Reading-up-time

For reviewing purposes of the problem statements, there is a "reading-up-time" of **10 minutes** prior to the official examination time. During this period it is **not** allowed to start solving the problems. This means explicitly that during the entire "reading-up-time" no writing utensils, e.g. pen, pencil, etc. at all are allowed to be kept on the table. Furthermore the use of carried documents, e.g. books, (electronic) translater, (electronic) dictionaries, etc. is strictly forbidden. When the supervisor refers to the end of the "reading-up-time" and thus the beginning of the official examination time, you are allowed to take your utensils and documents. Please **then**, begin with filling in the **complete** information on the titlepage and on page 3.

Good Luck!

LAST NAME	
FIRST NAME	
MATRIKEL-No.	
TABLE-No.	

Klausurunterlagen

Ich versichere hiermit, dass ich sämtliche für die Durchführung der Klausur vorgesehenen Unterlagen erhalten, und dass ich meine Arbeit ohne fremde Hilfe und ohne Verwendung unerlaubter Hilfsmittel und sonstiger unlauterer Mittel angefertigt habe. Ich weiß, dass ein Bekanntwerden solcher Umstände auch nachträglich zum Ausschluss von der Prüfung führt. Ich versichere weiter, dass ich sämtliche mir überlassenen Arbeitsunterlagen sowie meine Lösung vollständig zurück gegeben habe. Die Abgabe meiner Arbeit wurde in der Teilnehmerliste von Aufsichtsführenden schriftlich vermerkt.

Duisburg, den __________ ____________________________________

(Unterschrift der/des Studierenden)

Falls Klausurunterlagen vorzeitig abgegeben: _______________Uhr

Bewertungstabelle

Aufgabe 1	
Aufgabe 2	
Aufgabe 3	
Aufgabe 4	
Aufgabe 5	
Gesamtpunktzahl	
Anhebungsfaktor	
angehobene Punktzahl	
Bewertung gem. PO in Ziffern	

(Datum und Unterschrift 1. Prüfer, Univ.-Prof. Dr.-Ing. Söffker)

(Datum und Unterschrift 2. Prüfer, PD Dr.-Ing. Wend)

(Datum und Unterschrift des für die Prüfung verantwortlichen Prüfers, Söffker)

Fachnote gemäß Prüfungsordnung:

☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐
1,0	1,3	1,7	2,0	2,3	2,7	3,0	3,3	3,7	4,0	5,0
sehr gut		gut			befriedigend			ausreichend		mangelhaft

Bemerkung: ______________________________

Attention: Give your answers to ALL problems directly below the questions in the exam question sheet.

You are NOT allowed to use a pencil and also NOT red color (red color is used for corrections).

This exam "Control Technique" is taken by me as a
☐ mandatory (Pflichtfach)
☐ elective (Wahlfach)
☐ prerequsite (Auflage)
subject (Cross ONE option according to your own situation).

Maximum achievable points:	**80**
Minimum points for the grade 1,0:	**95%**
Minimum points for the grade 4,0:	**50%**

Problem 1 (13 points)
The following technical system is used to control a shaft with the help of two magnetic radial bearings and a single magnetic axial bearing. In this case the fully electromagnetic bearing must be controlled for all three planar movement directions (x_1, x_2, z) simultaneously (from: Roddeck, W.: *Einführung in die Mechatronik*. 2. Aufl. B. G. Teubner Verlag, Stuttgart, 2003) (refer to Fig. 1.1).

