Solutions for Exam
Control Theory
August 10th, 2007

1) a) $\begin{bmatrix} x_0 \\ Y(s) \end{bmatrix} = \begin{bmatrix} sI-A & -B \\ C & D \end{bmatrix} \begin{bmatrix} X(s) \\ U(s) \end{bmatrix}$

) $x(t) = \underbrace{e^{At}x_0}_{\text{free motion}} + \underbrace{\int_0^t e^{A(t-\tau)}Bu(\tau)d\tau}_{\text{forced response}}$

free motion
gives only instationary contributions

forced response
gives stationary and instationary contributions (assuming A is stable)

) $G(s) = C(sI-A)^{-1}B + D = \left[\frac{1}{s} \quad \frac{-s-3}{s^2+3s} \right]$

) $\det(\lambda I - A) = (\lambda+1)(\lambda-1)(\lambda-2) = 0$

$\Leftrightarrow \lambda_1 = 1;\ \lambda_2 = -1;\ \lambda_3 = 2$

$\hookrightarrow$ System is always unstable

) i) For all states x_i

$x_i \to 0$ for $t \to \infty$

ii) For all outputs y_j

$y_j \to 0$ for $t \to \infty$

2) a) Rank$[s_0 I - A \quad -B] < n$ $\rightarrow$ s_0 is (an eigenvalue and) an output decoupling zero

Rank$\begin{bmatrix} s_0 I - A \\ C \end{bmatrix} < n$ $\rightarrow$ s_0 is (an eigenvalue and) an input decoupling zero

Transmission zero $\tilde{s}_0$ $\Rightarrow$ $y_{stationary}(t) = 0$ for $u(t) = e^{\tilde{s}_0 t}$

Input decoupling zero s_0 $\Rightarrow$ eigenmotion $e^{s_0 t}$ is not affected by the input $u(t)$

Output — " — $\Rightarrow$ eigenmotion $e^{s_0 t}$ does not affect the output $y(t)$

b) Eigenvalues: $\lambda_1 = 1$; $\lambda_2 = -2$; $\lambda_3 = 3$

Due to positive values of λ_1 and λ_3 the system is <u>not</u> state stable, least of all asymptotically stable.

<u>State stability</u>: For all $\varepsilon > 0$ there exists a δ with $\|x_0\| < \delta$ such that $\|x(t)\| < \varepsilon$.

<u>Asymptotic state stability</u>: Additionally $\lim_{t \to \infty} \|x(t)\| = 0$ has to be fulfilled.

c) $G(s) = [0 \quad 0]$ $\Rightarrow$ No transmission for any frequency

2 d) A is of diagonal form and C comprises two 0-columns $\left([c \; \underset{\uparrow}{0} \; \underset{\uparrow}{0}] \right)$ => GILBERT: System is not fully observable

No, because $\lambda_1 = 1$ is not controllable (GILBERT).

No, the system is not fully observable so it is not possible to realize full state feedback.

e) With $\dot{v} = Fv$; $f = Hv$; $A_{ext} = \begin{bmatrix} A & NH \\ 0 & F \end{bmatrix}$; $C_{ext} = [C \; 0]$

the eigenvalues of the observer matrix $A_{ext} - L C_{ext}$ can be placed arbitrarily iff the system (A_{ext}, C_{ext}) is fully observable and no output decoupling zero exists respectively.

problem 3)

a) $\det(A - \lambda E) \overset{!}{=} 0$

$$\begin{vmatrix} -3-\lambda & 0 & 5 \\ 1 & 3-\lambda & -5 \\ -1 & 0 & 3-\lambda \end{vmatrix} = (-\lambda-3)(3-\lambda)(3-\lambda) + 5(3-\lambda)$$

$$= (-\lambda-3)(\lambda^2 - 6\lambda + 9) + 15 - 5\lambda$$

$$= -\lambda^3 + 6\lambda^2 - 9\lambda - 3\lambda^2 + 18\lambda - 27 + 15 - 5\lambda$$

$$= -\lambda^3 + 3\lambda^2 + 4\lambda - 12 \overset{!}{=} 0$$

Hurwitz $\Rightarrow$ not stable

- It is a SISO - System

b) $G(s) = C(A - sE)^{-1} \cdot B + D$

$$(A - sE)^{-1} = \begin{pmatrix} -3-s & 0 & 5 \\ 1 & 3-s & -5 \\ -1 & 0 & 3-s \end{pmatrix}^{-1} = \frac{1}{s^3 - 3s^2 - 4s + 12} \cdot$$

