Reading-up-time

For reviewing purposes of the problem statements, there is a "reading-up-time" of **10 minutes** prior to the official examination time. During this period it is **not** allowed to start solving the problems. This means explicitly that during the entire "reading-up-time" no writing utensils, e.g. pen, pencil, etc. at all are allowed to be kept on the table. Furthermore the use of carried documents, e.g. books, (electronic) translater, (electronic) dictionaries, etc. is strictly forbidden. When the supervisor refers to the end of the "reading-up-time" and thus the beginning of the official examination time, you are allowed to take your utensils and documents. Please **then**, begin with filling in the **complete** information on the titlepage and on page 3.

Good Luck!

LAST NAME	
FIRST NAME	
MATRIKEL-No.	
TABLE-No.	

Klausurunterlagen

Ich versichere hiermit, dass ich sämtliche für die Durchführung der Klausur vorgesehenen Unterlagen erhalten, und dass ich meine Arbeit ohne fremde Hilfe und ohne Verwendung unerlaubter Hilfsmittel und sonstiger unlauterer Mittel angefertigt habe. Ich weiß, dass ein Bekanntwerden solcher Umstände auch nachträglich zum Ausschluss von der Prüfung führt. Ich versichere weiter, dass ich sämtliche mir überlassenen Arbeitsunterlagen sowie meine Lösung vollständig zurück gegeben habe. Die Abgabe meiner Arbeit wurde in der Teilnehmerliste von Aufsichtsführenden schriftlich vermerkt.

Duisburg, den __________ ____________________________________

(Unterschrift der/des Studierenden)

Falls Klausurunterlagen vorzeitig abgegeben: _______________Uhr

Bewertungstabelle

Aufgabe 1	
Aufgabe 2	
Aufgabe 3	
Aufgabe 4	
Gesamtpunktzahl	
Anhebungsfaktor	
angehobene Punktzahl	
Bewertung gem. PO in Ziffern	

____

(Datum und Unterschrift 1. Prüfer, Univ.-Prof. Dr.-Ing. Söffker)

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(Datum und Unterschrift 2. Prüfer, PD Dr.-Ing. Wend)

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(Datum und Unterschrift des für die Prüfung verantwortlichen Prüfers, Söffker)

Fachnote gemäß Prüfungsordnung:

☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐
1,0	1,3	1,7	2,0	2,3	2,7	3,0	3,3	3,7	4,0	5,0
sehr gut		gut			befriedigend			ausreichend		mangelhaft

Bemerkung: ____

Attention: Give your answers to ALL problems directly below the questions in the exam question sheet.

You are NOT allowed to use a pencil and also NOT red color (red color is used for corrections).

Maximum achievable points:	**100**
Minimum points for the grade 1,0:	**95%**
Minimum points for the grade 4,0:	**50%**

Problem 1 (11 points)

a) (3 points)
Give a principal sketch for the practical realization of an observer-based state feedback control for a MIMO system and denote the related parts.

b) (6 points)
State the poles of the system

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u,$$

$$y = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$

Judge the I/O-stability of the system. Conclude to the state-stability of the system.

c) (2 points)
Define the state stability of a dynamical system with the eigenvalues
$\lambda_1 = 2.4$, $\lambda_2 = -2 + 3j$, and $\lambda_2 = -2 - 3j$.

$\sum$

Problem 2 (22 points)
In the following the system described by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -1 & a & 0 \\ 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} b_1 \\ 0 \\ b_2 \end{bmatrix} u,$$

$$y = \begin{bmatrix} 0 & c & 0 \\ 0 & 0 & 2c \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

has to be analyzed.

a) (5 points)
State the eigenvalues of the system. For which values of a is the system asymptotic stable according to Lyapunov?

b) (4 points)

For the system the relation $a = 1$ is given. Analyze the observability and controllability of the system stated in a). Under which conditions c, b_1, and b_2 is the system fully observable and fully controllable?

c) (8 points)

The system given by

$$A = \begin{bmatrix} 3 & 1 \\ 4 & 5 \end{bmatrix}, \; B = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \; C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

has to be controlled by an output feedback. The eigenvalues of the controlled system should be $\lambda_1 = -1$ and $\lambda_2 = -2$. State the controller gain matrix K of the output feedback.

d) (2 points)

For the system described by

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix}, \; C = \begin{bmatrix} 0 & -1 \end{bmatrix}$$

an observer-based stated feedback has to be realized. Is it possible to place the observer eigenvalues arbitrarily? Give reasons.

e) (3 points)

