LAST NAME	
FIRST NAME	
MATRIKEL-NO.	

Problem 1

(22 points)

a) (4 points)
Give the block diagram of a MIMO–System with state control and denote all matrices and variables.

b) (4 points)
Discuss the asymptotic stability of the system given by

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t)$$

in relation to the position of the eigenvalues.
For which parameters δ is the system given with

$$\boldsymbol{A} = \begin{bmatrix} -\delta & \omega \\ -\omega & -\delta \end{bmatrix}$$

asymptotic stable?

c) (5 points)
The modeling of a system gives the nonlinear MIMO description

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} x_2 \\ -kx_1 - cx_1^3 - dx_2(1 + x_2^2) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u \quad .$$

Give the linearized MIMO–description for the equilibrium $x_{1e} = 0$, $x_{2e} = 0$ in terms of the system matrix $\boldsymbol{A}$ and the input matrix $\boldsymbol{B}$.

d) (5 points)
The asymptotic stability of the system $\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t)$ has to be shown using the Ljapunov Matrix equation. State the related equation. Show additionally the asymptotic stability of a system $\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t)$ given with

$$\boldsymbol{A} = \begin{bmatrix} -\delta & \omega \\ -\omega & -\delta \end{bmatrix} \quad .$$

e) (4 points)
Consider the system given by

$$\dot{\boldsymbol{x}}(t) = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \boldsymbol{x}(t) + \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} u(t) \quad .$$

Is it possible to define the complete dynamics of the system by state control (is pole-assignment possible for all eigenvalues of the system)?

Problem 2

(18 points)

a) (4 points)
Declare the terms asymptotic stability, stability and 'boundary' stable considering the system matrix $\boldsymbol{A}$.
Is the system given by

$$\dot{\boldsymbol{x}}(t) = \begin{bmatrix} -1 & 0 & -1 \\ 0 & 1 & 0 \\ -1 & 0 & -1 \end{bmatrix} \boldsymbol{x}(t)$$

unstable, stable, asymptotic stable or 'boundary' stable?

b) (4 points)
A mechanical system is described by

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \\ \ddot{z}_1 \\ \ddot{z}_2 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -\frac{c_1}{m_1} & \frac{c_1}{m_1} & 0 & 0 \\ \frac{c_1}{m_2} & -\frac{c_1+c_2}{m_2} & 0 & 0 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \\ \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -\frac{1}{m_1} \\ 0 \end{bmatrix} u \,,$$

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} [z_1 \quad z_2 \quad \dot{z}_1 \quad \dot{z}_2]^T \,.$$

For which c_1, m_1, c_2, m_2 is the system asymptotic stable?

c) (4 points)
Under which condition is the system fully observable?

d) (6 points)
The system given in b) should be controlled by $u(t) = [0 \quad 0 \quad k_1 \quad k_2]\boldsymbol{x}(t)$.
Calculate the gains k_1, k_2 for an asymptotic stable closed-loop system (assume c_1, m_1, c_2, $m_2 > 0$). (**Hint**: One way is to use Hurwitz criteria).

Problem 3

(60 points)
The transfer function of a mechanical system with the parameter a is given by

$$F_{\mathrm{s}}(s) = \frac{Y(s)}{U(s)} = \frac{5}{(s+2)(s+a)} \ .$$

This plant should be controlled using a PID-element

$$u(t) = -k_1\, \dot{y}(t) - k_2 y(t) - k_3 \int_0^t y(\tau) d\tau$$

as controller.

a) (9 points)
Derive the matrices $\boldsymbol{A}$, $\boldsymbol{B}$ and $\boldsymbol{C}$ of a state space representation

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}u(t) \ \text{ and } \ y(t) = \boldsymbol{C}\boldsymbol{x}(t)$$

of the plant with the state space vector

$$\boldsymbol{x}(t) = [x_1(t) \ \ x_2(t)]^T \text{ and the measurement } y(t) = x_1(t) \ .$$

b) (6 points)
What are the eigenvalues of the uncontrolled system?

c) (6 points)
Is the uncontrolled system stable (state reason)?

d) (6 points)
Is the system fully controllable (state reason)?

e) (6 points)
Is the system fully observable (state reason)?

f) (12 points)
Due to the I-part of the controller the state space representation has to be enhanced by a third component:

$$\boldsymbol{x}_{\mathrm{e}}(t) = [x_1(t) \ \ x_2(t) \ \ x_3(t)]^T \ , \ x_3(t) = \int_0^t y(\tau) d\tau \ .$$

What is the system matrix $\boldsymbol{A}_{\mathrm{e}}$ of the closed-loop system $\dot{\boldsymbol{x}}_{\mathrm{e}}(t) = \boldsymbol{A}_{\mathrm{e}}\boldsymbol{x}_{\mathrm{e}}(t)$?

g) (7.5 points)
Which condition have to be fulfilled by the gains k_1, k_2, k_3 of the controller described above to make the system asymptotically stable?

h) (7.5 points)
Design the values of the gains k_1, k_2, k_3 so that the closed-loop system has the eigenvalues $\lambda_{1,2,3} = -1$.

Maximum achievable points:	**100**
Minimum points for the grade 1,0:	**95**
Minimum points for the grade 4,0:	**50**