$$\begin{pmatrix} (3-s)^2 & 0 & -5(3-s) \\ 8-s & (-3-s)(3-s)+5 & 5+5(-3-s) \\ 3-s & 0 & (-3-s)(3-s) \end{pmatrix}$$

$$G(s) = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \frac{1}{s^3 - 3s^2 - 4s + 12} \begin{bmatrix} (3-s)^2 & 0 & -5(3-s) \\ 8-s & (-3-s)(3-s)+5 & s+5(-3-s) \\ 3-s & 0 & (-3-s)(3-s) \end{bmatrix} \cdot \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

$$= \frac{1}{s^3 - 3s^2 - 4s + 12} \begin{bmatrix} (3-s)^2 & 0 & -5(3-s) \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$$

$$= \frac{-5(3-s)}{s^3 - 3s^2 - 4s + 12} = \frac{-5(3-s)}{(s+2)(s-2)(s-3)} = \frac{5}{(s+2)(s-2)}$$

c) $G(s_{0_i}) = 0 \quad \Rightarrow$ no transfer zero

$$P(s) = \begin{pmatrix} sE - A & -B \\ C & D \end{pmatrix}$$

$$\det(P(s_0)) \overset{!}{=} 0$$

$$\Leftrightarrow \begin{vmatrix} s+3 & 0 & -5 & 0 \\ -1 & s-3 & 5 & 1 \\ 1 & 0 & s-3 & 1 \\ 1 & 0 & 0 & 0 \end{vmatrix} \overset{!}{=} 0$$

$$\Leftrightarrow (s-3)\cdot\left(-5\cdot 1 - \cancel{(s-3)}\cdot 0\right) \overset{!}{=} 0$$

$$\Leftrightarrow s-3 \overset{!}{=} 0$$

$$s_0 = 3$$

$\Rightarrow$ decoupling zero

poles : $s_1 = -2$, $s_2 = 2$

d) KALMAN

$$Q_s = \left(B \quad AB \quad A^2B \right)$$

$$A \cdot B = \begin{bmatrix} -3 & 0 & 5 \\ 1 & 3 & -5 \\ -1 & 0 & 3 \end{bmatrix} \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ -2 \\ 3 \end{bmatrix}$$

$$A^2 \cdot B = \begin{bmatrix} -3 & 0 & 5 \\ 1 & 3 & -5 \\ -1 & 0 & 3 \end{bmatrix} \begin{bmatrix} 5 \\ -2 \\ 3 \end{bmatrix} = \begin{bmatrix} 0 \\ -16 \\ 4 \end{bmatrix}$$

$$Q_s = \begin{bmatrix} 0 & 5 & 0 \\ 1 & -2 & -16 \\ 1 & 3 & 4 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 & 4 \\ 1 & -2 & -16 \\ 0 & 5 & 0 \end{bmatrix} \rightarrow \begin{bmatrix} 1 & 4 & 3 \\ 1 & -16 & -2 \\ 0 & 0 & 5 \end{bmatrix} \xrightarrow{\cdot(-1)\ +} \begin{bmatrix} 1 & 4 & 3 \\ 0 & -20 & -8 \\ 0 & 0 & 5 \end{bmatrix}$$

$$\Rightarrow Rg(Q_s) = 3$$

$\Rightarrow$ fully controllable

d) KALMAN

$$Q_B = \begin{bmatrix} c \\ cA \\ cA^2 \end{bmatrix}$$

$$cA = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} -3 & 0 & 5 \\ 1 & 3 & -5 \\ -3 & 0 & 3 \end{bmatrix} = \begin{bmatrix} -3 & 0 & 5 \end{bmatrix}$$

$$cA^2 = \begin{bmatrix} -3 & 0 & 5 \end{bmatrix} \begin{bmatrix} -3 & 0 & 5 \\ 1 & 3 & -5 \\ -3 & 0 & 3 \end{bmatrix} = \begin{bmatrix} -6 & 0 & 0 \end{bmatrix}$$

$$Q_B = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 0 & 5 \\ -6 & 0 & 0 \end{bmatrix} \qquad Rg\,(Q_B) = 2 < n$$

not fully observable

=> the eigen value 3 is not observable

e) desired eigenvalues

$$(\lambda-2)(\lambda+2)(\lambda+1) = (\lambda^2-4)(\lambda+1) = \lambda^3+\lambda^2-4\lambda-4$$

controlled system

$$A - Bk = \begin{bmatrix} -3 & 0 & 5 \\ 1 & 3 & -5 \\ -1 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} \begin{bmatrix} -1 & -1 & k_3 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 0 & 5 \\ 1 & 3 & -5 \\ -1 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 \\ -1 & -1 & k_3 \\ -1 & -1 & k_3 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 0 & 5 \\ 2 & 4 & -5-k_3 \\ 0 & 1 & 3-k_3 \end{bmatrix}$$