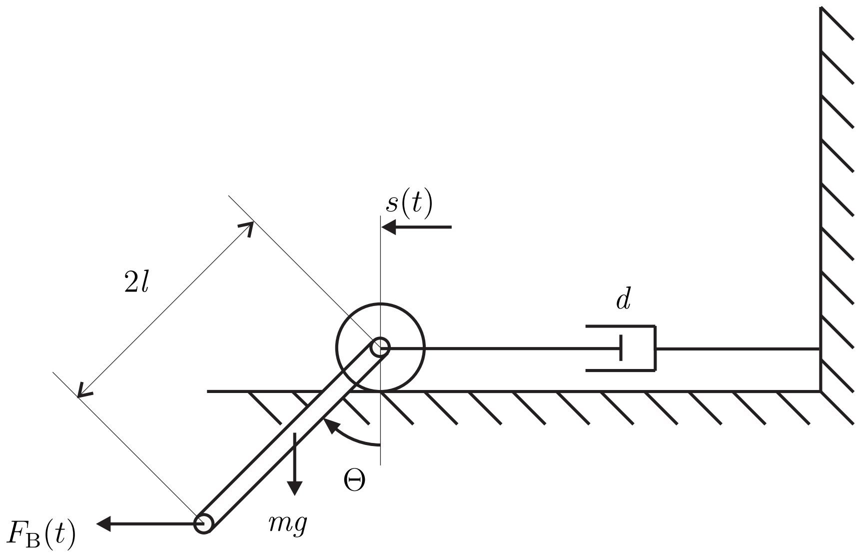
For the observer a feedback matrix $L = \begin{bmatrix} 0 \\ l \end{bmatrix}$ is given.

Calculate $l \in \mathbb{R}$ so that the eigenvalues of the observer are $\lambda_1 = -4$ and $\lambda_2 = -0.5$.

□

$\sum$ □

Problem 3 (27 points)



The mechanical system shown above is described by the following linearized differential equations

$$\begin{aligned} ml\ddot{s} + m(l^2 + r^2)\ddot{\Theta} + mgl\Theta &= 2lF_\mathrm{B}, \\ m\ddot{s} + ml\ddot{\Theta} + d\dot{s} &= F_\mathrm{B}. \end{aligned}$$

The measured variable is s.

a) (7 points)
Give the matrices A, B, and C of the system described above using the state vector

$$x = \begin{bmatrix} s \\ \Theta \\ \dot{s} \\ \dot{\Theta} \end{bmatrix}.$$

b) (5 points)
The characteristic equation of the system given in a) for certain parameters is

$$|\lambda I - A| \;=\; \lambda^4 \;+\; 2\;\lambda^3 \;+\; \lambda^2 \;+\; 2\;\lambda.$$

Give the Hurwitz determinants $H_i,\ i = 1\dots n$. What can be concluded with respect to the asymptotic stability of the system?

c) (7 points)
A reconstruction of the system given in a) leads to the matrices

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 10 & -3 & 0 \\ 0 & -1 & 0.1 & 0 \end{bmatrix}, \; B = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 100 \end{bmatrix}.$$

Assume $C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$ to realize an output-feedback with $u = -KCx$, with $K = [k_1 \; k_2]$.
For which k_1 is the closed-loop system asymptotically stable?

d) (3 points)
Give the transfer function matrix of the system

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 10 & -1 & 0 \\ 0 & -1 & 0.1 & 0 \end{bmatrix}, \ B \ = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 100 \end{bmatrix}, \text{ and C} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$$

and state the poles of $G(s)$. Use therefore the relation

$$(sI - A)^{-1} = \frac{1}{(s^2+s+1)} \begin{bmatrix} \frac{s^2+s+1}{s} & \frac{10}{s} & \frac{s^2+1}{s^2} & \frac{10}{s^2} \\ 0 & s+1 & \frac{1}{10s} & \frac{s+1}{s} \\ 0 & 10 & \frac{s^2+1}{s} & \frac{10}{s} \\ 0 & -1 & \frac{1}{10} & s+1 \end{bmatrix}.$$

e) (5 points)
The eigenvalues of a controlled system are
$\lambda_1 = 0 + \varepsilon_1$,
$\lambda_2 = -20 + j3$,
$\lambda_3 = -20 - j3$,
$\lambda_4 = -40 + j38.75$,
$\lambda_5 = -40 - j38.75$,
$\lambda_6 = -6.2531$,
$\lambda_7 = -0.8543$,
$\lambda_8 = -100.001$,
$\lambda_9 = 0.0000001 + \varepsilon_2$,
with $\varepsilon_i \in \mathbb{R}$, $i = 1, 2$.

The modes 2 and 3 are not controllable and the mode 9 is not observable, no further input/output decoupling zeros exist. Under which conditions is the state controlled system

i) asymptotically state stable,
ii) state unstable,
iii) boundary stable, or
iv) asymptotic I/O-stable?

