LAST NAME	
FIRST NAME	
MATRIKEL-NO.	

Problem 1

(each 2 points)

a) What are the eigenvalues of a MIMO system? Give the equation from which they are derived.

b) What are the poles of a MIMO system? Give the equation from which they are derived.

c) Declare the Hautus criterion for checking the observability of a linear dynamic system.

d) A 3rd order system is fully observable, fully stabilizable but not fully controllable. What kind of conclusion can be made about the position of the eigenvalues?

e) Which conditions are to be required for the eigenvalues of the matrix $\boldsymbol{A}$-$\boldsymbol{BK}$, in order to achieve an asymptotically stable controlled system. Which method is considered by this criterion in its basic approach?

Problem 2

(each 2 points)

a) The matrix $\boldsymbol{A}$-$\boldsymbol{BK}$ has by arbitrary gain matrix $\boldsymbol{K}$ the eigenvalues $\lambda_{1,2} = -2 \pm j\sqrt{5}$, while the remaining eigenvalues $\lambda_i, i = 3...n$ depend on $\boldsymbol{K}$. Investigate if the system $\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t)$ is fully controllable or not. Also investigate if the mentioned system is stabilizable or not.

b) The system

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t), \quad \boldsymbol{y} = \boldsymbol{C}\boldsymbol{x}(t) \quad \text{with}$$

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 \\ -a & a-10 \end{bmatrix} ; \quad \boldsymbol{B} = \begin{bmatrix} 1 \\ 2 \end{bmatrix} b \; ; \quad \boldsymbol{C} = \begin{bmatrix} 10 & 1 \end{bmatrix} c$$

has to be investigated.

Which values can the parameter a have in order to achieve an asymptotically stable controlled system? Declare additionally the Rosenbrock matrix of the system.

c) Which values should the parameters a and b have in order to achieve i) full observability and ii) detectability?

d) For the design of a linear optimal state feedback according to the quality criterion

$$J = \frac{1}{2} \int_0^\infty \boldsymbol{x}^T \begin{bmatrix} 9 & 0 \\ 0 & 3 \end{bmatrix} \boldsymbol{x} + u^2 \, dt$$

the elements P_{11}, P_{12} and P_{22} of $\boldsymbol{P}$ have to be assigned. What are in the case of $a = 12$ and $b = 1$, the three equations for P_{11}, P_{12} and P_{22}?

Problem 3

(each 3 points)

a) Give the modal matrix of the system
$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x}(t)$.

b) A system is described by $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{C}$. The dimension of the state vector $\boldsymbol{x}$ is 35. The modal matrix of $\boldsymbol{A}$ is $\boldsymbol{V}$. How can the observable and controllable modes be calculated numerically?

c) A system is described by

$$\boldsymbol{A} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix} \; ; \quad \boldsymbol{B} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \; ; \quad \boldsymbol{C} = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}.$$

Calculate the eigenvalues. Is the system stable, asymptotic stable or unstable?

d) Is the system observable?

e) Is the system controllable?

Problem 4

(each 3 points)

a) A system is given by the system matrix $\boldsymbol{A}$ with

$$\boldsymbol{A} = \begin{bmatrix} 3 & 4 & 5 \\ 1 & 2 & 3 \\ 1 & 0 & 1 \end{bmatrix} \; ; \quad \boldsymbol{B}^T = \begin{bmatrix} -1 & 0 & 1 \end{bmatrix} \; ; \quad \boldsymbol{C} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}.$$

Calculate the eigenvalues of $\boldsymbol{A}$. Is the system stable?

b) Is the system fully observable?

c) Is the system fully controllable?

d) Can the system be stabilized using $\boldsymbol{B}$?

e) Calculate the gain matrix $\boldsymbol{L}$ of an observer with the desired pole position at $\lambda_{1,2,3}$ = -1.

Problem 5

(each 3 points)

a) Draw the scheme of a linear MIMO system and denote the transfer elements, the signal flows and the inputs and outputs.

b) Draw additionally, the scheme of a Luenberger observer and denote the transfer elements, the signal flows and the inputs and outputs.

c) In which way the Luenberger observer can be physically realized? Denote the necessary hardware equipments and give hints about the necessary mathematical knowledge for its design. What can you emphasize about the dynamics of the observer?

d) The observer should be used for control purposes. The system is fully observable and fully controllable.
State the coupled dynamics of the model-based controller in state-space notation.

e) The system to be controlled is modeled by $\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u} + v(t)$, $\boldsymbol{y} = \boldsymbol{C}\boldsymbol{x}$ whereby $v(t) = a \sin \omega t$.

How can the disturbance be considered for the observer.
State the related equations and give the related error dynamics of the observer.

Problem 6

A rigid robot arm is driven by an electric motor. A gear couples the motor and the arm, which its elastic properties can not be neglected and should be modeled as a linear spring with constant stiffness coefficient c. The motor moment of inertia is J_1 and the arm moment of inertia is J_2. The different angels are the motor joint angle θ and the robot arm angle α. Between the input (motor angle θ) and the output (angle β), the ratio of the gear i is assumed. The dynamic behavior of the motor can be understood as a PT_1-behavior with the constants T and K. The control input $u(t)$ of the system is the motor voltage.
The equations of motion are

$$J_2\ddot{\alpha} + c(\alpha - \beta) = 0,$$

$$J_1\ddot{\theta} + \frac{c}{i}(\beta - \alpha) = e,$$

$$\dot{e} = -\frac{1}{T}e + \frac{K}{T}u.$$

The measured value is the angle β.

a) Give the equations of motion in a state-space notation. State the matrices $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$ in detail. Use therefore $\boldsymbol{x}^T = [\alpha \quad \beta \quad \dot{\alpha} \quad \dot{\beta} \quad e]$ as state vector.

   Hint: Use $\omega^2 = \frac{c}{J_2}$, $\Omega^2 = \frac{c}{i^2 M}$ as abbreviations.

b) Give the characteristic polynomial $p(\lambda) = \det[\lambda\boldsymbol{I} - \boldsymbol{A}]$ of the system to be controlled.

c) Calculate the eigenvalues of $\boldsymbol{A}$.

d) State if the system to be controlled is asymptotic stable or not.

e) Is the system fully controllable?

f) Is the system fully observable?

Maximum achievable points:	**100**