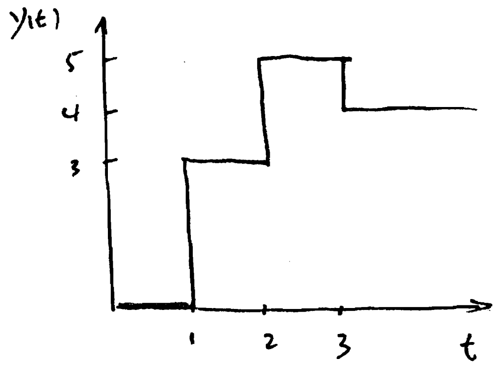
1.

a) System : the system is in interaction with the real world, whereby from the real world inputs are acting to the system and outputs are acting from the system to the environment.

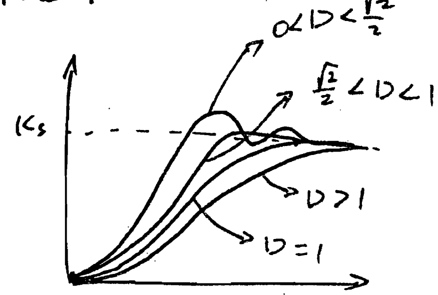
Inputs : Variable acting direct to the system.

b)



c) The poles denote those frequencies for which the transfer behavior probably get "∞" depending on the damping.
The zeros denote those frequencies, which cannot be transfered though the system.

Double pole :



d) ODE:

$$y(t) = K\left(u(t-T_t) + T_D\dot{u}(t-T_t)\right)$$

$$TF: \; G(s) = \frac{Y(s)}{U(s)} = K(1+T_D s)e^{-sT_t}$$

e)

$$\dot{x} = Ax + bu$$
$$y = cx + du$$

x: state vector, $x \in R^n$
u: Input vector, $u \in R^1$
y: Output vector, $y \in R^1$
A: system matrix, $A \in R^{n \times n}$
b: Input matrix, $b \in R^{n \times 1}$
c: output matrix, $c \in R^{1 \times n}$
d: feed through matrix, $d \in R^{1x}$

2. a)

$$G(s) = \frac{K\left(1+\frac{1}{T_3 s}\right)}{T_2 s^2 + T_1 s + 1}, \quad U(s) = \mathcal{L}\{1(t)\} = \frac{1}{s}$$

$$\Rightarrow H(s) = G(s)\cdot U(s) = \frac{K\left(1+\frac{1}{T_3 s}\right)}{s\left(T_2 s^2 + T_1 s + 1\right)}$$

Initial value:

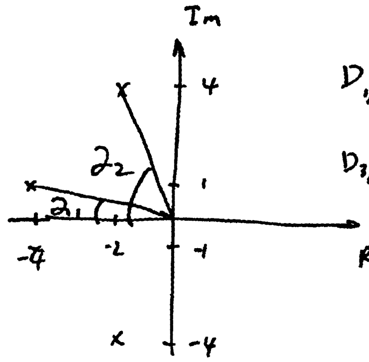
$$\lim_{t\to 0^+} y(t) = \lim_{s\to\infty} s\cdot H(s) = 0$$

b) Eigenvalue: $\lambda = -\delta \pm j\omega$

Damping ratio: $D = \frac{\delta}{\sqrt{\delta^2+\omega^2}}$

$$\Rightarrow D_{1/2} = \frac{\sqrt{5}}{5} \approx 0.4472 \quad (-2 \pm j4)$$

$$D_{3/4} = \frac{4\sqrt{17}}{17} \approx 0.97012 \quad (-4 \pm j1)$$



$D_{1/2} = \cos\vartheta_1$

$D_{3/4} = \cos\vartheta_2$

$\Rightarrow$ the eigenvalues $-2 \pm j4$ have the smallest damping coefficient!

c) This is not a linear system, because of the second order term $(\dot{y}\cdot x)$

d) $F_a(s) = G(s)\cdot F(s) = \frac{1}{T}\,\frac{1}{s\,(T_1 s+1)\,(T_2 s+1)}$

$$\Rightarrow f_a(t) = \mathcal{L}^{-1}\{F_a(s)\} = \frac{1}{T}\left(1 - \frac{T_1}{T_1-T_2}e^{-\frac{t}{T_1}} + \frac{T_2}{T_1-T_2}e^{-\frac{t}{T_2}}\right)$$

plant: $G = \frac{K(T_D s + 1)}{T_I s + 1}$ (PDT_1)

controller: $G_r = P$ (P)

reference transfer function:

$$G_w = \frac{G \cdot G_r}{1 - G G_r} = \frac{K(T_D s+1) \cdot P}{(T_I s + 1)\left(1 - \frac{K(T_D s+1)P}{T_I s+1}\right)}$$