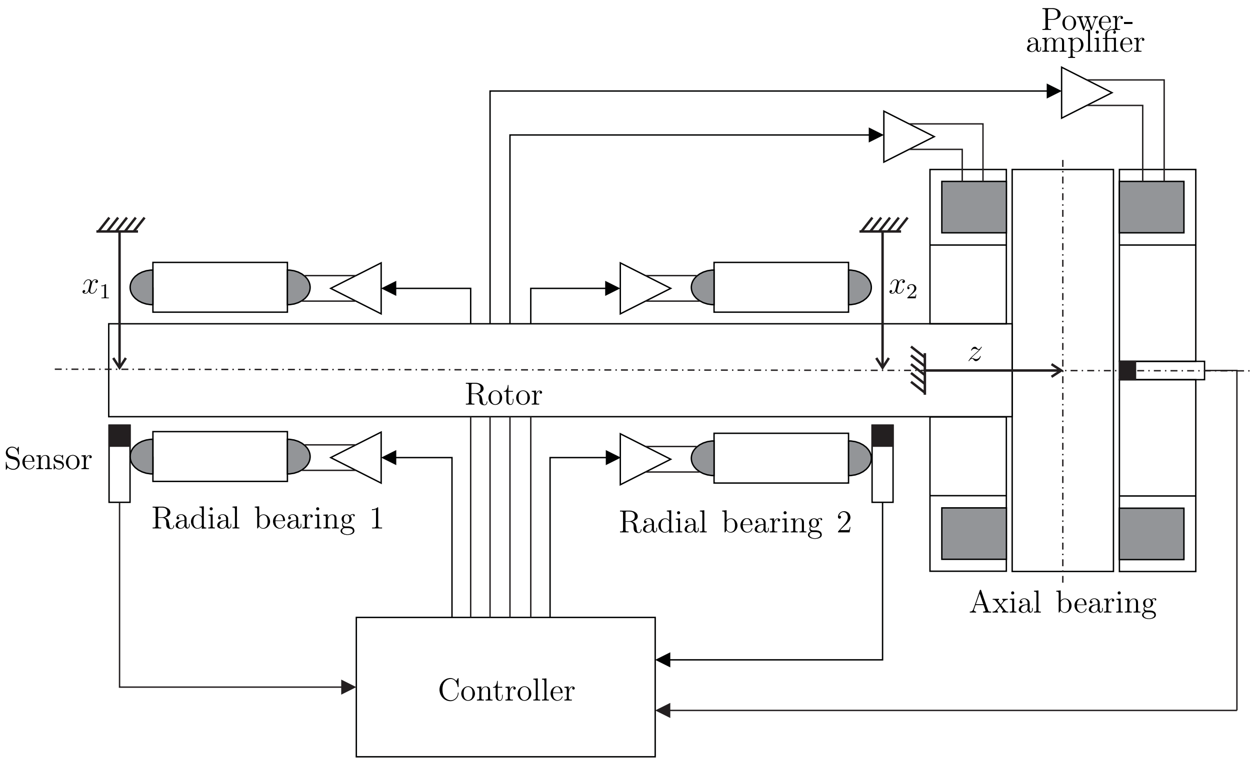

Figure 1.1: Control of an axle by two magnetic radial bearings and a magnetic axial bearing (Roddeck, W.: *Einführung in die Mechatronik*, pp. 425)

The position of the rotor at the measuring points has to be controlled in the static state when only exposed to the gravitational field of the earth as well as in a rotating state related to the indicated central positions.

a) (3 points)
Draw the control loops of the three coordinate directions x_1, x_2, and z shown in the Fig. 1.1 separately. Label the elements, the signals, and additionally the control error and the control signal with both, the nomenclatures of control technique and with the terms used in the illustrated example.

b) (1 point)
Classify the input/output behavior described by the equation $4y + 2\dot{y} = 3\dot{u} + 5\ddot{u}$ with u defining the output and y the input of the system.

c) (2 points)
Regarding the input/output behavior described in problem 1b), determine the values of the time constant T of the dynamics and the integrational time constant T_{I}?

d) (2 points)
The step responses of a front chassis of a new experimental vehicle are measured as illustrated in Fig. 1.2.

