LAST NAME	
FIRST NAME	
MATRIKEL-NO.	

Problem 1

(15 points, 3 points each)

a) Define the difference between a MIMO and a SISO control system with respect to the design of the feedback gains.

b) State the state space description of the SISO system with the transfer function

$$F(s) = \frac{Y(s)}{U(s)} = k\frac{s^2+5}{(s^2+2s+5)(s-1)}.$$

c) State the conditions for the matrices P, Q of the Lyapunov equation $A^TP + PA = -Q$ to guarantee the asymptotic stability of the system

$$\dot{x} = Ax.$$

d) For the system $\dot{x} = Ax$ with the system matrix

$$A = \frac{1}{2}\begin{bmatrix} -1 & 1 & 0 \\ -1 & 1 & 4 \\ -1 & -1 & -2 \end{bmatrix}$$

and the matrices

$$Q = 2\begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix} \text{ and } P = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 1 & 3 \end{bmatrix},$$

where Q is the weighting matrix and P is the solution matrix of the Lyapunov equation. Check and state about the quality of the stability of A (asymptotically stable, (boundary) stable, unstable).

e) Is the system described with $A = \begin{bmatrix} 0 & 1 & 0 \\ a & 1 & a \\ 3 & 0 & 0 \end{bmatrix}$ stable? Calculate the stability condition(s) with respect to the parameter a.

Problem 2

(15 points, 3 points each)

a) Calculate the damping ratio of the eigenvalue pair

$$\lambda_{1,2} = -1 \pm j.$$

b) The system

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned}$$

with the matrices

$$A = \begin{bmatrix} 0 & 1 \\ -a & a-10 \end{bmatrix}, \quad B = b\begin{bmatrix} 1 \\ 2 \end{bmatrix}, \quad \text{and } C = c\begin{bmatrix} 10 & 1 \end{bmatrix}$$

is given.

What is the domain of parameter a if the uncontrolled system is asymptotically stable?

c) What are the domains of parameters a, b if the system described in task b) is fully controllable?

d) What are the domains of parameters a, c if the system described in task b) is fully observable?

e) For which values of parameters k_1, k_2 of a state feedback $u = -k_1 x_1 - k_2 x_2$ has the controlled system described in task b) with $a = 12$, $b = 1$ the eigenvalues $\lambda_{1,2} = -2$?

Problem 3

(30 points)

The system

$$\dot{x} = Ax + Bu$$

with the matrices

$$A = \begin{bmatrix} 3+2p+q & -1-p & -3-p-q \\ 4+2q & -1 & -4-2q \\ 2+2p & -1-p & -2-p \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 1+b \\ 1 & 1 \\ 1 & 1+b \end{bmatrix}, \quad C = \begin{bmatrix} -2 & 1 & 2 \\ 2 & -1 & -1 \\ 1 & 0 & -1 \end{bmatrix}$$

is given, whereby $p, q, b \in \mathbb{R}$.

a) (15 points)
Check the stability of the system in dependence on the parameters p and q.
For which values of p and q is the system state stable? Use the Hurwitz criterion and pay attention to the distinction of cases for p and q.

b) (4 points)
Assume $q = -3$ and $p = 0$.
For which values of b can the system be stabilized by state feedback?

c) (4 points)
The system should be controlled by state feedback $u = -Kx$. Calculate the missing elements of the state feedback matrix

$$K = \begin{bmatrix} -183/11 & k_{12} & 7/11 \\ 16 & k_{22} & 0 \end{bmatrix}$$

so that the controlled system has the eigenvalues $\lambda_1 = -1$, $\lambda_2 = -2$, and $\lambda_3 = -16$ for $b = 1$, $q = -3$, and $p = 0$.

d) (4 points)
Choosing $b = 1$, $q = -3$, and $p = 0$.
Would it be possible to realize a state feedback such that the controlled system has the eigenvalues $\lambda_1 = 0$, $\lambda_2 = -1$, and $\lambda_3 = -4$? Prove your statement by calculation.

e) (3 points)
Assume the system is fully observable. State the poles of the system for $b = 1$, $q = -3$, and $p = 0$.

Problem 4

(40 points)

The system

$$\begin{aligned}\dot{x} &= Ax + Bu\\ y &= Cx\end{aligned}$$

with the matrices $A = \begin{bmatrix} -8 & -8 \\ -8 & -20 \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, and $C = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ is given.

a) (6 points)
Determine the transfer function matrix and state the zeros, poles, and eigenvalues of the system.

b) (3 points)
Is the system fully controllable? If not, which eigenvalues are controllable?

c) (3 points)
Is the system fully observable? If not, which eigenvalues are observable?

d) (6 points)
The system is controlled by state feedback with the matrix
$K = \begin{bmatrix} k_1 & k_2 \end{bmatrix}$.
Which conditions have to be fulfilled for the elements of K, so that the system is asymptotically stable (use Hurwitz!)?

e) (16 points)
For the design of an optimal controller the matrices

$$Q = \begin{bmatrix} 84 & 96 \\ 96 & 112 \end{bmatrix}, \quad P = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}, \quad \text{and } R = \begin{bmatrix} 0.25 \end{bmatrix}$$

are given.
Calculate $K^\star$ of a linear quadratic controller (assume $p_{12} = p_{21} = p_{22} > 0$).

f) (6 points)
Which condition has to be fulfilled for the elements of K, so that the system controlled by state feedback is fully observable?

Maximum achievable points:	**100**
Minimum points for the grade 1,0:	**95**
Minimum points for the grade 4,0:	**50**