Disturbance transfer function:

$$G_d = \frac{1}{1 - G G_r} = \frac{1}{1 - \frac{K(T_D s+1)P}{T_I s + 1}}$$

charactoristic equation:

$$(T_I - KPT_D)s + 1 - KP = 0$$

$\Rightarrow$ ①

$1 - KP > 0 \wedge T_I - KPT_D > 0$

$\Rightarrow KP < 1 \wedge KP < \frac{T_I}{T_D}$

assumming: $\frac{T_I}{T_D} \geq 1$

$\Rightarrow 0 < KP < 1$

②

$1 - KP < 0 \wedge T_I - KPT_D < 0$

$\Rightarrow KP > 1 \wedge KP > \frac{T_I}{T_D}$

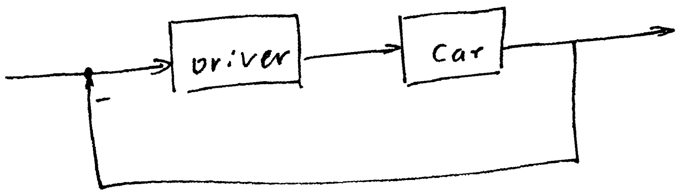
assumming $\frac{T_I}{T_D} \geq 1$

$\Rightarrow KP > \frac{T_I}{T_D}$

The system is asymptotically stable when $0 < KP < 1$ or $KP > \frac{T_I}{T_D}$ (assuming $\frac{T_I}{T_D} \geq 1$)

3

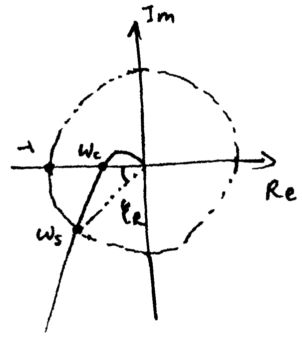
a) Blockdiagram



Stability:

phase margin:

$$\varphi_R = \pi + \varphi(w_s) > 0^\circ$$



Amplitude margin:

$$A_R = \left| \frac{1}{G_0(jw_c)} \right| > 1$$

Good Behavior:

$$30^\circ < \varphi_R < 60^\circ, \quad 2 < A_R < 6$$

b) phase margin:

$$|G_0(jw)| = \left| \frac{1}{(jw)^2 + 10jw + 1} \right| \cdot \left| \frac{2}{1+jw} \right| \cdot \left| e^{-1.5jw} \right| \overset{!}{=} 1$$

$$\Rightarrow (-11w_s^2+1)^2 + (-w_s^3 + 11w_s)^2 = 4$$

$$\Rightarrow w_s \approx 0.172 \ (\text{Rad/s})$$

$$\varphi(w_s) = \operatorname{atan2}(w_s^3 - 11w_s, \ 11w_s^2 - 1) + (-1.5 \cdot w_s)$$
$$\approx -85^\circ$$

$$\Rightarrow \varphi_R = 180^\circ + \varphi(w_s) = 95^\circ$$

$\Rightarrow$ The system is stable.

c) $\lim_{t\to\infty} \| x(t) - x_0 \| = 0$ (x_0 : equilibrium position)

or

$\lim_{t\to\infty} \| x(t) \| = 0$ ($x_0 = 0$)

d) If the open loop is asymptotically stable, then the ~~tot~~ closed-loop is only asymptotically stable, if the frequency response locus of the open loop does neither revolve around or pass through the critical point $(-1, j0)$

Assumptions :

a) negative feedback

b) gain $K > 0$

c) G_0 stable

d) max. two poles in origin

e) order of numerator < order of deteminator.

f) no pole / zero cancellation.

e) :

System 1 : linear

Input frequency = output frequency

System 2 : linear

Input frequency = output frequency.