$$|\lambda E - (A-Bk)| \overset{!}{=} 0$$

$$\Leftrightarrow \begin{vmatrix} \lambda+3 & 0 & -5 \\ -2 & \lambda-4 & 5+k_3 \\ 0 & -1 & \lambda-3+k_3 \end{vmatrix} = (\lambda+3)(\lambda-4)(\lambda-3+k_3) - 10 + (5+k_3)(\lambda+3)$$

$$= (\lambda^2 \underbrace{-4\lambda+3\lambda}_{-\lambda} -12)(\lambda+(-3+k_3)) - 10 + 15 + 5\lambda + k_3\lambda + 3k_3$$

$$= \lambda^3 + (-3+k_3)\lambda^2 - \lambda^2 \cancel{-\lambda(-3+k_3)} \cancel{-12\lambda} + 36 - 12k_3 + 5 + 3k_3 \cancel{+5\lambda} \cancel{+k_3\lambda}$$

$$= \lambda^3 + (-3+k_3-1)\lambda^2 + (3-k_3-12+5\cancel{+k_3})\lambda + 41 - 9k_3$$

$$= \lambda^3 + (-4+k_3)\lambda^2 + (-4)\lambda + 41 - 9k_3$$

Problem 3e) comparison of coefficients

$$-4 + k_3 = 1$$

$$\Rightarrow \underline{\underline{k_3 = 5}}$$

Test:

$$41 - 9 \cdot 5 \overset{!}{=} -4$$
$$41 - 45 = -4$$
$$-4 = -4 \quad \checkmark$$

The controlled system is not stable

$\rightarrow Re(\lambda_1) > 0$! A practical realization usually does make no sense.

$$A_c = A - Bk = \begin{bmatrix} -3 & 0 & 5 \\ 2 & 4 & -10 \\ 0 & 1 & -2 \end{bmatrix}$$

$$Q_B = \begin{bmatrix} c \\ cA_c \\ cA_c^2 \end{bmatrix}$$

$$cA_c = [1 0 0] \begin{bmatrix} -3 & 0 & 5 \\ 2 & 4 & -10 \\ 0 & 1 & -2 \end{bmatrix} = [-3 \quad 0 \quad 5]$$

$$cA_c^2 = [-3 0 5] \begin{bmatrix} -3 & 0 & 5 \\ 2 & 4 & -10 \\ 0 & 1 & -2 \end{bmatrix} = [9 \quad 5 \quad -25]$$

$$Q_B = \begin{bmatrix} 1 & 0 & 0 \\ -3 & 0 & 5 \\ 9 & 5 & -25 \end{bmatrix} \qquad R_g(Q_B) = 3 = n$$

fully observable

The uncontroled system is not fully observable

$\rightarrow$ State feedback is not possible

A positive desired eigenvalue makes no sense.

Problem 4)

a) $\det(\lambda E - A) \overset{!}{=} 0$

$$\begin{vmatrix} \lambda & 0 & -1 & 0 \\ 0 & \lambda & 0 & -1 \\ -1 & 0 & \lambda & 0 \\ -4 & -2 & 0 & \lambda - d \end{vmatrix} \overset{!}{=} 0$$

$$-1 \begin{vmatrix} \lambda & 0 & -1 \\ -1 & 0 & \lambda \\ -4 & -2 & 0 \end{vmatrix} + (\lambda - d) \begin{vmatrix} \lambda & 0 & -1 \\ 0 & \lambda & 0 \\ -1 & 0 & \lambda \end{vmatrix} \overset{!}{=} 0$$

$$-1(2\lambda^2 - 2) + (\lambda - d)[\lambda^3 - \lambda] \overset{!}{=} 0$$

$$-2\cancel{\lambda^2} + 2 + \cancel{\lambda^4} \cancel{- \lambda^2} \cancel{- d\lambda^3} + d\lambda \overset{!}{=} 0$$

$$\lambda^4 - d\lambda^3 - 3\lambda^2 + d\lambda + 2 \overset{!}{=} 0$$

Hurwitz: not stable

a) $(\lambda^4 - d\lambda^3 - 3\lambda^2 + d\lambda + 2) : (\lambda + 1) = \lambda^3 + (-d-1)\lambda^2 + (d-2)\lambda + 2$

$$
\begin{array}{l}
-(\lambda^4 + \lambda^3) \\
\hline
(-d-1)\lambda^3 - 3\lambda^2 \\
-((-d-1)\lambda^3 + (-d-1)\lambda^2) \\
\hline
(d-2)\lambda^2 + d\lambda \\
-((d-2)\lambda^2 + (d-2)\lambda) \\
\hline
2\lambda + 2 \\
-(2\lambda + 2) \\
\hline
\underline{\underline{0}}
\end{array}
$$