$\sum$

Problem 4 (40 points)
In Fig. 4.1 a test rig of a new pneumatic actuator consisting of a fluidic muscle paired with a linear spring is shown. Six of these actuators are used in a hexapod for research of stresses and strains on the human cervical spline. In the illustrated configuration with the upper end of the actuator fixed to the mounting frame, the actuator force $F_\mathrm{m} = f(z, p)$ as a function of the stroke z and the pressure p of the fluidic muscle can be investigated in a single axes configuration with the position spindle representing the cervical spine.

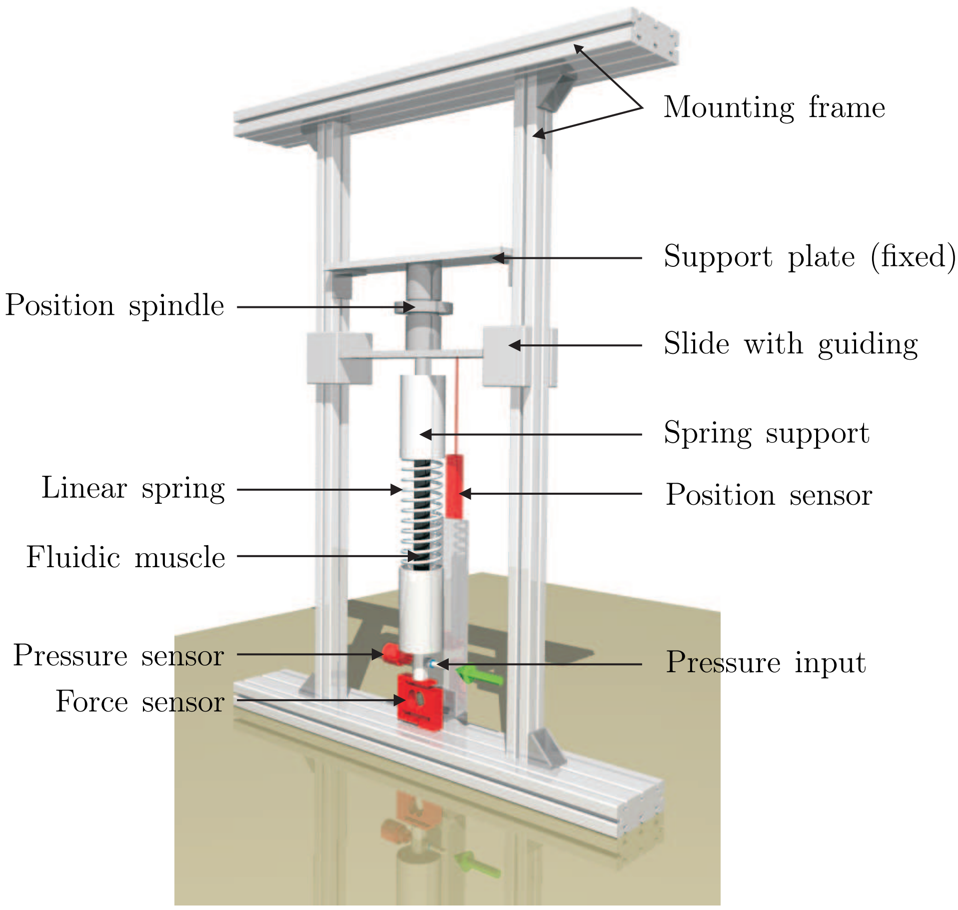


Figure 4.1: Test rig for force-pressure-stroke measurement of a pneumatic actuator

Using a fluidic muscle has several advantages compared to usual pneumatic actuators in modern robotics. A major advantage is the high power/weight ratio, a major disadvantage the nonlinearity in its force-stroke as well as force-pressure relations as stated in the following equations

$$\ddot{z} = \frac{1}{m_\mathrm{s}} \left(F_\mathrm{m} - F_\mathrm{spring} - f_\mathrm{c} \tanh \frac{\dot{z}}{\epsilon} + f_\mathrm{v} \dot{z} - m_\mathrm{s} g \right), \text{ and}$$

$$\dot{p} = \frac{\chi}{V(z)} \left(RT \dot{m}_\mathrm{g} - \frac{dV}{dz} p \right).$$

These equations are describing the overall system behavior including all nonlinearities. When using the position z, the velocity $\dot{z}$, and the pressure p of the slide respectively the muscle, a flatness-based control law is suitable to face the nonlinearities.