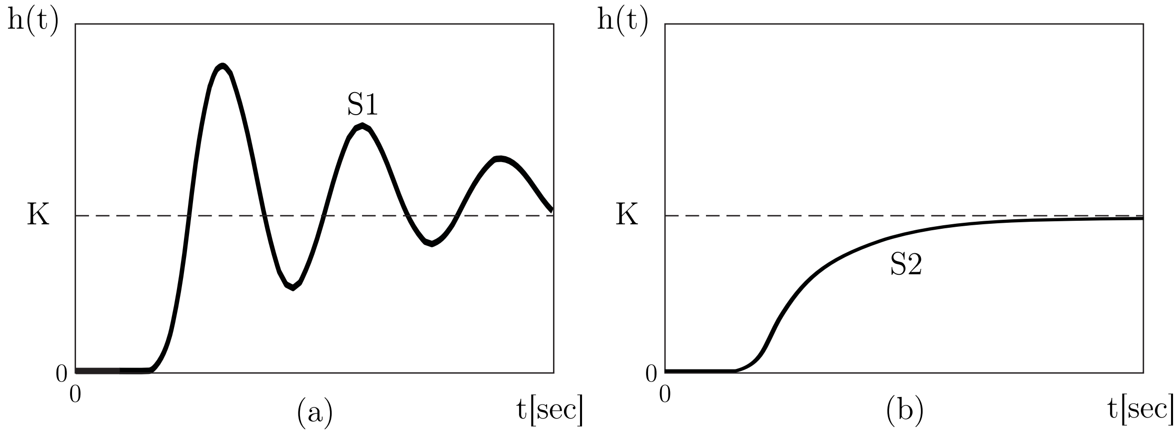

Figure 1.2: Step responses

Give the describing differential equations of the underlying systems with u as output and y as input.

e) (2 points)
Which central physical difference is illustrated in the transfer behavior given in Fig. 1.2 (a) and (b)?

Which parameter describes (depending on the value) the ability of the system to have a transfer behavior such as illustrated in Fig. 1.2(a) as well as Fig. 1.2(b) precisely?

f) (2 points)
The simplified description of the dynamic behavior of a valve-controlled hydraulic cylinder can be expressed by the equations

$$\dot{p}_{\mathrm{A}} = \frac{E}{V_{\mathrm{A}}}\left(Q_{\mathrm{A}} - A_{\mathrm{A}}\dot{x}\right),$$
$$\dot{p}_{\mathrm{B}} = \frac{E}{V_{\mathrm{B}}}\left(Q_{\mathrm{B}} - A_{\mathrm{B}}\dot{x}\right), \text{ and}$$
$$m\ddot{x} = p_{\mathrm{A}}A_{\mathrm{A}} - p_{\mathrm{B}}A_{\mathrm{B}} - F_{\mathrm{ext}}$$

where
$p_{\mathrm{A,B}}$: pressure in chamber A or B
$A_{\mathrm{A,B}}$: pressurized area in chamber A or B
$V_{\mathrm{A,B}}$: volume of the chamber A or B
$Q_{\mathrm{A,B}}$: volume flow of the chamber A or B
E : bulk modulus
m : mass
x : position of cylinder rod
F_{ext} : external applied force

and the signal $\dot{x}$ to be measured.
The matrices A and C of the state space representation are

$$A = \begin{bmatrix} 0 & 0 & -\frac{EA_{\mathrm{A}}}{V_{\mathrm{A}}} \\ 0 & 0 & -\frac{EA_{\mathrm{B}}}{V_{\mathrm{B}}} \\ \frac{A_{\mathrm{A}}}{m} & -\frac{A_{\mathrm{B}}}{m} & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}.$$

Considering $m = 0.5$, $A_{\mathrm{A}} = A_{\mathrm{B}} = V_{\mathrm{A}} = 2$, and $V_{\mathrm{B}} = 1$, calculate the eigenvalues of the system as a function of E.

g) (1 point)
In principle, which serious advice for the controller design for the control of a pure integral system (IT_0-transfer behavior) can you state, when the controller design particularly addresses the realization of stationary accuracy and oscillations should not be additionally stimulated. State reasons.