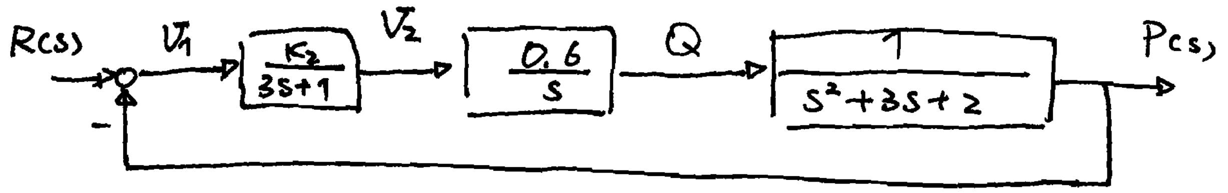
Problem 4.

a). $$\frac{P(s)}{V_1(s)} = \frac{0{,}2\,(0{,}1s+1)}{s(10s+1)(s+1)(s+2)}$$

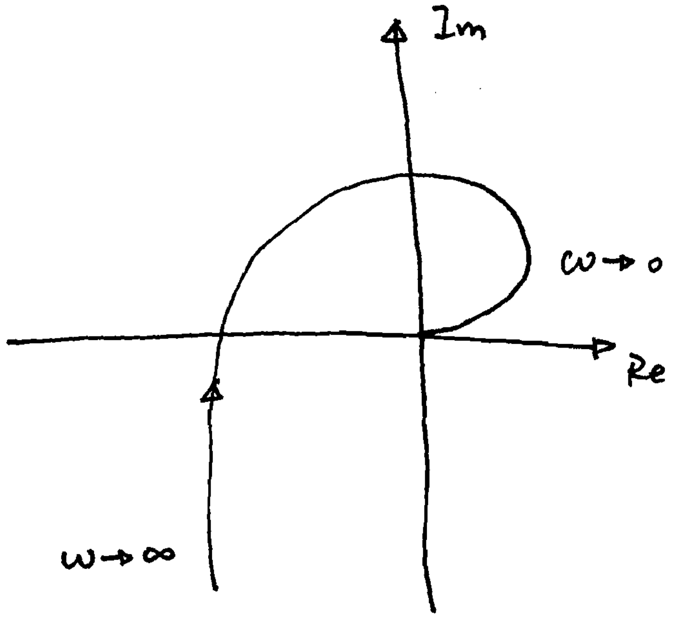
Zeros: $s_{1,0} = -10$

Poles: $s_1 = 0, \quad s_2 = -0.1, \quad s_3 = -1, \quad s_4 = -2$

c). $$G_0(s) = \frac{0.6\,K_2}{s(3s+1)(s+1)(s+2)}$$



d)



e). $\mathrm{Im}\left[G_0(j\omega)\right] = 0 \Rightarrow \omega^2 = 0.2$

$$\mathrm{Re}\left[G_0(j\omega)\right]\Big|_{\omega^2=0.2} = -\frac{25}{42} = -0.5952 > -1$$

$\Rightarrow$ System is stable.

4. b)

