LAST NAME	
FIRST NAME	
MATRIKEL-NO.	

Problem 1

(each part 2 points)

a) Draw the scheme of a MIMO system described by $\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u}$, $\boldsymbol{y} = \boldsymbol{C}\boldsymbol{x}$ and describe the transmission blocks as well as the matrices.

b) Give the general solution for the output of a homogeneous vector differential equation $\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x}$, $\boldsymbol{y} = \boldsymbol{C}\boldsymbol{x}$.

c) The dynamics of a mechanical spring-mass-damper system is described by $m\ddot{x}_{\mathrm{m}} + d\dot{x}_{\mathrm{m}} + Kx_{\mathrm{m}} = u(t)$, with the input force $u(t)$. The measured value is $\dot{x}_{\mathrm{m}}$. State the correspondent state-space-description.

d) A system to be controlled is described by the matrices $\boldsymbol{A}$ and $\boldsymbol{B}$. State the equations to calculate the feedback matrix $\boldsymbol{K}$ using poleplacement approach.

e) Is the system described by the system matrix

$$\boldsymbol{A} = \begin{bmatrix} -3+4j & 0 \\ 0 & -3-4j \end{bmatrix}$$

stable?

Problem 2

(each part 2 points)

a) Calculate the eigenvalues of the observer applied to the system described using the matrices

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}, \; \boldsymbol{C} = [c_1 \quad c_2], \; \boldsymbol{L} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \quad \text{with} \quad c_2 = 0\,.$$

State the conditions for stable observer-dynamics.

b) Give the Rosenbrock matrix of the system given in a) with $\boldsymbol{D} = 0$, $c_2 = 0$ and $\boldsymbol{B} = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$ with $b_2 = 0$. Calculate the invariant zeros and the blocking zeros.

c) Is the system given in a) observable? State the conclusions for arbitrary c_1, c_2!

d) Calculate the feedback matrix $\boldsymbol{K} = [k_1 \quad k_2]$ for $\boldsymbol{B} = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$ and for the desired poles $\lambda_1 = -\frac{4}{3}$, $\lambda_2 = -\frac{3}{2}$.

e) Is the controlled system calculated in d) observable? State the conditions for c_1, c_2!

Problem 3

(each part 2 points)

a) Is the system described by

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u} \; ; \; \boldsymbol{y} = \boldsymbol{C}\boldsymbol{x} \quad \text{with} \quad \boldsymbol{A} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -a & 0 \end{bmatrix}, \; \boldsymbol{B} = \begin{bmatrix} 0 \\ 0 \\ b \end{bmatrix}, \; \boldsymbol{C} = [c \; 0 \; 0]$$

stable? Calculate the eigenvalues and the eigenvectors!

b) Is the system given in a) fully controllable?
If not, which eigenvalues are controllable?

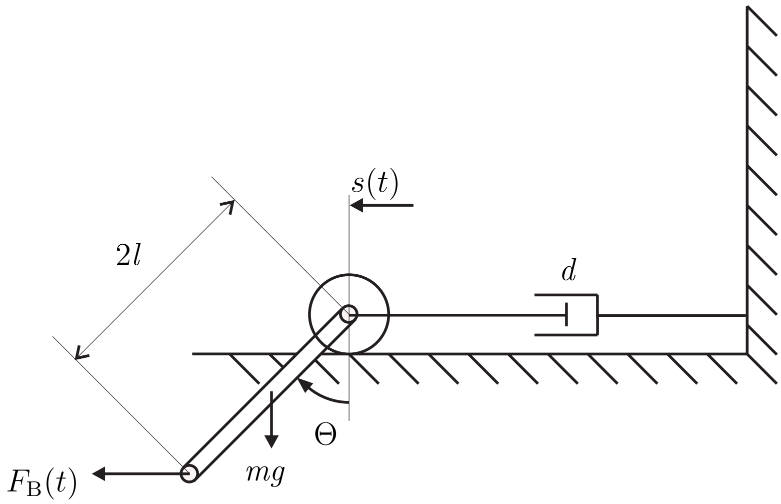
c) Is the system given in a) fully observable?
If not, which eigenvalues are observable?

d) The input matrix is changed to $\boldsymbol{B} = \begin{bmatrix} b_1 \\ 0 \\ b_2 \end{bmatrix}$. Can an arbitrary desired dynamics be achieved by state control?

e) Calculate $\boldsymbol{K} = [k_1 \; k_2 \; k_3]$ for $\lambda_1 = -1$, $\lambda_{2,3} = -\frac{10}{3}$ for $\boldsymbol{B} = \begin{bmatrix} b_1 \\ 0 \\ b_2 \end{bmatrix}$ with $b_1 = 0$.
Relate the solution to the result of question b).