$\square$

$\sum \square$

Problem 2 (18 points)

The block diagram of a suspended ball (refer to Fig. 2.1) is given in Fig. 2.2. The ball is modeled as a mass point with the mass m. The position of the ball is controlled by a magnetic force f, which is generated by the actuator of the system. The force depends on the position x of the ball and the current i in the coil of the actuator. The position of the ball is measured by a sensor.

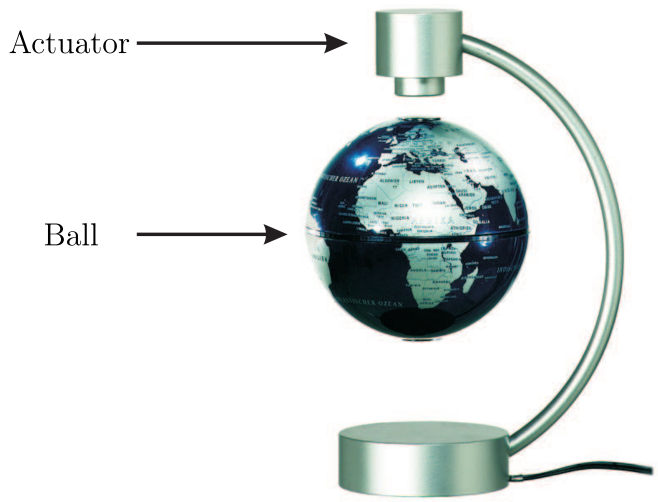

Figure 2.1: Suspended ball (www.techgalerie.de)

Based on the sensor signal, the control signal i_r is calculated by a controller. For minimizing noise signals from the sensor, a filter can be used additionally.