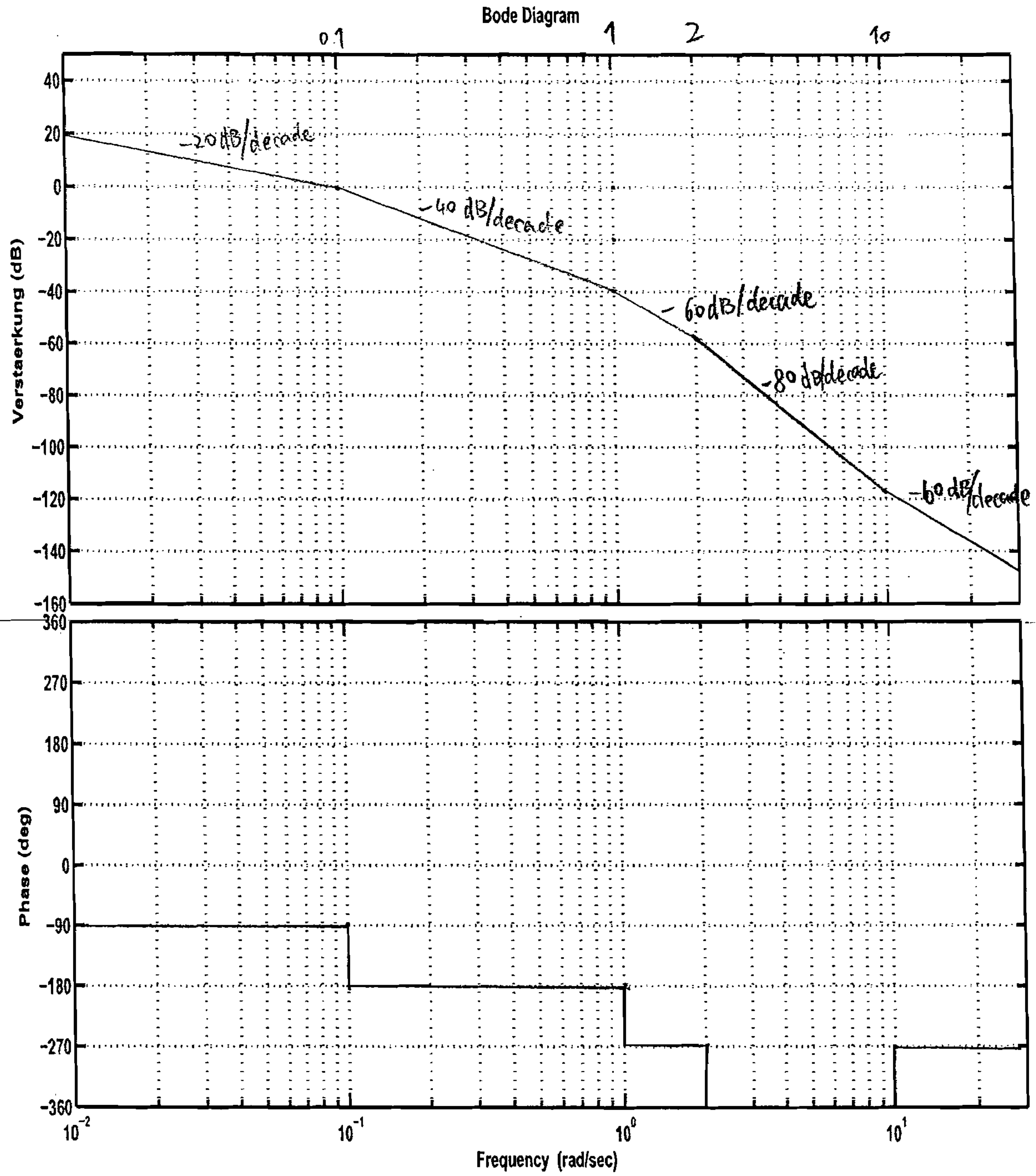


Abbildung 4.2: Bode Diagramm

Problem 5.

a) (1) $Y = Y_3$

(2) $Y_1 = G_1 U_1 = G_1 U$

(3) $Y_2 = G_2 U_2 = G_2 (G_1 U - G_4 U_4)$

(4) $Y_3 = G_3 U_3 = G_3 (G_5 U + G_2 U_2)$

(5) $Y_4 = G_4 U_4 = G_4 Y_3$

(6) $Y_5 = G_5 U$

$$\Rightarrow \quad G_s(S) = \frac{G_3 G_5 + G_1 G_2 G_3}{1 + G_2 G_3 G_4}$$

Problem 5

b).

$$G_S(s) = \frac{(s+1)}{s^3+6s^2+11s+6+\frac{1}{K}}$$

characteristic equation $C(s) = s^3+6s^2+11s+6+\frac{1}{K}$

Hurwitz - Criterion.

$a_3 > 0$

$a_2 > 0$

$a_1 > 0$

$a_0 > 0$

$H_1 > 0$

$H_2 > 0 \Rightarrow K > \frac{1}{60}$

$H_3 > 0$ if $H_2 > 0$

$\Rightarrow$ for $K > \frac{1}{60}$ the controlled system is asymptotic stable.

The system is asymptotic stable. $\Rightarrow$ I/O Stable.

