LAST NAME	
FIRST NAME	
MATRIKEL-NO.	

Attention: Give your answers to problem 1 and problem 2 directly below the questions in the exam question sheet.

Problem 1

(15 points)

a) (1 point)
A system description is given with

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -3 & a & b \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}, \text{and} \qquad C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}.$$

It is noted, that no direct transmission between the inputs and the outputs/measurements is given. State the Rosenbrock system matrix $P(s)$.

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b) (3 points)
Calculate the invariant zeros for the system given in a) depending on the parameters a, b, and c determine the input/output decoupling zeros.

☐

c) (3 points)
Calculate the characteristic polynom of the system matrix A given in a). Under which conditions is the system described with A asymtotic stable?

☐

d) (6 points)
For which parameters a, b is the system given in a) fully controllable? Assuming full controllability a state observer should be designed with $\lambda_1 = -5, \lambda_2 = -10$, and $\lambda_3 = 5$. Calculate the gains l_1, l_2, l_3 depending on the system parameters a, b.

☐

e) (2 points)
Declare the principal strategy for practical realizing of a model-based full state feedback by drawing a related sketch. Please define the whole processer-based realization using elements for A/D, D/A - conversion, the microcontroller, and measurement devices. Give a statement about the relation between the number of independent measurement channels, the rank of C, and the related dimensions of matrices and vectors to be multiplied and added.

□

Problem 2

(15 points)

a) (3 points)
The system described with

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 3 & 0 \\ 1 & 1 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 1 \\ -1 & 0 \\ 0 & 0 \end{bmatrix}, \quad \text{and} \quad C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

should be controlled using a state feedback $K = \begin{bmatrix} k_1 & k_2 & k_3 \\ k_4 & k_5 & k_6 \end{bmatrix}$. Two questions:

i) Is it for practical purposes necessary to control this system with respect to the time behavior of this system? Check the stability of the system and then state reason.

☐

ii) Is it possible to place the eigenvalues of this system to arbitrary locations?

□

b) (4 points)
Calculate the gains k_1, k_2, k_6 so that the eigenvalues of the controlled system are $\lambda_1 = -1.0, \quad \lambda_2 = -2.0$, and $\lambda_3 = -3.0$ (let k_3, k_4, and k_5 be 0).

c) (2 points)
State the formulas to calculate the gains of a Linear Quadratic Optimal Observer.

☐

d) (3 points)
Determine the transfer function matrix of the system given in a).

e) (3 points)
For a given system, the weighting matrices

$$Q = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix} \tag{2.1}$$

and

$$R = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix} \tag{2.2}$$

are given.
State the related integral $J \rightarrow Min$ in detail as well as the dimensions of x, B, C.

Problem 3

(30 points)

The transfer function of a mechanical system with the parameter a is given by

$$G_s(s) = \frac{Y(s)}{U(s)} = \frac{K_f}{(s+2)(s+a)} \quad , \quad \text{with} \quad a = 0.$$

This plant should be controlled using a PT_1 element

$$T_1\, \dot{u}_o(t) + u_o(t) = u_i(t)$$

as controller. The measured value is $y(t)$.

a) (6 points)
Derive the matrices A, B, and C of the state space representation

$$\dot{x}(t) = Ax(t) + Bu(t) \text{ and } y(t) = Cx(t)$$

of the plant with the state space vector

$$x(t) = [\dot{y}(t) \;\; y(t)]^T.$$

b) (4 points)
Calculate the eigenvalues of the uncontrolled system. Is the uncontrolled system stable (state reason)?

c) (5 points)
For which parameter K is the system fully controllable (state reasons)?
For which parameter K is the system fully observable (state reasons)?

d) (5 points)
Derive the state space matrices A_1, B_1, and C_1 for the PT_1–controlled system, and use u and $\dot{u}$ as separated inputs.

e) (6 points)
For the following steps (parts e and f of the problem) the controlled system is assumed as

$$\dot{x} = A_3 x + B_3 u \text{ and } y = C_3 x \quad \text{with}$$

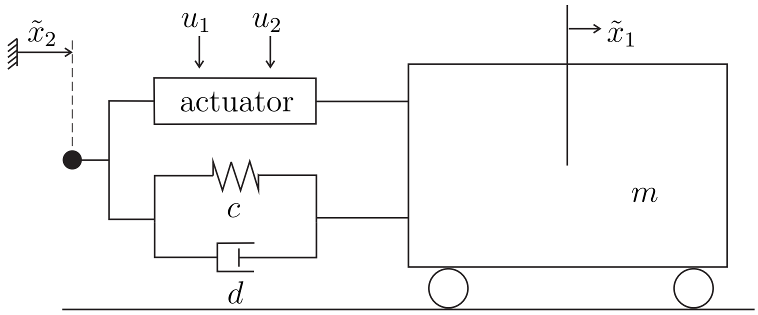
$$A_3 = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -K_f & \frac{-1}{T_1} & -1 \end{bmatrix}, \qquad B_3 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \text{ and } \qquad C_3 = \begin{bmatrix} K_f & \frac{K_f}{T_1} & 0 \end{bmatrix}.$$