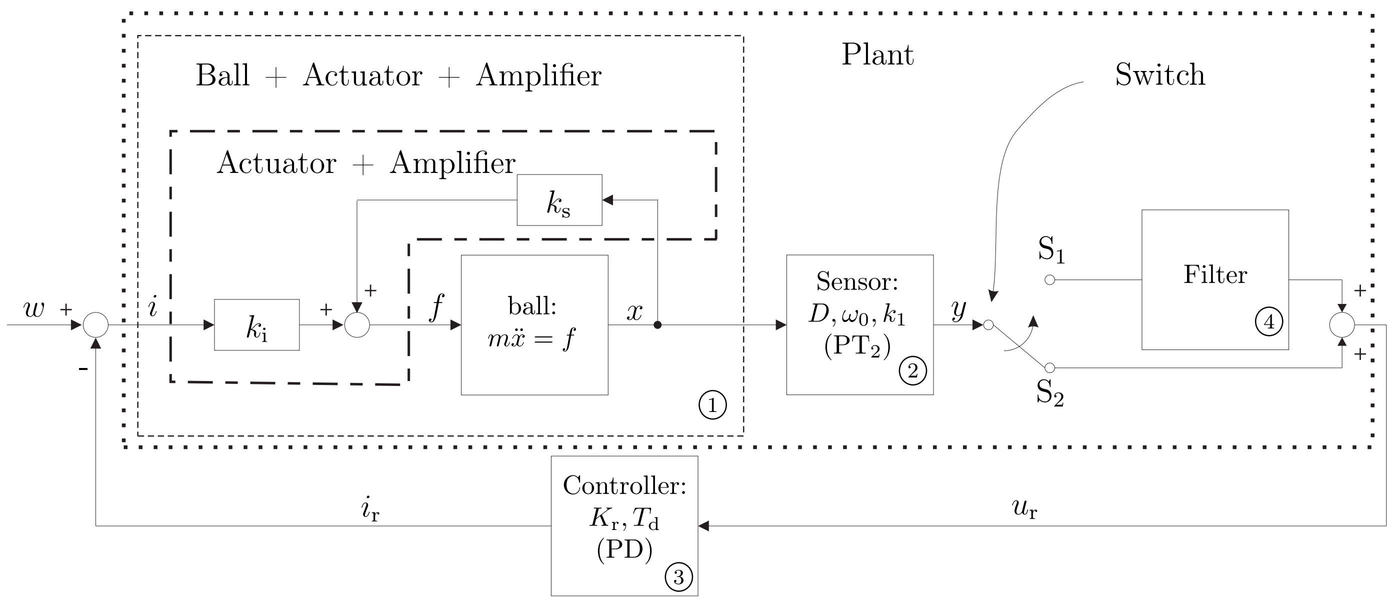

Figure 2.2: Block diagram of a suspended ball

a) (5 points)
In Fig. 2.2, the actuator dynamics is assumed to be linear. However, measurements show the relation

$$f = k_0 \left(\frac{(i_\mathrm{b} + i)^2}{(s_0 - x)^2} - \frac{(i_\mathrm{b} - i)^2}{(s_0 + x)^2} \right),$$

where:
- k_0 : system constant,
- i_b : bias current (constant),
- s_0 : air gap (constant),
- i : coil current (control signal), and
- x : displacement of the ball.

Determine the parameters k_i and k_s from the linear relation

$$f(i, x) = k_\mathrm{i} i + k_\mathrm{s} x$$

with respect to the working point ($i_0 = 0,\ x_0 = 0$).

b) (5 points)
Draw the step responses of the elements ② and ③, qualitatively. Give the differential equations of the elements ①, ②, and ③.

c) (3 points)
Classify the transfer behavior of the plant without filter (switch positon S_2).

d) (3 points)
Assume the sensor can be described by the following differential equation

$$10^{-8}\ddot{y} + 10^{-4}\dot{y} + y = x.$$

Is the plant behavior stable without filter (switch positon S_2) ($m, k_\mathrm{s} > 0$)? State reasons.

e) (2 points)
For the purpose of signal conditioning, a filter (element ④) can be activated (switch position S_1). The transfer function of the filter is measured as shown in Fig. 2.3.

