LAST NAME	
FIRST NAME	
MATRIKEL-NO.	

Problem 1

(3 points each)

a) Illustrate in detail the typical graphical description of the state-space vector differential equation describing the Multi-Input-Multi-Output (MIMO) time behavior of dynamical systems.

b) Give the definition of the transfer function matrix and of the Rosenbrock matrix.

c) Is the system described with $\boldsymbol{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & a \end{bmatrix}$ stable? Calculate the stability condition(s) with respect to the parameter a.

d) Calculate $G(j\omega)$ for the system description given with

$$\boldsymbol{A} = \begin{bmatrix} 0 & 2 \\ 0 & -2 \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} 1 & 0 \\ 1 & -1 \end{bmatrix}, \quad \boldsymbol{C} = \begin{bmatrix} 1 & 1 \end{bmatrix}, \quad \text{and} \quad \boldsymbol{D} = \begin{bmatrix} 0 & 0 \end{bmatrix}.$$

e) A system is stated as BIBO-stable. Does this include the state stability of the system; or: is it possible that the BIBO-stable system is not state-stable? Explain your answer using the properties of decoupling zeros.

Problem 2

(3 points each)

a) A system description is given with

$$\boldsymbol{A} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -3 \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & b \end{bmatrix}, \quad \boldsymbol{C} = \begin{bmatrix} c & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad b \neq 0,\ c \neq 0.$$

No direct transmission between the inputs and the outputs/measurements is given. State the Rosenbrock system matrix $\boldsymbol{P}(s)$.

b) What are the eigenvalues of the system described by $\boldsymbol{A}$? Is the system BIBO-stable (state reason)? Is the system state-stable (state reason)?

c) Calculate the invariant zeros for the system given in a).

d) Give the input/output decoupling zeros for the system given in a).

e) Is the system stabilizable? Is it possible to realize full state feedback?

Problem 3

(30 points)

A system is given by

$$\boldsymbol{A} = \begin{bmatrix} -1 & 0 & 1 \\ 0 & -1 & 1 \\ 0 & 0 & 1 \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \text{and} \quad \boldsymbol{C} = \begin{bmatrix} 2 & 4 & -2 \end{bmatrix}.$$

There is no direct transmission between the inputs and the outputs / measurements.

a) (3 points)
Calculate the eigenvalues of the system. Is the system stable?

b) (6 points)
Calculate the transfer function matrix for the system and calculate the poles. Has the system more poles than eigenvalues, more eigenvalues than poles, or are the numbers equal?
Hint: For a transfer function matrix $\boldsymbol{G} = \begin{bmatrix} \frac{1}{(s+a)(s+b)} & \frac{1}{(s+a)} \end{bmatrix}$, there are only two poles: $s_1 = -a, s_2 = -b$.

c) (4 points)
Check the controllability of the system. Is the system fully controllable?

d) (4 points)
Is the system fully observable? Please draw the conclusion and state reasons without calculation (use the results from part b) and c)).

e) (8 points)
A second measurement, which is $y_2 = 2x_1 + cx_2 - 3x_3$, is realized. Determine the new output matrix $\boldsymbol{C}_{new}$. Calculate the condition for full observability with respect to parameter c for the new system $(\boldsymbol{A}, \boldsymbol{C}_{new})$ with Gilbert criteria.

f) (5 points)
Determine the weighting matrices $\boldsymbol{Q}$ and $\boldsymbol{R}$ with the cost function given by

$$\begin{aligned} J &= \frac{1}{2}\int_0^\infty [\boldsymbol{x}^T\boldsymbol{Q}\boldsymbol{x} + \boldsymbol{y}^T\boldsymbol{R}\boldsymbol{y}]dt \longrightarrow Minimum \\ &= \frac{1}{2}\int_0^\infty [x_1^2(t) + x_1(t)x_2(t) + x_2^2(t) + x_3^2(t) + y_1^2(t) + y_2^2(t)]dt \longrightarrow Minimum. \end{aligned}$$

Declare the calculation steps and give the equations to calculate the matrix $\boldsymbol{L}$ for the linear quadratic optimal observer, when the system is observable.

Problem 4

(40 points)

A new approach for controlling hybrid mechatronical systems, consisting of electric and combustion engines, has to be analyzed. The mathematical model

$$\dot{\boldsymbol{x}} = \begin{bmatrix} 0 & 2 & 3 \\ 2 & 3 & 0 \\ 1 & 1 & 0 \end{bmatrix} \boldsymbol{x} + \begin{bmatrix} 0 & 1 \\ -1 & 0 \\ 0 & 0 \end{bmatrix} \boldsymbol{u}$$

$$y = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} \boldsymbol{x} + \begin{bmatrix} 0 & 0 \end{bmatrix} \boldsymbol{u} \, .$$

describes this system.

a) How many inputs, how many outputs has the system?
(2 points)

b) Is the system stable? If not, is it possible to place the eigenvalues arbitrarily?
(4 points)

c) Can all states of the system be estimated (be reconstructed) (state reason)?
(3 points)

For practical purpose, it is necessary to control the observed system by state-feedback control.

d) Which state ($\boldsymbol{x}$ or $\hat{\boldsymbol{x}}$) has to be controlled (state reason)?
(2 points)

e) Calculate the feedback matrix

$$\boldsymbol{K} = \begin{bmatrix} k_1 & k_2 & k_3 \\ k_4 & k_5 & k_6 \end{bmatrix}$$

in such a way that the eigenvalues of the controlled system are $\lambda_1 = -1$, $\lambda_2 = -2$, and $\lambda_3 = -3$ (let k_3, k_4, and k_5 be 0).
(11 points)

For the following analysis, the modified system

$$\dot{\boldsymbol{x}} = \begin{bmatrix} -2 & 0 \\ 0 & -2 \end{bmatrix} \boldsymbol{x} + \begin{bmatrix} \sqrt[3]{\pi} & 1 \\ -1 & -\arccos(\frac{\pi}{3}) \end{bmatrix} \boldsymbol{u}$$

$$\boldsymbol{y} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \boldsymbol{x} + \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \boldsymbol{u}$$

is considered.
An observer has to be applied to the system.

f) Sketch the blockdiagram of the observed and the controlled system.
(4 points)

g) Is it possible to design an observer for the given system? If so, assume the matrices $\boldsymbol{Q}$ and $\boldsymbol{R}$ as identity matrices with appropriate dimensions and state the cost function J (simplify the expression as much as possible without solving the integral). State the equation for observer design and calculate the matrix $\boldsymbol{L}$.

Hint: Assume for $\boldsymbol{P}$ the following structure

$$\boldsymbol{P} = \begin{bmatrix} p & 0 \\ 0 & p \end{bmatrix}.$$

(10 points)

h) Calculate the eigenvalues of the observer.
(2 points)

i) Why is it reasonable to apply an observer to an observable system? State 2 reasons!
(2 points)

Maximum achievable points:	**100**
Minimum points for the grade 1,0:	**95**
Minimum points for the grade 4,0:	**50**