The system should be additionally controlled using a state feedback $u = -Kx$, where $K = [k_1 \;\; k_2 \;\; k_3]$. Which conditions have to be fulfilled by the gains k_1, k_2, k_3 of the controller described above to make the system asymptotically stable?

f) (4 points)
Design the values of the gains k_1, k_2, k_3 so that the closed-loop system has the eigenvalues $\lambda_{1,2,3} = -1$.

Problem 4

(40 points)



The dynamics of the positioning system shown in the figure should be examined. Regarding only the mechanical part, it can be described by

$$m\ddot{\tilde{x}}_1 + d(\dot{\tilde{x}}_1 - \dot{\tilde{x}}_2) + c\tilde{x}_1 = 0;$$

with $\tilde{x}_2 = 0$.
After rearranging the description into relative coordinates, it results in

$$\ddot{x}_1 = -\frac{d}{m}\dot{x}_1 - \frac{c}{m}x_1 \, .$$

Further considering a novel actuator, whose inputs are voltages, the system can be (approximatively) specified by

$$\begin{bmatrix} \dot{x}_1 \\ \ddot{x}_1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{c}{m} & -\frac{d}{m} \end{bmatrix} \begin{bmatrix} x_1 \\ \dot{x}_1 \end{bmatrix} + \begin{bmatrix} b_1 & 0 \\ 0 & b_2 \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} .$$

In addition the output is given by

$$y = \begin{bmatrix} c_1 & 0 \\ 0 & c_2 \end{bmatrix} \begin{bmatrix} x_1 \\ \dot{x}_1 \end{bmatrix} .$$

a) (3 points)
Calculate the eigenvalues of the system (only by using the parameters c, d, and $m > 0$).

b) (3 points)
Make a statement about the state stability of the system, assuming c, d, and $m > 0$.

c) (7 points)
The parameters are given by $m = 2$ kg, $c = 0.5$ kg/s^2, $d = 0.25$ kg/s.

Make use of the Hautus criterion for the following tasks:

Is the system controllable for $b_1 = 1$?
If so, is it still controllable for $b_1 = 1$ and $b_2 = 0$?

Is the system observable for $c_2 = 1$?
If so, is it still observable for $c_2 = 1$ and $c_1 = 0$?

d) (6 points)
Set $b_1 = b_2 = 1$. Calculate a state feedback

$$K = \begin{bmatrix} k_1 & k_2 \\ 1 & 1 \end{bmatrix}$$

so that the eigenvalues of the closed loop system are $\lambda_1 = -1$, $\lambda_2 = -2$.

e) (6 points)
Set $c_1 = c_2 = 1$. Find an identity observer

$$L = \begin{bmatrix} 1 & l_1 \\ 1 & l_2 \end{bmatrix}$$

with the eigenvalues $\lambda_1 = -3$, $\lambda_2 = -4$.

f) (5 points)
Derive the error equation for the observer dynamics in general and for this special system (with the observer matrix calculated in part e)).

Is this observer able to compensate the differences in the initial conditions between the state $x(t = 0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ and the observer state error $e(t = 0) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$ for $t \to \infty$? Calculate the eigenvalues of the observer dynamics.

g) (4 points)
Now, use the observed states for feedback.
Set up the system equation with this feedback (in general, not for this special system).

Build up the vector equation of the complete system i.e. the system with feedback and the observer error-equation.

Build it up in general and for this system, using the feedback matrix calculated in part d) and using the observer matrix calculated in part e); if you have not been able to calculate numbers, do it in an appropriate manner seperately.

Give the eigenvalues of this extended system and name the principle behind by which you calculate the eigenvalues.

h) (1 points)
How to choose the observer eigenvalues for practical applications?

i) (3 points)
Calculate the transfer function matrix with the given values and

$$B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

Give the final shortened form.

j) (2 points)
How does the first input u_1 affect the second output y_2? Give the related transfer function.

Maximum achievable points:	**100**
Minimum points for the grade 1,0:	**95**
Minimum points for the grade 4,0:	**50**