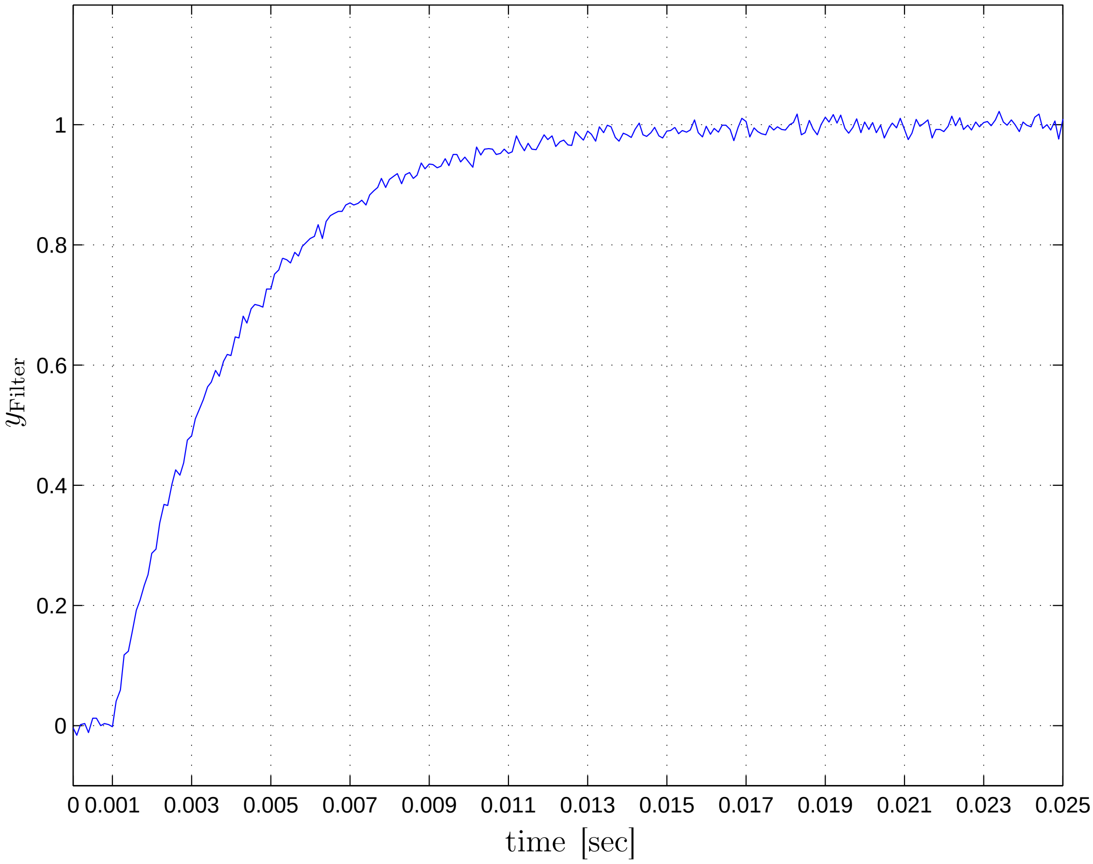

Figure 2.3: Experimental estimated transfer function of the filter

Classify the transfer behavior of the filter by neglecting the messurement noise. Determine the parameters of the underlying differential equation from the Fig. 2.3.

☐

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Problem 3 (12 points)

a) (3 points)
Calculate the inverse Laplace transform $u(t)$ of the function

$$U(s) = \frac{s+1}{s^2(s^2+2s+1)}.$$

b) (4 points)
A PIDT$_1$T$_t$-System can be described by the differential equation

$$T_1\dot{y} + y = K\left[u(t-T_\mathrm{t}) + T_\mathrm{D}\dot{u}(t-T_\mathrm{t}) + \frac{1}{T_\mathrm{I}}\int_0^t u(\tau - T_\mathrm{t})d\tau\right].$$

State the input/output behavior of the system in form of the corresponding transfer function. Determine the cut-off frequencies in terms of the parameters K, T_1, T_D, T_I, and T_t depending on the conditions

$$K, T_1, T_\mathrm{D}, T_\mathrm{I}, T_\mathrm{t} \;>\; 0, \quad T_\mathrm{I} \;<\; 4T_\mathrm{D} \quad \text{and} \quad \frac{1}{T_1} \;>\; \frac{1}{\sqrt{T_\mathrm{D}T_\mathrm{I}}}.$$

Draw the Bode diagram qualitatively and indicate the cut-off frequencies as well as the gradients of the approximated courses.

c) (2 points)
A transfer element with PDT_1-behavior is connected to a transfer element with PI-behavior as controller in positive feedback.
Determine the transfer function of the open-loop system $G_0(s)$.

d) (3 points)
A control system with negative feedback has to be analyzed. The system consists of a stable PT_2-element with one conjugate-complex pole pair as well as a controller with PIT_1-behavior.

According to the qualitatively drawn root locus of the system, explain that the system becomes unstable for large gains.

$\sum$

Problem 4 (20 points)
For an electric drive a control for the position of the rotor φ has to be designed. The angular velocity $\omega(t) = \dot{\varphi}(t)$ of the rotor dependending on the terminal voltage $u(t)$ can be stated approximately by

$$T_2 \iint \omega(t)\ dt\ dt + T_1 \int \omega(t)\ dt + \omega(t) = k_1 \int u(t)\ dt + k_2 \iint u(t)\ dt\ dt.$$

The position control has to be realized by a controller with the transfer behavior

$$\frac{1}{T_{\mathrm{I}}} \int u(t)\ dt = k_{\mathrm{R}}\ \varphi(t).$$

The control loop is closed by negative feedback.

