LAST NAME	
FIRST NAME	
MATRIKEL-NO.	

Klausurunterlagen

Ich versichere hiermit, dass ich sämtliche für die Durchführung der Klausur vorgesehenen Unterlagen erhalten, und dass ich meine Arbeit ohne fremde Hilfe und ohne Verwendung unerlaubter Hilfsmittel und sonstiger unlauterer Mittel angefertigt habe. Ich weiß, dass ein Bekanntwerden solcher Umstände auch nachträglich zum Ausschluss von der Prüfung führt. Ich versichere weiter, dass ich sämtliche mir überlassenen Arbeitsunterlagen sowie meine Lösung vollständig zurück gegeben habe. Die Abgabe meiner Arbeit wurde in der Teilnehmerliste von Aufsichtsführenden schriftlich vermerkt.

Duisburg, den __________ ______________________________
(Unterschrift der/des Studierenden)

Falls Klausurunterlagen vorzeitig abgegeben: __________Uhr

Bewertungstabelle

	a.	b.	c.	d.	e.	f.	g.	h.	Summe
Aufgabe 1									
Aufgabe 2									
Aufgabe 3									
Aufgabe 4									
Aufgabe 5									
Aufgabe 6									
	i.	j.	k.	l.	m.	n.	o.		
Aufgabe 6									
Gesamt-punktzahl									
Anhebungs-faktor									
angehobene Punktzahl									
Bewertung gem. PO in Ziffern									

(Datum und Unterschrift 1. Prüfer, Univ.-Prof. Dr.-Ing. Söffker)

(Datum und Unterschrift 2. Prüfer, PD Dr.-Ing. Wend)

(Datum und Unterschrift des für die Prüfung verantwortlichen Prüfers Söffker)

Bewertung in Ziffern: __________

Fachnote gemäß Prüfungsordnung in Worten: ____________________

Bemerkung: ____

Problem 1

(15 points)

a) (2 points)
Define by equation the input and output decoupling zeros. Explain the difference between the decoupling zeros and transmission zeros.

b) (3 points)
A system description is given with

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -3 & 0 \\ 0 & 0 & 2 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 \\ 0 & b \\ 0 & 1 \end{bmatrix}, \quad \text{and} \quad C = \begin{bmatrix} 0 & 0 & c \end{bmatrix}, \quad (c \neq 0).$$

Is is noted that no direct transmission between the inputs and the outputs/measurements is given.

Calculate the eigenvalues of the system described by A. Is the system state stable? Is the system asymptotically stable? Denote the difference between state stability and asymptotic stability.

c) (3 points)
Calculate the invariant zeros for the system given in b).

d) (2 points)
Is the system in b) fully observable?

e) (3 points)
Is the system in b) stabilizable? Is it possible to realize full state feedback (measurement according to the output matrix C in b))?

f) (2 points)
Assume a system is fully controllable, fully observable and unstable. An optimal controller is developed with LQR (Linear Quadratic Regulator) method. The weighting matrices Q and R are chosen as positive definite. Is the controller able to stabilize the unstable system? Is the controlled system (system and controller) asymptotically stable? State reasons.

Problem 2

(15 points)

A new energy storage device is modeled as a MIMO system and is to be analyzed for controllability.
The model of this system is described by

$$A = \begin{bmatrix} 1 & 4 & 6 \\ 4 & -5 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ and}$$
$$C = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 0 & 1 \end{bmatrix}.$$

a) (2 points)
In the following an observer-based control method will be used. Prove for the given system that the observer eigenvalues can be placed arbitrarily.

In the following an observer with the observer feedback matrix $L = \begin{bmatrix} l_2 & l_1 & 0 \\ 0 & l_2 & l_1 \end{bmatrix}^T$ will be used.

b) (8 points)
Calculate l_1 and l_2 such that the eigenvalues of the observer are
$\lambda_1 = -1$, $\lambda_2 = -3$, and $\lambda_3 = -4$.

c) (2 points)
Is the system asymptotically stable? State your answer by calculation.

d) (3 points)
Draw the scheme of an observer-based state feedback control and label all blocks (system, observer, state control, ...) as well as the related signals (states, measurement, ...) for the system with reference input w and initial conditions $x(0), \hat{x}(0)$. Additionally state the order/dimensions of each used vector and matrix. Consider w to have the dimension $(k \times 1)$.

Problem 3

(30 points)

The system

$$\begin{aligned} \dot{x} &= Ax + Bu, \\ y &= Cx \end{aligned}$$

with the matrices

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ p & q & r \end{bmatrix}, \quad B = \begin{bmatrix} b_1 \\ 0 \\ b_2 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & c_1 & c_2 \end{bmatrix}$$

is given, whereby $p,\ q,\ b\ \in\ \mathbb{R}$.

a) (5 points)
Check the stability of the system in dependence on the parameters p, q, and r.

b) (6 points)
Assume $q = -1$, $p = 0$, and $r = 2$.
For which values of b_1 and b_2 can the system be stabilized by state feedback?

c) (8 points)
The system should be controlled by state feedback $u = -Kx$. Calculate the missing elements of the state feedback matrix

$$K = \begin{bmatrix} k_1 & k_2 & k_3 \end{bmatrix}$$

so that the controlled system has the eigenvalues $\lambda_1 = -10$, $\lambda_2 = 10$, and $\lambda_3 = 0$ for $b_1 = 2$, $b_2 = 1$, $r = 0$, $q = -1$, and $p = 1$.
(Note: Determine only the final simplified equations for k_1, k_2, k_3.)

d) (7 points)
Assume $p = 1$, $q = 1$, and $r = -1$.
Calculate the right- and left-eigenvectors of the system. (Note: One of the eigenvalues is $\lambda = 1$.)

e) (4 points)
Under which conditions in terms of b_1, b_2, c_1, and c_2 is the system in d) fully controllable and observable?

Problem 4

(30 points)

The system

$$\begin{aligned}\dot{x} &= Ax + Bu \\ y &= Cx\end{aligned}$$

with the matrices $A = \begin{bmatrix} 1 & 4 \\ 2 & a \end{bmatrix}$, $B = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, and $C = \begin{bmatrix} 0 & 1 \end{bmatrix}$ is given.

a) (1 point)
For which values of a is the system fully controllable?

b) (1 point)
For which values of a is the system fully observable?

For the following tasks use $a = 1$.

c) (5 points)
Determine the transfer function matrix and state the zeros, poles, and eigenvalues of the system.

d) (5 points)
The system is controlled by state feedback with the matrix $K = \begin{bmatrix} k_1 & k_2 \end{bmatrix}$. Which conditions have to be fulfilled for the elements of K so that the system is asymptotically stable?

e) (10 points)
For the design of an optimal controller the matrices

$$Q = \begin{bmatrix} 24 & 8 \\ 8 & 24 \end{bmatrix}, \; P = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}, \text{ and } R = [0.5]$$

are given. Calculate the control gain matrix K^* of a linear quadratic controller (assume $p_{12} = p_{21} \geq 0$, $p_{11} = p_{22} \geq 0$).

f) (3 points)
Which condition has to be fulfilled for $K = \begin{bmatrix} k_1 & k_2 \end{bmatrix}$ so that the system controlled by state feedback is fully observable?

g) (5 points)
Design an observer $L = \begin{bmatrix} l_1 \\ l_2 \end{bmatrix}$ with the eigenvalues $\lambda_1 = -1$, $\lambda_2 = -2$.

Maximum achievable points:	**90**
Minimum percentage for the grade 1,0:	**95%**
Minimum percentage for the grade 4,0:	**50%**