Problem 5

c). PDT$_1$ element, $H(s) = \qquad = \frac{1+3s}{1+s}$

closed loop system:

$$G_c(s) = \qquad = \frac{(1+3s)(s-3)}{(1+3s)(s-3) + (1+s)(s^3+9s^2+23s+5)}$$

control error:

$e(t) = w(t) - y(t)$, $\quad E(s) = W(s) - Y(s)$

$w(t) = 1(t)$, $\quad W(s) = \frac{1}{s}$

$$E(s) =$$
$$= \frac{1}{s}\left[1 - \frac{(1+3s)(s-3)}{(1+3s)(s-3)+(1+s)(s^3+9s^2+23s+5)}\right]$$

steady-state error: $e(\infty)$

$$e(\infty) = \lim_{t\to\infty} e(t) = \lim_{s\to 0} sE(s)$$
$$= 2.5$$

a) Plant: PT_4

Controller: PD

b) no zeros

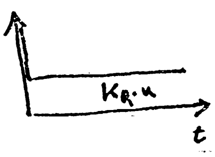
poles: $s_1 = 1$ $\omega_{0_1} = 1 s^{-1}$

$s_2 = 2$ $\omega_{0_2} = 2 s^{-1}$

$s_{3/4} = -4 \pm 10i$ $\omega_{0_{3/4}} = \sqrt{116}$ $D_{3/4} = \frac{4}{\sqrt{116}}$