Problem 4

(15 points)



The mechanical system shown in the figure is described by the following linearized equations of motion

$$\begin{aligned} ml\ddot{s} + m(l^2 + r^2)\ddot{\Theta} + mgl\Theta &= 2lF_\mathrm{B}\ , \\ m\ddot{s} + ml\ddot{\Theta} + d\dot{s} &= F_\mathrm{B}\ . \end{aligned}$$

a) (2 points)
Derive the state space model with the state space vector

$$x = \begin{bmatrix} s \\ \Theta \\ \dot{s} \\ \dot{\Theta} \end{bmatrix} .$$

b) (3 points)
Check the stability of the system with $m = 10$ kg, $d = 10$ kg/s, $l = 2$ m, $r = 0.2$ m, $g = 10$ m/s^2 (using the Hurwitz criteria).

c) (4 points)
Consider a state feedback with the feedback matrix $\boldsymbol{K} = [k_1 \quad 120 \quad -12 \quad 0.8]$. For which k_1 is the closed-loop system stable?

d) (4 points)
Use the same feedback matrix as in part c). Determine k_1 in the way that the closed-loop system has the eigenvalues
$\lambda_{1,2} = -11.4731 \pm 11.3320j$,
$\lambda_{3,4} = -0.4296 \pm 6.1709j$.
(Round up the results to the next integer number.)

e) (2 points)
The eigenvalues of the (uncontrolled) system are
$\lambda_1 = 0$,
$\lambda_2 = 93.6011$,
$\lambda_3 = 6.2531$,
$\lambda_4 = -0.8543$.

What can you state about the stability of the corresponding modes regarding

i) Ljapunov-stability?
ii) state stability?

Problem 5

(15 points)
A mechanical system is modelled using the differential equation

$$m\ddot{x} + d\dot{x} + kx = f(t)\ . \tag{5.1}$$

a) (1 point)
Build up the state-space model. The displacement x should be measured.

b) (2 points)
Use the values $m = 3$ kg, $d = 60$ kg/s, $k = 12$ kg/s^2
for calculating the eigenvalues of the system matrix.

c) (2 points)
Calculate the related (right-) eigenvectors.

d) (2 points)
Check the controllability for the input matrix resulting from (5.1) (with input $f(t)$) and for the input matrix $\boldsymbol{B} = \begin{bmatrix} b \\ 0 \end{bmatrix}$. Which condition has to be fulfilled?

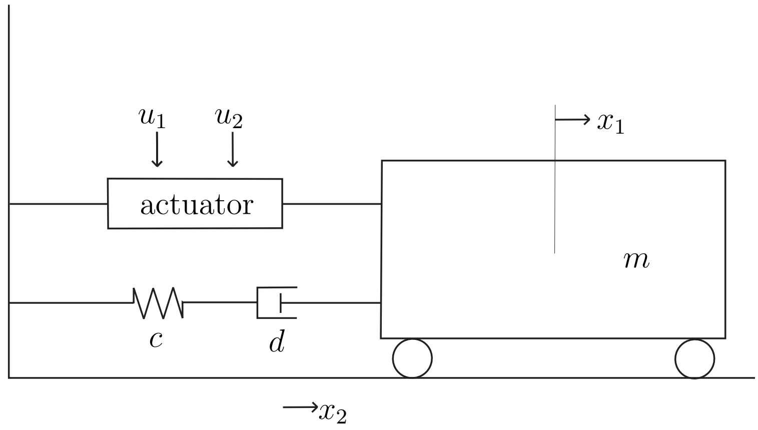
e) (2 points)
A disturbance n is acting on the second state. Write down the state space model considering this additional input.
Give an approximation for the disturbance n and write down a general model for the disturbance, using the system matrix $\boldsymbol{S}$ for the approximation.

