (4 Points)

Determine the weighting matrices Q and R with the cost function given by

$$J = \int_0^\infty [x^T Q x + y^T R y] dt$$
$$= \int_0^\infty [{x_1}^2(t) + x_1(t)x_2(t) + {x_2}^2(t) + {y_1}^2(t) + 0.25 y_1(t) y_2(t) + {y_2}^2(t)] dt.$$

Declare the calculation steps with the necessary equations and conditions to calculate the matrix L for the linear quadratic optimal observer, assuming the system is fully observable.

$Q = \begin{bmatrix} 1 & \frac{1}{2} \\ \frac{1}{2} & 1 \end{bmatrix}$ $R = \begin{bmatrix} 1 & \frac{1}{8} \\ \frac{1}{8} & 1 \end{bmatrix}$

steps to calculate L:

① P is a symmetric matrix.

assume $P = \begin{bmatrix} P_1 & P_2 \\ P_2 & P_3 \end{bmatrix}$

② calculate P matrix using Riccati equation

$AP + PA^T - PC^T R^{-1} C P + Q = 0$

③ calculate L using

$L = PC^T R^{-1}$

3g) (4 Points)

The system matrix A of a linear dynamical system is defined as $A = \begin{bmatrix} 0 & 9 \\ 2 & 8 \end{bmatrix}$. Calculate the solution matrix P using Ljapunov approach (with the weighting matrix $Q = \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$) and determine the Ljapunov stability of the system according to the solution matrix P.

Iff for arbitrary matrix $Q = Q^T > 0$, the Ljapunov equation $A^T P + PA = -Q$ has an unique solution $P = P^T > 0$, the system is asymptotically stable.

assume $P = \begin{bmatrix} P_1 & P_2 \\ P_2 & P_3 \end{bmatrix}$

$$\begin{bmatrix} 0 & 2 \\ 9 & 8 \end{bmatrix} \begin{bmatrix} P_1 & P_2 \\ P_2 & P_3 \end{bmatrix} + \begin{bmatrix} P_1 & P_2 \\ P_2 & P_3 \end{bmatrix} \begin{bmatrix} 0 & 9 \\ 2 & 8 \end{bmatrix} = - \begin{bmatrix} 4 & 0 \\ 0 & 2 \end{bmatrix}$$

$$\Rightarrow \begin{cases} P_1 = \frac{2}{3} \\ P_2 = -1 \\ P_3 = 1 \end{cases} \Rightarrow P = \begin{bmatrix} \frac{2}{3} & -1 \\ -1 & 1 \end{bmatrix}$$

check the positive definiteness of P:

$$|\lambda I - P| = 0 \Rightarrow \lambda_{1,2} = \frac{5 \pm \sqrt{37}}{6}$$

$\lambda_1 < 0 \Rightarrow P$ is not positive definite.

$\Rightarrow$ The system is not asymptotically stable.

$\square$

$\sum \square$