a) (6 points)
Classify the transfer behavior $G_{\mathrm{P}}(s) = \frac{\varphi(s)}{u(s)}$ of the plant as well as the transfer behavior $G_{\mathrm{C}}(s) = \frac{u(s)}{\varphi(s)}$ of the controller. State the transfer function of the open-loop system $G_{\mathrm{O}}(s)$ and classify the corresponding transfer behavior.

b) (2 points)
State the transfer function $G(s)$ of the entire system (electric drive with position control (negative feedback)) and classify the transfer behavior.

c) (6 points)
Three systems are controlled by a controller with P-behavior (negative feedback). For each transfer function of the open loops, the zeros, and the poles are given as follows:

System 1:
Zeros: $s_{01} = -3 + i$; $s_{02} = -3 - i$; $s_{03} = 2$
Poles: $s_1 = -1 + i$; $s_2 = -1 - i$; $s_3 = 0.1$

System 2:
Zeros: $s_{01} = -2 + 2i$; $s_{02} = -2 - 2i$
Poles: $s_1 = 1 + i$; $s_2 = 1 - i$; $s_3 = -0.1$; $s_4 = -1$

System 3:
Zeros: $s_{01} = -2$; $s_{02} = -1$
Poles: $s_1 = 2 + i$; $s_2 = 2 - i$

Determine by using qualitively drawn root loci which system to prefer in terms of stability and maximal damping. Give the transfer function $G(s)$ of this system.

d) (6 points)
A plant $G_\mathrm{P}(s)$ with the dynamic transfer behavior

$$G_\mathrm{P}(s) = \frac{4 + \frac{1}{s}}{s^2 + 3s + 2}$$

has to be controlled by a negative feedback controller. The controller transfer function is given by

$$G_\mathrm{C}(s) = \frac{4}{2s + 1}.$$

Determine the transfer function of the open-loop system as well as the zeros and the poles. Furthermore, draw both Bode diagram and Nyquist plot for the corresponding system, qualitatively. Determine the gain margin and the phase margin of the system graphically by using the Bode diagram.

$\square$

$\sum \square$

Problem 5 (17 points)
A hydraulic motor and an amplifier are connected in series (refer to Fig. 5.1).

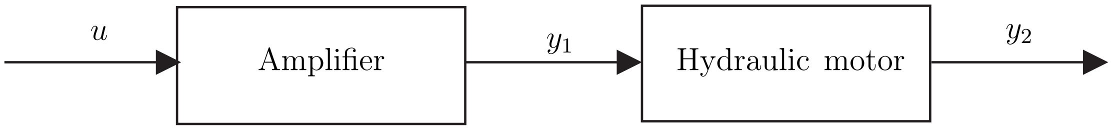

Figure 5.1: System

The behavior of the hydraulic motor can be stated approximately by

$$G_{\mathrm{H}}(s) = \frac{\frac{1}{s}}{2s^2 + 4s + 2}.$$

The transfer behavior of the amplifier can be described as

$$G_{\mathrm{A}}(s) = \frac{10 + 2s}{s^2 + 2s + 1}.$$

a) (9 points)
Give the state space representation and the eigenvalues as well as at least two eigenvectors for the entire system.

☐

In the following, a simplified description of the entire system is given by

$$G(s) = \frac{10 + \frac{1}{T_I s}}{s^3 + 4s^2 + T_1 s + 2}.$$

In order to control the system, a controller with the transfer behavior

$$G_C(s) = K_C(s + 2)$$

has to be connected to the system (negative feedback).

b) (4 points)
Give the Hurwitz matrix for the parameters $K_C = 0.2$ and $T_1 = 3$ for the closed-loop system.

In the following, for the closed-loop system, the simplified transfer function

$$G(s) = \frac{10s + 1}{s^4 + 4s^3 + 5s^2 + T_1 s + 1}$$

has to be used.

c) (4 points)
For which values of the parameter T_1 is the closed-loop system stable? (Hint: $9 < \sqrt{84} < 10$)

$\square$

$\sum \square$