c) input-output stable

cf. step response



d) 4 separate branches

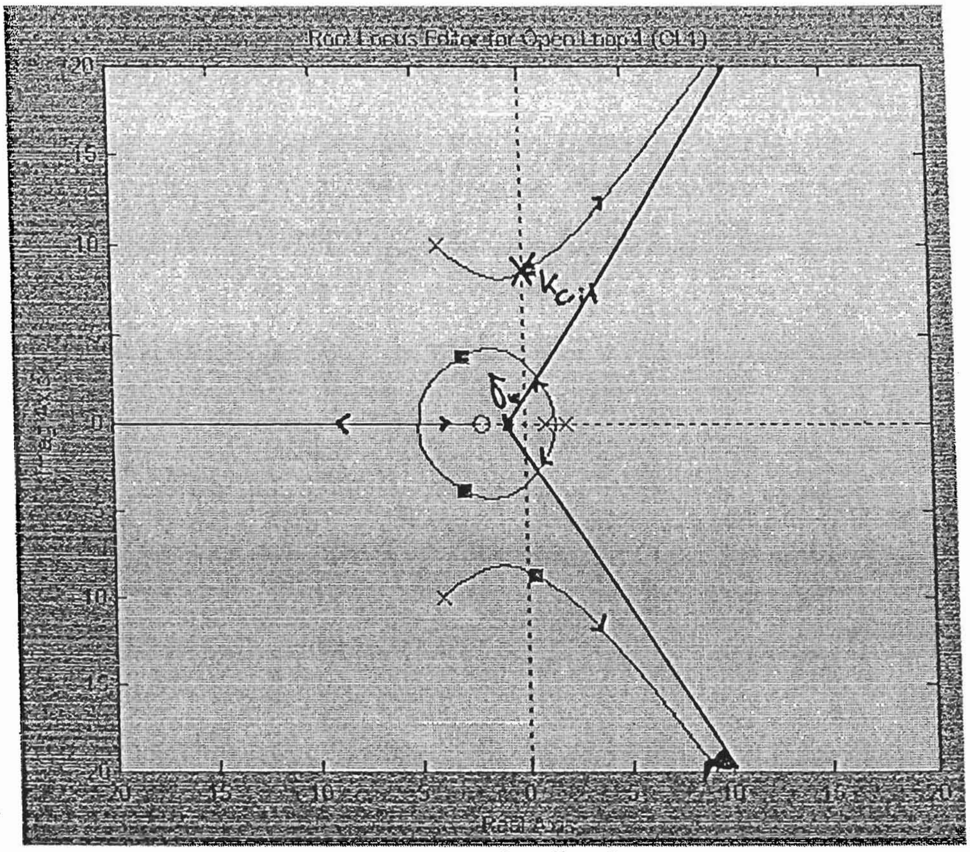
3 branches go to infinity

e) $\varphi_1 = 180°$, $\varphi_2 = 300°$, $\varphi_3 = 420°$ for neg. feedback

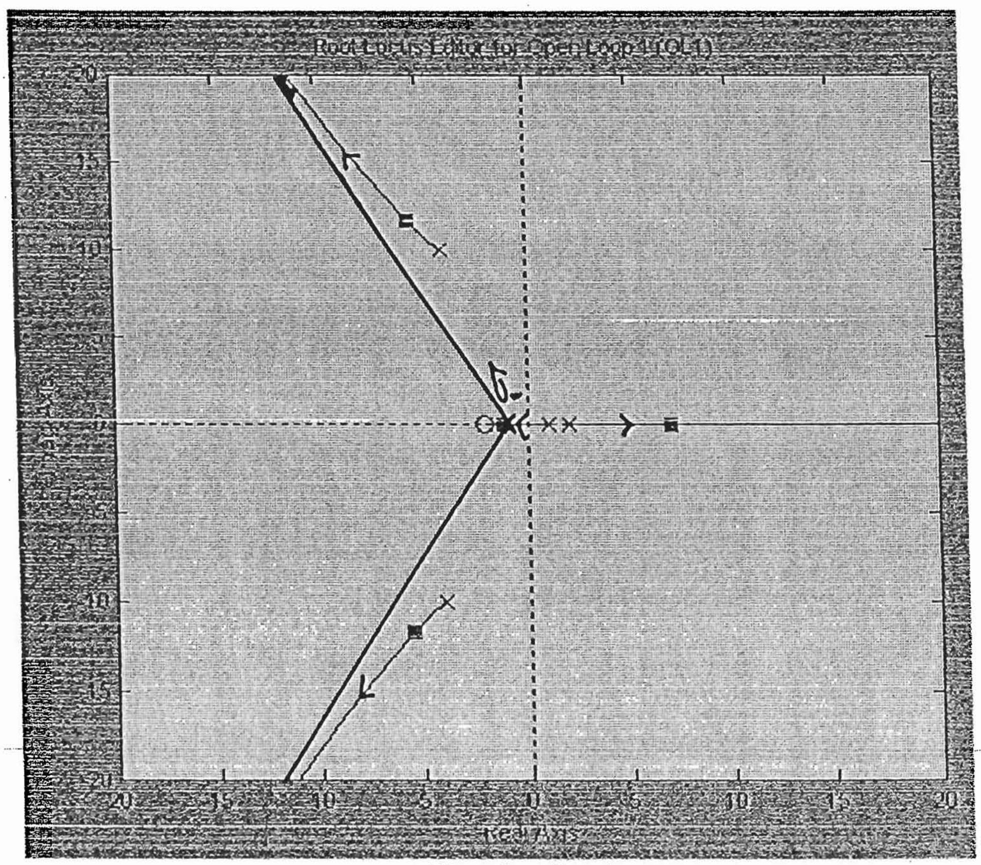
$\varphi_1 = 120°$, $\varphi_2 = 240°$, $\varphi_3 = 360°$ for pos. feedback

$$\delta_w = \frac{1 + 2 - 4 + 10i - 4 - 10i + 2}{3} = -1$$

g) neg. feedback (stability is possible) f)



negative feedback



positive feedback

6 h) stable ($s_i < 0$)

$$G_p(s) = -\frac{1}{2}\frac{1}{(s+5)^2(s+2)} = -\frac{1}{2}\frac{1}{25\cdot 2(\frac{1}{5}s+1)^2(\frac{1}{2}s+1)} = \underbrace{-\frac{1}{100}}_{k}\frac{1}{(\frac{1}{5}s+1)^2(\frac{1}{2}s+1)}$$