f) (2 points)
Note a general extended state space model which includes the approximation and the model from part e).
Include the matrices/values given in parts a) and b) in this general extended model.

g) (4 points)
Take the special extended state space model for this system (from part f)) and use for the approximation $1 \cdot \boldsymbol{I}$ (with suitable dimension) and for the model $0.1 \cdot \boldsymbol{I}$ (with suitable dimension).
Check the observability of the extended system.
In case the system is fully observable, design the feedback so that the eigenvalues of the extended observer are
$\lambda_1 = -1,\ \lambda_2 = -2,\ \lambda_3 = -3.$

Problem 6

(40 points)



The positioning system shown in the picture should be examined.
Regarding only the mechanical part, it can be described by

$$\begin{aligned} m\ddot{x}_1 + d(\dot{x}_1 - \dot{x}_2) &= 0 \, , \\ cx_2 &= d(\dot{x}_1 - \dot{x}_2) \, . \end{aligned}$$

After rearranging and a differentiation, it results in

$$\dddot{x}_1 = -\frac{c}{d}\ddot{x}_1 - \frac{c}{m}\dot{x}_1 \, .$$

Further considering the novel actuator, whose inputs are voltages, the system can be (approximative) specified by

$$\begin{bmatrix} \dot{x}_1 \\ \ddot{x}_1 \\ \dddot{x}_1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -\frac{c}{m} & -\frac{c}{d} \end{bmatrix} \begin{bmatrix} x_1 \\ \dot{x}_1 \\ \ddot{x}_1 \end{bmatrix} + \begin{bmatrix} 0 & 0 \\ b_1 & 0 \\ 0 & b_2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} \, .$$

In addition the output is given by

$$\boldsymbol{y} = \begin{bmatrix} c_1 & 0 & 0 \\ 0 & c_2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ \dot{x}_1 \\ \ddot{x}_1 \end{bmatrix} \, .$$

a) (5 points)
Calculate the eigenvalues of the system (only by using the parameters, not detailed values).

b) (3 points)
Make a statement about the state stability of the system, assuming $c, d, m > 0$.

c) (6 points)
The parameters are given by $m = 2$ kg, $c = 0.5$ kg/s^2, $d = 0.25$ kg/s.

Is the system controllable for $b_1 = 1$?
If so, is it still controllable for $b_1 = 1$ and $b_2 = 0$?

Is the system observable for $c_2 = 1$?
If so, is it still observable for $c_2 = 1$ and $c_1 = 0$?

d) (4 points)
Set $b_1 = b_2 = 1$. Create a state feedback so that the eigenvalues of the closed loop system are $\lambda_1 = -1,\ \lambda_2 = -2,\ \lambda_3 = -3$.
Suggestion:
Set all elements of the second row of the feedback matrix equal to 1.

e) (4 points)
Set $c_1 = c_2 = 1$. Find an observer with the eigenvalues
$\lambda_1 = -4,\ \lambda_2 = -5,\ \lambda_3 = -6$.
Suggestion:
Set all elements of the first column of the observer matrix equal to 1.

f) (4 points)
Derive the error equation for the observer dynamics in general and for this special system (with the observer matrix calculated in part e)).

g) (4 points)
Now, use the observed states for feedback.
Set up the system equation with this feedback (in general, not for this special system).

Build up the equation of the complete system i. e. the system with feedback and the observer error-equation.

Build it up in general and for this system, using the feedback matrix calculated in part d) and using the observer matrix calculated in part e).

Give the eigenvalues of this extended system.
Name the method by which you calculate the eigenvalues.

h) (2 points)
How to choose the observer eigenvalues for practical applications?

i) (6 points)
Calculate the transfer function matrix

(with the given values and $\boldsymbol{B} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, \boldsymbol{C} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$).

Give the final shortened form.
Quote the poles and the transmission zeros of this MIMO system.

j) (2 points)
How does the first input u_1 affect the second output y_2? Give the related transfer function.

Maximum achievable points:	**100**
Minimum points for the grade 1,0:	**95**
Minimum points for the grade 4,0:	**50**