LAST NAME	
FIRST NAME	
MATRIKEL-NO.	

Problem 1

(3 points each)

a) Denote by vector equation in detail a state-space description in frequency domain describing a Multi-Input-Multi-Output (MIMO) dynamical system.

b) Distinguish (i.e. label) the two different parts of the solution equation of an inhomogenous MIMO-system. Which part(s) describe the stationary and the instationary behavior of the system?

c) Calculate $\boldsymbol{G}(s)$ for the system description given with

$$\boldsymbol{A} = \begin{bmatrix} 0 & 3 \\ 0 & -3 \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}, \quad \boldsymbol{C} = \begin{bmatrix} 1 & 1 \end{bmatrix} \text{and } \boldsymbol{D} = \begin{bmatrix} 0 & 0 \end{bmatrix} .$$

d) Calculate the stability condition(s) for the system described with $\boldsymbol{A} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 3 & 0 & z \end{bmatrix}$ with respect to the parameter z.

e) A linear system is stated as asymptotically stable after Ljapunov. An input occurs and excites the system in all modes, causing also the outputs to change their time behavior. After some time the input vanishes. What happens i) with the time behavior of the system states and ii) with the time behavior of the outputs (after the vanishing of the input)?

Problem 2

(3 points each)

a) Define by equation the input and output decoupling zeros. Explain the difference to transmission zeros?

b) A system description is given with

$$\boldsymbol{A} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 3 \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} 0 & 0 \\ 0 & 1 \\ 0 & b \end{bmatrix}, \text{and } \boldsymbol{C} = \begin{bmatrix} c & 0 & 0 \end{bmatrix}, \; b \neq 0, \; c \neq 0.$$

It is noted, that no direct transmission between the inputs and the outputs/measurements is given.

State the eigenvalues of the system described by $\boldsymbol{A}$. Is the system state stable? Is the system asymptotically stable? Denote the difference between state stability and asymptotic state stability.

c) Calculate the transmission zeros for the system given in b).

d) Is the system observable? Is the system stabilizable? Is it possible to realize full state feedback?

e) Give the observability condition for the disturbance observer and explain the condition in the context of output decoupling zeros.

Problem 3

(30 points)

The following state space representation is given

$$\boldsymbol{A} = \begin{bmatrix} -3 & 0 & 5 \\ 1 & 3 & -5 \\ -1 & 0 & 3 \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \quad \text{and} \quad \boldsymbol{C} = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}.$$

a) (5 points)
Check the stability of the system. Is it a SISO or a MIMO-system?

b) (6 points)
Quote the transfer function and show the detailed calculation. The correct solution is

$$G(s) = \frac{5}{(s+2)(s-2)}. \tag{3.1}$$

(!Hint: see fig. 3.1)

$$A^{-1} = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}^{-1} = \frac{1}{\det(A)} \begin{pmatrix} ei-fh & ch-bi & bf-ce \\ fg-di & ai-cg & cd-af \\ dh-eg & bg-ah & ae-bd \end{pmatrix}$$

Figure 3.1: Calculation of the inverse matrix

c) (5 points)
Calculate the poles and zeros of the system. Classify the determined zeros (transfer zeros, decoupling zeros).

d) (6 points)
Determine the controllability and observability of the system. Which eigenvalues are not controllable or not observable?

e) (8 points)
Calculate the parameter k_3 of the matrix

$$\boldsymbol{K} = \begin{bmatrix} -1 & -1 & k_3 \end{bmatrix}$$

for a state feedback controller. The desired eigenvalues are $\lambda_1 = 2$, $\lambda_2 = -1$, and $\lambda_3 = -2$. Give a statement about the stability and the observability of the controlled system.

Reflect about the sense and practical realization about this subtask and denote your opinion. (The calculation has to be done in either case.)

Problem 4

(40 points)

A mechanical system is described by

$$\begin{aligned} \dot{\boldsymbol{x}} &= \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u} \\ \boldsymbol{y} &= \boldsymbol{C}\boldsymbol{x} \end{aligned}$$

with the matrices

$$\boldsymbol{A} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 4 & 2 & 0 & d \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \quad \text{and } \boldsymbol{C} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}.$$

a) (8 points)
Calculate the eigenvalues (two eigenvalues are given with λ_1 = -1 and $\lambda_2 = 1$). For which d is the system state stable?

b) (6 points)
The system from a) is controlled by state feedback with the matrix

$$\boldsymbol{K} = \begin{bmatrix} 0 & k_2 & k_3 & 0 \end{bmatrix}.$$

For which parameters of d, k_2, and k_3 is the system asymptotic stable (use Hurwitz!).

c) (6 points)
How many inputs and outputs does the system from a) have? Is the system for $d = 0$ fully observable?

d) (8 points)
A new system

$$\begin{aligned}\dot{\boldsymbol{x}} &= \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u}\\ \boldsymbol{y} &= \boldsymbol{C}\boldsymbol{x}\end{aligned}$$

with the matrices

$$\boldsymbol{A} = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \quad \boldsymbol{B} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad \text{and } \boldsymbol{C} = \begin{bmatrix} 0 & 1 \end{bmatrix}$$

is given. Additionally, for the design of an optimal observer the matrices

$$\boldsymbol{Q} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad \boldsymbol{P} = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}, \quad \boldsymbol{R} = \begin{bmatrix} 0.1 \end{bmatrix}$$

are given. Calculate $\boldsymbol{L}$ of a linear quadratic optimal Luenberger Observer (Assume: $p_{12} = p_{21} = p_{22} > 0$).

e) (4 points)
For the system from d) the matrix $\boldsymbol{L}$ is given with

$$\boldsymbol{L} = \begin{bmatrix} 25 \\ 10 \end{bmatrix}$$

State and calculate the system matrix of the observation error and give the equations of dynamic for the whole observer equation ($\dot{\hat{x}} = ...$) in a general (variables) and a special (with the given values) form.

f) (2 points)
Assume $\boldsymbol{K}$ as

$$\boldsymbol{K} = \begin{bmatrix} 3 & 3 \end{bmatrix}.$$

State the system matrix of the controlled system from d) without the observer.

g) (6 points)
Give the system matrix of the system with observer error ($\begin{pmatrix} \dot{x} \\ \dot{e} \end{pmatrix} = ...$) and calculate the eigenvalues.

Maximum achievable points:	**100**
Minimum points for the grade 1,0:	**95**
Minimum points for the grade 4,0:	**50**