LAST NAME	
FIRST NAME	
MATRIKEL-NO.	

Problem 1

(3 points each)

a) Illustrate in detail the typical graphical description of the state-space vector differential equation for the Luenberger observer including the observed system.

b) Give the mathematical definitions of transmission zeros and of decoupling zeros.

c) The system described with $\boldsymbol{A} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & a \end{bmatrix}$ is controlled by using $\boldsymbol{B} = \begin{bmatrix} b \\ 0 \\ 0 \end{bmatrix}$ and $\boldsymbol{K} = \begin{bmatrix} 0 & 0 & k \end{bmatrix}$. Calculate the stability condition(s) with respect to the parameters a and b.

d) A system is given by

$$3\ddot{x} + 4\dot{x} + 5x = 6u;$$

the output (measurement) is $y = 2x - 3\dot{x}$.
Calculate $\boldsymbol{G}(s)$ based on the state-space description.

e) A system is stated as state-stable. Does this include the BIBO-stability of the system; or: is it possible that the state-stable system is not BIBO-stable? Explain your answer using properties of decoupling zeros.

Problem 2

(3 points each)

a) A system description is given with

$$\boldsymbol{A} = \begin{bmatrix} -3 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -1 \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & -b \end{bmatrix}, \quad \boldsymbol{C} = \begin{bmatrix} c & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}, \quad b \neq 0, c \neq 0.$$

No direct transmission between the inputs and the outputs/measurements is given. State the Rosenbrock system matrix $\boldsymbol{P}(s)$.

b) What are the eigenvalues of the system described by $\boldsymbol{A}$? Is the system state-stable (state reason)? Is the system asymptotically stable in the sense of Ljapunov (state reason)?

c) Calculate the invariant zeros for the system given in a).

d) Give the input/output decoupling zeros for the system given in a).

e) Under which assumptions is the system fully observable? Is it possible to realize an identity observer?

Problem 3

(30 points)

The dynamics of a quarter car system can be approximately modeled by a spring-mass-damper system as shown in Fig. 3.1. The corresponding differential equations are given by

$$m_a\ddot{x}_1 + d_1(\dot{x}_1 - \dot{x}_d) + d_2(\dot{x}_1 - \dot{x}_2) + k_1(x_1 - x_d) + k_2(x_1 - x_2) = -f_u, \quad (3.1)$$
$$m_r\ddot{x}_2 + d_2(\dot{x}_2 - \dot{x}_1) + k_2(x_2 - x_1) = f_u. \quad (3.2)$$

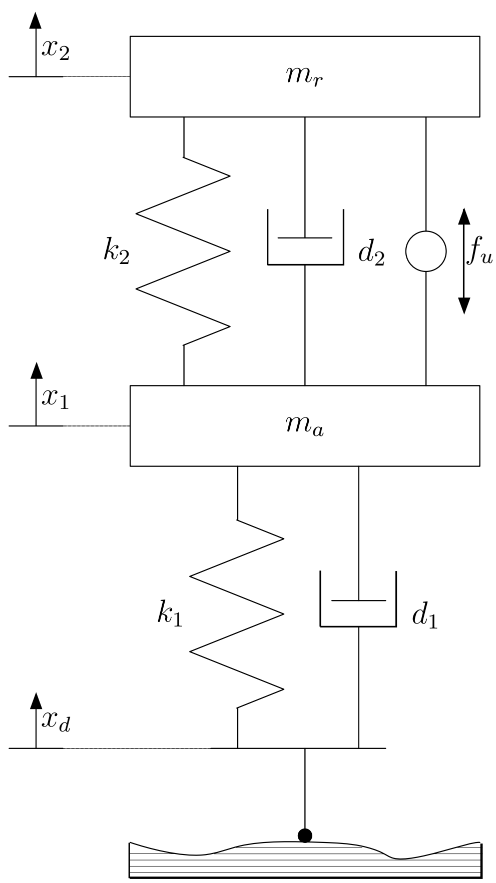

Figure 3.1: Quarter car model

a) (4 points)
The states are defined as $[\ x_1 \ \ x_2 \ \ x_3 \ \ x_4\ ]^T = [\ x_1 \ \ x_2 \ \ \dot{x}_1 \ \ \dot{x}_2\ ]^T$. The displacements x_1, x_2 are measured. Set up the state space model of the system with the parameters $m_a = m_r = 1$, $k_1 = k_2 = 2$, $d_1 = 3$, $d_2 = 1$, $x_d = 0$, and $\dot{x}_d = 0$.

