LAST NAME	
FIRST NAME	
MATRIKEL-NO.	

Problem 1

(3 points each)

a) State the typical mathematical description of the vector equations describing the Multi-Input-Multi-Output (MIMO) time behavior of dynamical systems and declare the matrix elements in detail.

b) State the typical mathematical description of the vector equation describing the MIMO-behavior in frequency domain and declare the matrix elements in detail.

c) Calculate $\boldsymbol{G}(s)$ for the system given by

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} 1 & 0 \\ 1 & -1 \end{bmatrix}, \quad \boldsymbol{C} = \begin{bmatrix} 2 & 0 \end{bmatrix}, \quad \text{and} \quad \boldsymbol{D} = \begin{bmatrix} 0 & 0 \end{bmatrix}.$$

d) Is the system described by $\boldsymbol{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & a \\ 3 & 0 & 0 \end{bmatrix}$ stable? Calculate the stability condition(s) with respect to the parameter a.

e) The right eigenvectors of a system description are given with

$$\tilde{\boldsymbol{x}}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad \text{and} \quad \tilde{\boldsymbol{x}}_2 = \begin{bmatrix} -3 \\ a \end{bmatrix}.$$

The related measurement matrix is $\boldsymbol{C} = \begin{bmatrix} a & -2 \end{bmatrix}$. Calculate the condition for the observability of the eigenmotions of the system.

Problem 2

(3 points each)

a) A system is given by

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & a \\ 3 & 0 & 0 \end{bmatrix}, \qquad \boldsymbol{B} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \text{and} \qquad \boldsymbol{C} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}.$$

It is noted that no direct transmission between the inputs and the outputs / measurements is given. Give the Rosenbrock system matrix $\boldsymbol{P}(s)$.

b) Calculate the invariant zeros of the system given in a).

c) Define the input/output decoupling zeros, and the conditions for input/output decoupling zeros?

d) A system is described with the system matrix $\boldsymbol{A}$ and the measurement matrix $\boldsymbol{C}$. Declare the calculation steps and give the equations to calculate the feedback matrix / gain for the linear quadratic optimal observer. Which conditions have to be fulfilled to realize the observer?

e) Declare the strategy for practical realization of a model-based state reconstruction scheme (= observer) by a sketch. Please define the whole processor-based observer realization using elements for A/D, D/A - conversion, the microcontroller, measurement devices. Give a statement about the relation between the number of independent measurement channels and the rank of $\boldsymbol{C}$, the number of states to be reconstructed and the related dimensions of matrices and vectors to be multiplied and added. No direct transmission between the inputs and the outputs/measurements is given.

Problem 3

(30 points)

a) Transfer the differential equation

$$\ddot{y}(t) + (0.32b)\dot{y}(t) + 9y(t) = \int_0^t u(\tau)d\tau$$

to a state space representation and calculate its eigenvalues. The measured value is y.

b) For which real values b is this system state stable? Give b so that all eigenvalues are real values.

The following state space representation is given

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 14 & 0.5 \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} 0 \\ 0 \\ 4 \end{bmatrix}, \quad \text{and} \quad \boldsymbol{C} = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}.$$

c) Check the controllability of the system by the use of its eigenvectors. Is the system fully controllable?

d) Check the observability of the system by the use of its eigenvectors. Is the system fully observable.

e) The system has to be asymptotically stable. Is it possible to realize this with a state control (state reason)? Is it possible to estimate the system states by the use of an observer (state reason)? Can the control be improved by the use of a state observer (state reason)?

Problem 4

(40 points)

The system

$$\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u} \\ \boldsymbol{y} &= \boldsymbol{C}\boldsymbol{x} \end{aligned}$$

with the matrices

$$\boldsymbol{A} = \begin{bmatrix} 0 & 0 & 125/16 \\ 8 & 0 & 5/4 \\ 0 & 16 & -2 \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} -125/128 & 0 \\ 0 & -125/16 \\ 0 & 0 \end{bmatrix}, \quad \text{and } \boldsymbol{C} = \begin{bmatrix} 0 & 0 & 8 \end{bmatrix}$$

is given.

a) Give the equilibrium state $\boldsymbol{x}_\mathrm{g}$ of the system.
Examine the stability of the state $\boldsymbol{x}_\mathrm{g}$.

b) Can all states of the system be estimated (reconstructed) (state reason)?

c) How many inputs has the system?
The eigenvalues of the system can be shifted arbitrary by using the 2nd input only. Is this also possible by using the 1st input only (state reason)?

d) A state feedback has to be designed. Use the information of part c). That means, reduce the dimension (i.e. remove column(s) and / or row(s)) of the matrix $\boldsymbol{B}$ and of the input vector $\boldsymbol{u}$, so that for the following design only the 2nd input is used. Calculate the desired eigenvalues of the controlled system as follows:

1. Change the sign of all unstable eigenvalues.
2. Raise the damping of all conjugated complex eigenvalues to 4/3 of there original damping ($d_\mathrm{desired} = 4/3\, d_\mathrm{original}$) without changing there distances to the imaginary axis.

Now, calculate the related feedback matrix $\boldsymbol{K}$ for the feedback $u = -\boldsymbol{K}\boldsymbol{x}$.

e) Now the necessary observer dynamics has to be designed to be able to realize the state feedback. The observer should be faster than the system that has to be observed.

1. Which is the "fastest" eigenvalue of the observed system?
2. How do you have to choose the eigenvalues of the observer to make it faster than the observed system?
3. By using the transformation

$$\boldsymbol{x}_\mathrm{o} = \boldsymbol{T}_\mathrm{o}^{-1}\boldsymbol{x} \tag{4.1}$$

with

$$\boldsymbol{T}_\mathrm{o} = \begin{bmatrix} 1/1024 & 0 & 0 \\ 0 & 1/128 & 0 \\ 0 & 0 & 1/8 \end{bmatrix} \tag{4.2}$$

the matrices $\boldsymbol{A}$ and $\boldsymbol{C}$ can be transformed to

$$\boldsymbol{A}_\mathrm{o} = \begin{bmatrix} 0 & 0 & 1000 \\ 1 & 0 & 20 \\ 0 & 1 & -2 \end{bmatrix}, \quad \boldsymbol{C}_\mathrm{o} = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}. \tag{4.3}$$

Use this as a help to design the observer. The desired eigenvalues of the observer should be λ_1 = -20, λ_2 = -30, and λ_3 = -40. Determine the observer feedback matrix $\boldsymbol{L}$.

f) How many eigenvalues has the uncontrolled system examined in part a) (state reason but do not calculate)?
State the mathematical overall model that comprises the controlled system and the observer (state vector $\boldsymbol{x}_\mathrm{f} = \begin{bmatrix} \boldsymbol{x} \\ \hat{\boldsymbol{x}} \end{bmatrix}$). Give the eigenvalues of this complete description.

Maximum achievable points:	**100**
Minimum points for the grade 1,0:	**95**
Minimum points for the grade 4,0:	**50**