j) $$G_o(s) = -\frac{1}{2}\cdot k_p \cdot \frac{1}{(s+5)^2(s+2)} = 1$$

$$\Rightarrow -100 < k_p < 980$$

k) $$G_o(s) = -\frac{1}{2}k_p\frac{1}{(s+5)^2(s+2)} = 1$$

$$\Leftrightarrow k_p = +400$$

$\hookrightarrow$ pos. feedback

6 l) $$G_{o_1} = k_p \cdot \frac{1}{(s-1)(s-4)}$$

$$G_{o2} = \frac{s\cdot k_D}{(s-1)(s-4)}$$

both open loop systems are unstable $s_i > 0$

m) $$\frac{1}{s-1} + \frac{1}{s-4} = 0$$

$$\Rightarrow s = 2{,}5$$

$$\frac{1}{s-1} + \frac{1}{s-4} = \frac{1}{s}$$

$$\Rightarrow s_1 = 2 \wedge s_2 = -2$$

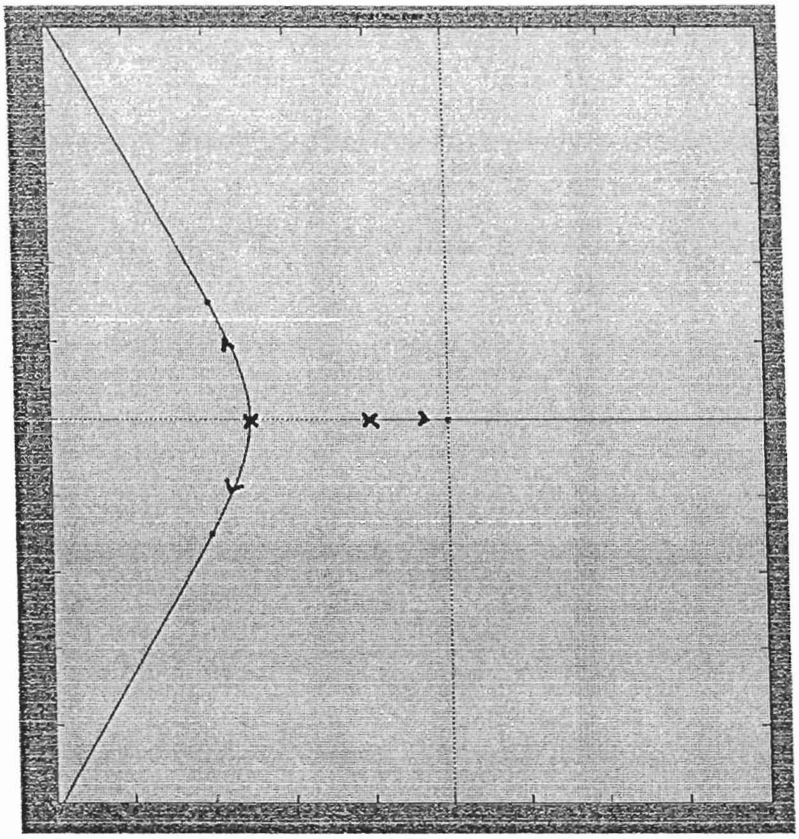
o) System 1: stable for large k ($k > k_{crit}$)

System is not able to oscillate for small and large k.

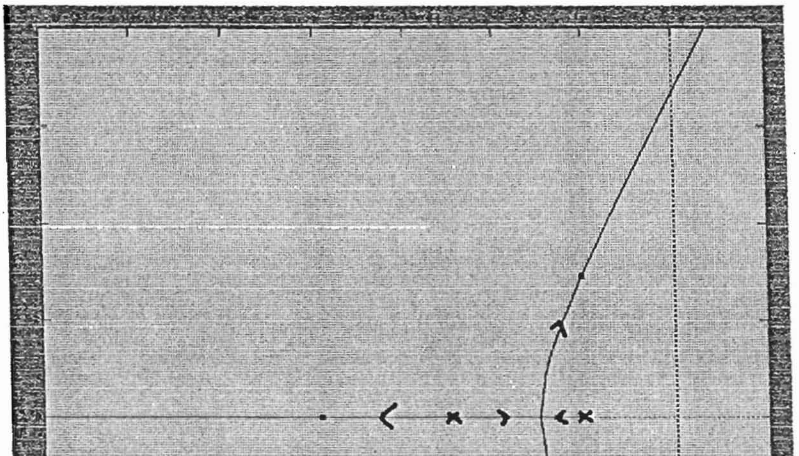
system 2: unstable (independent from k)

System is able to oscillate for large k.

i)