$(\lambda^3 + (-d-1)\lambda^2 + (d-2)\lambda + 2) : (\lambda - 1) = \lambda^2 - d\lambda - 2$

$$
\begin{array}{l}
-(\lambda^3 - 1\lambda^2) \\
\hline
-d\lambda^2 + (d-2)\lambda \\
-(-d\lambda^2 + d\lambda) \\
\hline
-2\lambda + 2 \\
-(-2\lambda + 2) \\
\hline
\underline{\underline{0}}
\end{array}
$$

$$\lambda_{3,4} = \frac{d}{2} \pm \sqrt{\frac{d^2}{4} + 2}$$

The system is independent from d not stable.

b) $A - Bk$

$$= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 4 & 2 & 0 & d \end{bmatrix} - \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \end{bmatrix} \begin{bmatrix} 0 & k_2 & k_3 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 4 & 2 & 0 & d \end{bmatrix} - \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & k_2 & k_3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\det\left(\lambda E - (A - Bk)\right)$$

$$= \begin{vmatrix} \lambda & 0 & -1 & 0 \\ 0 & \lambda & 0 & -1 \\ -1 & +k_2 & +k_3+\lambda & 0 \\ -4 & -2 & 0 & \lambda - d \end{vmatrix} = -1 \begin{vmatrix} \lambda & 0 & -1 \\ -1 & k_2 & k_3+\lambda \\ -4 & -2 & 0 \end{vmatrix} + (\lambda - d) \begin{vmatrix} \lambda & 0 & -1 \\ 0 & \lambda & 0 \\ -1 & k_2 & k_3+\lambda \end{vmatrix}$$

$$= -1\left[(-2) - 4k_2 - \left(-2\lambda(k_3+\lambda)\right)\right] + (\lambda - d)\left[\lambda^3 + \lambda^2 k_3 - \lambda\right]$$

$$= \cancel{2} + \cancel{4k_2} \cancel{- 2\lambda^2} \cancel{- 2k_3\lambda} + \cancel{\lambda^4} + \cancel{k_3\lambda^3} \cancel{- \lambda^2} \cancel{- d\lambda^3} - \cancel{dk_3\lambda^2} + \cancel{d\lambda}$$

$$= \lambda^4 + (k_3 - d)\lambda^3 + (-2 - 1 - dk_3)\lambda^2 + (-2k_3 + d)\lambda + (2 + 4k_2)$$

$k_3 - d > 0$

$\boxed{1}$ $\underline{\underline{k_3 > d}}$

$-3 - d k_3 > 0$

$\boxed{3}$ $\underline{\underline{d k_3 < -3}}$

$-2k_3 + d > 0$

$\boxed{2}$ $\underline{\underline{d > 2k_3}}$

So from

$\boxed{1}$ $k_3 > d$

$\boxed{2}$ $k_3 < \frac{d}{2}$

$\longrightarrow$ solution not possible!

With the given parameters
the system can not be stabilized

c) The system has two outputs and one input.

$$Q_B = \begin{bmatrix} C \\ CA \\ CA^2 \\ CA^3 \end{bmatrix}$$

$$CA = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 4 & 2 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$CA^2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 4 & 2 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 4 & 2 & 0 & 0 \end{bmatrix}$$

$$CA^3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 4 & 2 & 0 & 0 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 4 & 2 & 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 4 & 2 \end{bmatrix}$$

$$Q_B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \cancel{1} & \cancel{0} & \cancel{0} & \cancel{0} \\ \cancel{4} & \cancel{2} & \cancel{0} & \cancel{0} \\ \cancel{0} & \cancel{0} & \cancel{1} & \cancel{0} \\ \cancel{0} & \cancel{0} & \cancel{4} & \cancel{2} \end{bmatrix} \qquad Rg(Q_B) = 4 = n$$

$\Rightarrow$ fully observable

Problem 4)

d) $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $A^T = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $B^T = [1 \ 0]$

