LAST NAME	
FIRST NAME	
MATRIKEL-NO.	

Problem 1

(each 3 Points)

a) A system description is given by

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 3 & 2 & a \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \qquad \text{and} \qquad C = \begin{bmatrix} 0 & 0 & c \end{bmatrix}; \qquad (c \neq 0).$$

It is noted that no direct transmission between the inputs and the outputs/measurements is given. Calculate the transfer function matrix $G(s)$.

b) Under which conditions is the system given in a) asymptotically stable?

c) Calculate the input/output decoupling zeros of the system given in a).

d) A system is described by the system matrix A and the input matrix B. Declare the calculation steps and give the equations to calculate the feedback matrix / gain for the linear quadratic optimal regulator/controller. Which conditions have to be fulfilled to realize the state feedback? Are these conditions satisfied by the system described in a)? State reasons.

e) Declare the principal strategy for the practical realization of a model-based state reconstruction scheme (observer). Please define the whole processor-based observer realization using elements of A/D, D/A - conversion, the microcontroller, and measurement devices.

An observer can be used to realize state feedback control. Which additional task has to be realized on the microcontroller due to state feedback ?

Problem 2

(15 points)

Given is the standard description of a linear MIMO system.

a) The system described by

$$A = \begin{bmatrix} 0 & 2 & 3 \\ 2 & 3 & 0 \\ 1 & 1 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

should be observed using the observer feedback matrix $L = \begin{bmatrix} 0 & 2l_1 \\ l_2 & l_1 \\ 0 & l_1 \end{bmatrix}$.

i) Is it for practical purposes necessary to build an observer for the given system to realize full state-control? (1 point)

ii) Is it possible to place the observer eigenvalues arbitrarily? (2 points)

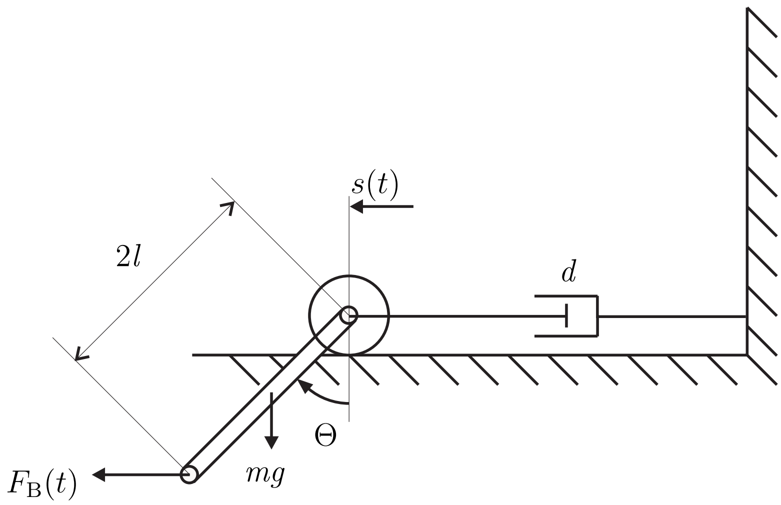
b) Calculate $\{l_1, l_2\} \in \mathbb{Z}$ so that the eigenvalues of the observer are $\lambda_1 = -1$, $\lambda_2 = -2$, and $\lambda_3 = -3$. (6 points)
Remark: $\sqrt{\frac{17^2}{4^3} - 4.5} = 0.125$

c) Draw the scheme of an observer-based state feedback control and label the blocks (system, observer, state control, ...) as well as the related signal names (states, measurement, ...). Additionally state the order/dimensions of each used vector and matrix. (4 points)

d) State the equations to calculate left and right eigenvectors. (2 points)

Problem 3

(30 points)



The mechanical system shown above is described by the following linearized differential equations

$$\begin{aligned} ml\ddot{s} + m(l^2 + r^2)\ddot{\Theta} + mgl\Theta &= 2lF_B, \\ m\ddot{s} + ml\ddot{\Theta} + d\dot{s} &= F_B. \end{aligned}$$

a) (2 points)
Derive a model description in the form $M\ddot{z} + D\dot{z} + Kz = Vf(t)$ and also the state space model with the state space vector

$$x = \begin{bmatrix} s \\ \Theta \\ \dot{s} \\ \dot{\Theta} \end{bmatrix}.$$

b) (3 points)
Check the asymptotic stability of the system with $m = 1$, $d = 10$, $l = 1$, $r = 1$, $g = 1$ using the Hurwitz criteria.

c) (4 points)

Assuming measurements are available with $C = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$ to realize feedback by $u = -KCx$, with $K = [k_1 \quad 0 \quad 10 \quad 1]$. For which k_1 is the closed-loop system stable?

d) (4 points)
Use the same feedback matrix as in part c) with $k_1 = 1$. Determine the observer feedback matrix element l_{33} of the observer matrix L

$$\begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & l_{33} & 0 \\ 0 & -1 & 10 & 40 \end{bmatrix}$$

that the observer has the eigenvalues

$\lambda_1 = \lambda_2 = -1$,
$\lambda_3 = -3$ and
$\lambda_4 = -40$.

e) (2 points)
The eigenvalues of a controlled system are
$\lambda_1 = 0 + \varepsilon_1$,
$\lambda_2 = 20 + j3$,
$\lambda_3 = 20 - j3$,
$\lambda_4 = -40 + j38.75$,
$\lambda_5 = -40 - j38.75$,
$\lambda_6 = 6.2531$,
$\lambda_7 = -0.8543$,
$\lambda_8 = -100.001$,
$\lambda_9 = 0.0000001 + \varepsilon_2$,
with $\varepsilon \in \mathbb{R}$.

The modes 2 and 3 are not observable, no further input/output decoupling zeros exist. Under which conditions is the system

i) asymptotically stable,
ii) state unstable,
iii) I/O (BIBO) stable?

Problem 4

(40 points)

a) (8 points)
Is the system described by

$$\dot{x} = Ax + Bu, \; y = Cx$$

with $A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & -a & 0 \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ b \\ 0 \end{bmatrix}$, $C = \begin{bmatrix} 0 & 0 & c \end{bmatrix}$, $(a \neq 0, b \neq 0, c \neq 0)$
asymptotically stable? Calculate the eigenvalues and the eigenvectors.

b) (6 points)
Is the system given in a) fully controllable?
If not, which eigenvalues are controllable?

c) (6 points)
Is the system given in a) fully observable?
If not, which eigenvalues are observable?

d) (3 points)
The input matrix is changed to $B = \begin{bmatrix} b_1 \\ b_2 \\ 0 \end{bmatrix}$, $(b_1 \neq 0, b_2 \neq 0)$. Under which condition(s) can an arbitrary desired dynamics be achieved by state feedback control?

e) (9 points)
Calculate the state feedback matrix $K = \begin{bmatrix} 0 & k_1 & k_2 \end{bmatrix}$ for desired eigenvalues of the controlled system $\lambda_1 = -1, \; \lambda_{2,3} = -\frac{10}{3}$ with $B = \begin{bmatrix} b_1 \\ b_2 \\ 0 \end{bmatrix}$ and $b_1 = 0, \; b_2 \neq 0$.

f) (4 points)
Calculate the transfer function matrix $G(s)$ for $B = \begin{bmatrix} 0 \\ b \\ 0 \end{bmatrix}$ and $D = 0$. State the poles of the system described by $G(s)$.

g) (4 points)
Describe in general the relation between poles and eigenvalues. Explain the details using the poles and eigenvalues discussed within this problem.

Maximum achievable points:	**100**
Minimum points for the grade 1,0:	**95**
Minimum points for the grade 4,0:	**50**