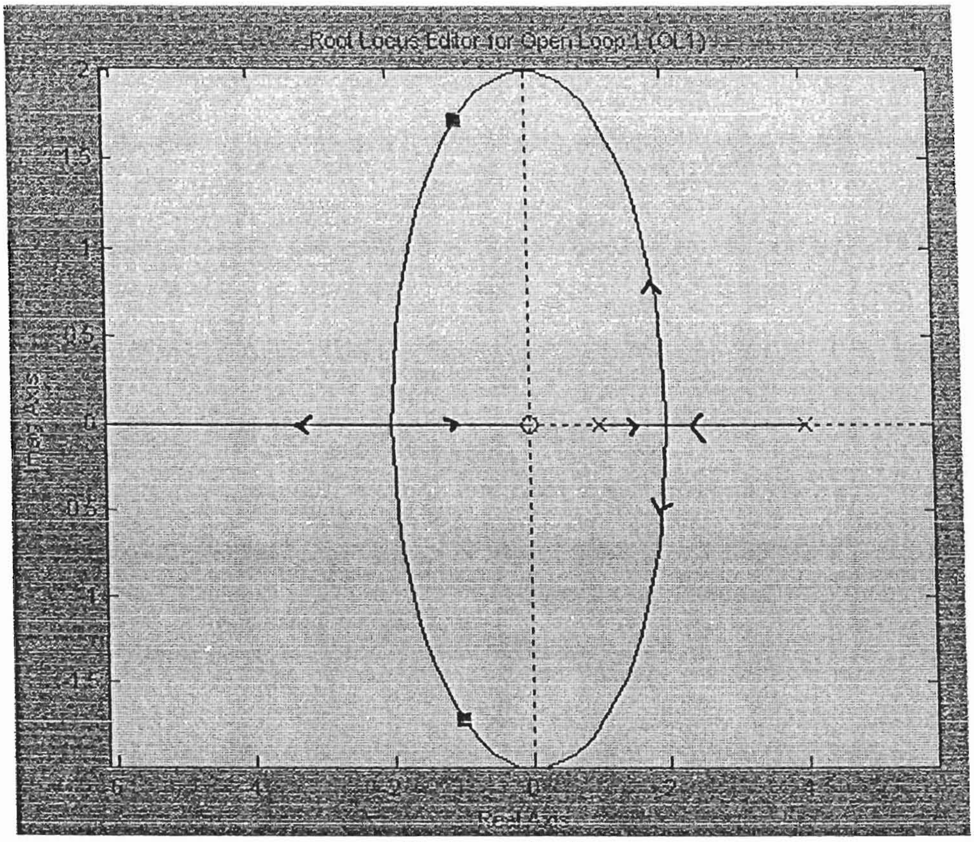


negative feedback

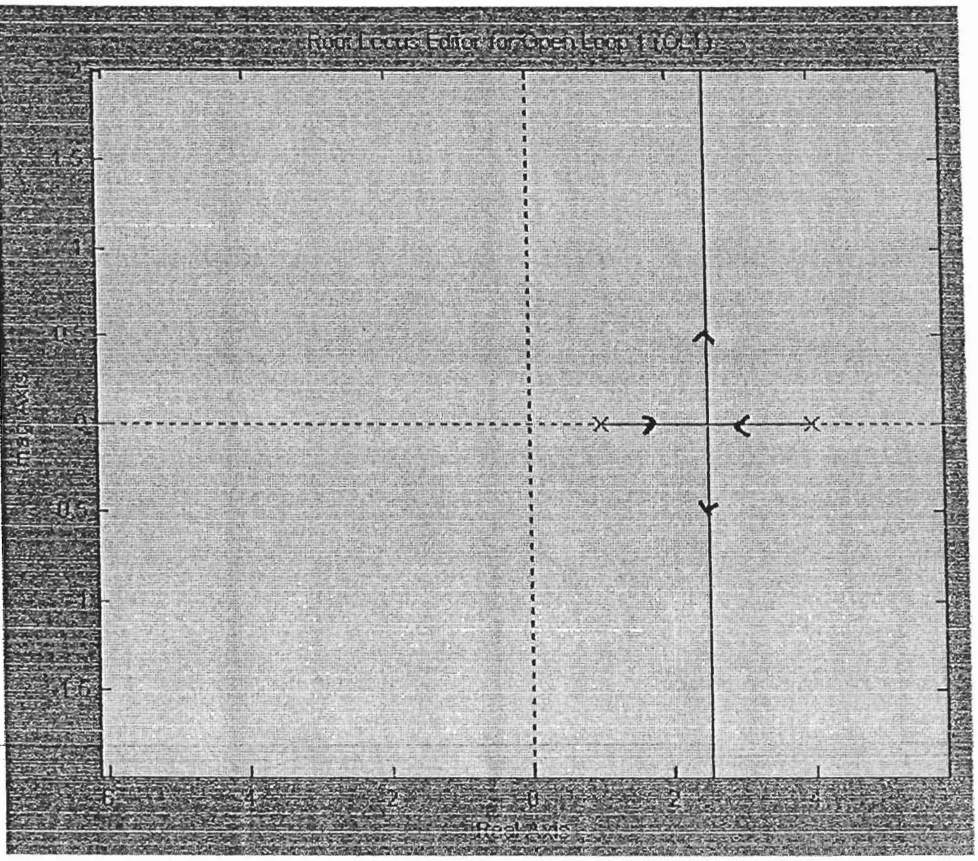


positive feedback

n)



with D-controller



with P-controller