b) (6 points)
Calculate the eigenvalues of the system. Is the system stable? State reason.
Hint: Two eigenvalues of the system are known, $\lambda_1 = \lambda_2 = -2$.

c) (4 points)
Is the system fully observable?

d) (8 points)
With the input f_u the system is fully controllable. Calculate the feedback matrix

$$\boldsymbol{K} = \begin{bmatrix} k_1 & k_2 & k_3 & k_4 \end{bmatrix}$$

in such a way that the eigenvalues of the controlled system are $\lambda_{1,des} = -1$, $\lambda_{2,des} = -2$, $\lambda_{3,des} = -3$, and $\lambda_{4,des} = -4$ (let $k_1 = 2$ and $k_3 = 1$).

e) (2 points)
How will the system behavior change with the new eigenvalues in d) comparing with the original eigenvalues in b)?
(If you did not get the eigenvalues in b), please assume that the original eigenvalues are $\lambda_{1,2} = -2$, $\lambda_{3,4} = -3.41 \pm \frac{\sqrt{\pi}}{6.8} i$).

f) (6 points)
Based on the differential equations, take the states, parameters, and measurements given in a) except for $d_1 = 0$ and $x_d \neq 0$.
Set up the new state space model of the system.
Calculate the transfer function from the road disturbance x_d to the output $y_1 = x_1$.

Hint: The inverse matrix of $[s\boldsymbol{I} - \boldsymbol{A}_{new}]$ is given by

$$[s\boldsymbol{I} - \boldsymbol{A}_{new}]^{-1} =$$

$$= \frac{1}{s^4 + 2s^3 + 6s^2 + 2s + 4} \begin{pmatrix} s^3 + 2s^2 + 2s & 2s & s^2 + s + 2 & s + 2 \\ 2s - 2 & s^3 + 2s^2 + 4s + 2 & s + 2 & s^2 + s + 4 \\ -4s^2 - 2s - 4 & 2s^2 & s^3 + s^2 + 2s & s^2 + 2s \\ 2s^2 - 2s & -2s^2 - 4 & s^2 + 2s & s^3 + s^2 + 4s \end{pmatrix}.$$

Problem 4

(40 points)

The system

$$\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u} \\ \boldsymbol{y} &= \boldsymbol{C}\boldsymbol{x} \end{aligned}$$

with the matrices

$$\boldsymbol{A} = \begin{bmatrix} -1 & 12 & -3 \\ 0 & 2 & 0 \\ 0 & 10 & -3 \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \\ -1 & 0 \end{bmatrix}, \quad \text{and } \boldsymbol{C} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

is given.

a) (10 points)
Determine the transfer function matrix and state the zeros, poles, and eigenvalues.

b) (4 points)
For the design of an optimal observer the matrices

$$\boldsymbol{Q} = \begin{bmatrix} 1 & -1 & 0 \\ -1 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \boldsymbol{R} = \begin{bmatrix} 2 \end{bmatrix}$$

are given. State the cost function J and simplify to a short form. The integral itself should not be calculated.

c) (8 points)
The matrix $\boldsymbol{L}$ is given with

$$\boldsymbol{L} = \begin{bmatrix} -1 \\ 2 \\ -2 \end{bmatrix}.$$

State and calculate the equation of the observer and determine the eigenvalues of the observer.

d) (6 points)
Determine the damping (of all modes) of the system and of the observer. Draw the eigenvalues in the complex s-plane. Is the observer suitable for the given system? (state reason)
If you have no solution for subtask a) or c) assume the following values.
system: $\lambda_1 = -5,\ \lambda_2 = -3,\ \lambda_3 = 3$; observer: $\lambda_1 = 0,\ \lambda_2 = -1 + 4.5i,\ \lambda_3 = -1 - 4.5i$

e) (8 points)
The given system from subtask a) has to be controlled. The matrix $\boldsymbol{K}$ is given with

$$\boldsymbol{K} = \begin{bmatrix} 2 & -1 & -4 \\ 1 & 4 & 2 \end{bmatrix}.$$

Is the controlled system stable?

f) (4 points)
Give the representation of the observed and controlled system in a graphical form.

Maximum achievable points:	**100**
Minimum points for the grade 1,0:	**95**
Minimum points for the grade 4,0:	**50**