$c = [0 \ 1]$ $\quad c^T = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$Q = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ $\quad R = [0{,}1]$ $\quad R^{-1} = [10]$

$$A P + P A^T - P C^T R^{-1} C P + Q = 0$$

$$\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\begin{bmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{bmatrix} + \begin{bmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$

$$- \begin{bmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} 10 \, [0 \ 1] \begin{bmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$$

$$\Leftrightarrow \begin{bmatrix} P_{12} & P_{22} \\ P_{11} & P_{12} \end{bmatrix} + \begin{bmatrix} P_{12} & P_{11} \\ P_{22} & P_{12} \end{bmatrix} - \underbrace{\begin{bmatrix} P_{12} \\ P_{22} \end{bmatrix} [P_{12} \ P_{22}]}_{} 10 + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$$

$$\begin{bmatrix} 2P_{12} & P_{11}+P_{22} \\ P_{11}+P_{22} & 2P_{12} \end{bmatrix} - 10 \begin{bmatrix} P_{12}^2 & P_{12}P_{12} \\ P_{12}P_{22} & P_{22}^2 \end{bmatrix} = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$\left.\begin{aligned} 2P_{12} - 10 P_{12}^2 &= -1 \\ 2P_{12} - 10 P_{22}^2 &= -1 \end{aligned}\right\} P_{12} = P_{22}$$

$$P_{11} + P_{22} - 10 P_{12}P_{22} = 0$$

d)

$$2P_{12} - 10P_{12}^2 = -1$$

$$\Rightarrow P_{12} - 0{,}2 P_1 - 0{,}1$$

$$P_{12_{1,2}} = 0{,}1 \pm \sqrt{0{,}01 + 0{,}1}$$

$$\Rightarrow P_{12} = P_{22} = 0{,}1 + \sqrt{0{,}11}$$

$$\Rightarrow P_{11} = -0{,}1 - \sqrt{0{,}11} + 10\left(0{,}1 + \sqrt{0{,}11}\right)^2$$

$$= -\cancel{0{,}1} - \cancel{\sqrt{0{,}11}} + \cancel{0{,}1} + \cancel{2}\sqrt{0{,}11} + 1{,}1$$

$$= 1{,}1 + \sqrt{0{,}11}$$

$$L = P\, C^T R^{-1}$$

$$= 10 \cdot \begin{bmatrix} 1{,}1 + \sqrt{0{,}11} & 0{,}1 + \sqrt{0{,}11} \\ 0{,}1 + \sqrt{0{,}11} & 0{,}1 + \sqrt{0{,}11} \end{bmatrix} \begin{bmatrix} 0 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 + 10\sqrt{0{,}11} \\ 1 + 10\sqrt{0{,}11} \end{bmatrix}$$

e) $\dot{e} = (A - L\,C_1)\,e$

$\dot{\hat{x}} = (A - L\,C_1)\,\hat{x} + Bu + Ly$

$\dot{\hat{x}} = \left(\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 25 \\ 10 \end{bmatrix}\begin{bmatrix} 0 & 1 \end{bmatrix}\right)\hat{x} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u + \begin{bmatrix} 25 \\ 10 \end{bmatrix} y$

$= \begin{bmatrix} 0 & -24 \\ 1 & -10 \end{bmatrix}\hat{x} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} u + \begin{bmatrix} 25 \\ 10 \end{bmatrix} y$

f) $A - Bk$

$= \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} - \begin{bmatrix} 1 \\ 0 \end{bmatrix}\begin{bmatrix} 3 & 3 \end{bmatrix}$

$= \begin{bmatrix} -3 & -2 \\ 1 & 0 \end{bmatrix}$

g) $\begin{pmatrix} \dot{x} \\ \dot{e} \end{pmatrix} = \begin{pmatrix} A - Bk & Bk \\ 0 & A - LC_1 \end{pmatrix}\begin{pmatrix} x \\ e \end{pmatrix} = \begin{pmatrix} -3 & -2 & 3 & 3 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -24 \\ 0 & 0 & 1 & -10 \end{pmatrix}\begin{pmatrix} x \\ e \end{pmatrix}$

$\det(\lambda E - (A - Bk))$

$= \begin{vmatrix} \lambda + 3 & 2 \\ -1 & \lambda \end{vmatrix} = \lambda^2 + 3\lambda + 2$ $\qquad \lambda_{1,2} = -1{,}5 \pm \sqrt{2{,}25 - 2}$

$\lambda_1 = -2 \qquad \lambda_2 = -1$

$\det(\lambda E - (A - LC_1))$

$= \begin{vmatrix} \lambda & +24 \\ -1 & \lambda + 10 \end{vmatrix} = \lambda^2 + 10\lambda + 24$ $\qquad \lambda_{3,4} = -5 \pm \sqrt{25 - 24}$

$\lambda_3 = -4 \qquad \lambda_4 = -6$