However, when using a slightly modified state vector with z, $\dot{z}$, and the pressure depending muscle force F_m, a linear state space representation can be obtained by the following equations

$$F_\mathrm{m} = m_\mathrm{s}\ddot{z} + f_\mathrm{v}\dot{z} - cz \text{ and}$$
$$F_\mathrm{m}(s) = \frac{1}{1+\frac{s}{K_\mathrm{p}}} F_\mathrm{d}(s)$$

with F_d denoting the desired actuator force, m_s the total mass of the slide, f_v the viscous friction coefficient, c the spring constant, and K_p the control gain.

a) (5 points)
Give the A, B, C, and D matrices corresponding to the state vector $x = \begin{bmatrix} F_\mathrm{m} & z & \dot{z} \end{bmatrix}^T$ in terms of the variables m_s, f_v, c, and K_p with the force F_d as the input. The position z of the slide has to be measured.

☐

Through experiments, the viscous friction coefficient f_v has been obtained, and the total mass m_s of the slide is also known. With both values inserted into the system matrices of a), the A, B, C, and D matrices yield

$$A = \begin{bmatrix} -K_\mathrm{p} & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 2 & 1 \end{bmatrix}, \qquad B = \begin{bmatrix} K_\mathrm{p} \\ 0 \\ 0 \end{bmatrix}, \qquad C = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}, \text{ and } \qquad D = 0.$$

If not stated differently, use the matrices above for the following tasks.

b) (3 points)
Is the system fully controllable for all control gains K_p? Give the controllability matrix, and state your conclusion on the range of K_p for which the system is fully controllable.

c) (3 points)
Alternatively, direct measurement of the muscle force F_{m} besides the end effector position z is available. Which output should be preferred in terms of observability? Give the observability matrix for both cases and recommend your choice of measurement with respect to the two alternatives.

d) (3 points)
In case that the velocity $\dot{z}$ of the slide is measured and the control gain is set to $K_\mathrm{p} = 0$, determine the eigenvalues of the system matrix A. Is the system asymptotic stable? State your reason referring to the eigenvalues of the system matrix A.

e) (5 points)
Due to a redesign of the slide and improved lubrication, the system matrices are changed to

$$A = \begin{bmatrix} -K_\mathrm{p} & 0 & 0 \\ 0 & 0 & 1 \\ 1 & 4 & 3 \end{bmatrix}, \qquad B = \begin{bmatrix} K_\mathrm{p} \\ 0 \\ 0 \end{bmatrix}, \qquad C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, \text{ and } \qquad D = 0.$$

with the eigenvalues $\lambda_1 = -1$, $\lambda_2 = 0$, $\lambda_3 = 4$. Complete the following eigenvectors

$$v_1 = \begin{bmatrix} v_{11} \\ 1 \\ v_{13} \end{bmatrix}, \qquad v_2 = \begin{bmatrix} v_{21} \\ 1 \\ v_{23} \end{bmatrix}, \qquad v_3 = \begin{bmatrix} v_{31} \\ 1 \\ v_{33} \end{bmatrix},$$

and use GILBERT-criterion to determine the eigenvalue(s), that cannot be observed ($K_\mathrm{p} = 0$).

f) (4 points)
The system in task e) is subject to a pole placement design problem. Determine the state feedback matrix $K = \begin{bmatrix} k_1 & k_2 & k_3 \end{bmatrix}$ for the desired poles $s_{1,\mathrm{d}} = -2$, $s_{2,\mathrm{d}} = -1$, and $s_{3,\mathrm{d}} = -3$ with $\dot{z}$ as output and $K_\mathrm{p} = 1$.

g) (6 points)
Give the state space representation of the system described in task a) by stating the A, B, C, and D matrices for the state vector $x = \begin{bmatrix} z & \dot{z} & \ddot{z} \end{bmatrix}^T$.

Use $m_\mathrm{s} = 2$, $f_\mathrm{v} = 1$, $c = 2$, $K_\mathrm{p} = 2$, F_d as the input, and z as the output. What is the advantage of this description?

Measurements show that the viscous friction coefficient f_v and the mass m_s of the slide was not determined accurately. Thus, a slightly modified state space representation is introduced by

$$A = \begin{bmatrix} 0 & 0 & 10 \\ 1 & 0 & -11 \\ 0 & 1 & -2 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \qquad C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}, \text{ and} \qquad D = 0.$$

h) (2 points)
The eigenvalues of the system matrix A are $\lambda_1 = 0.7629$, $\lambda_{2,3} = -1.3815 \pm 3.3466i$. What general advice can you give concerning the values for the poles of any observer with respect to the real and to the imaginary parts?

i) (4 points)
Determine the observer matrix $L = \begin{bmatrix} l_1 & l_2 & l_3 \end{bmatrix}^T$ for the given system with the observer poles $s_{1,\mathrm{o}} = -3$, $s_{1,\mathrm{o}} = -2$, and $s_{1,\mathrm{o}} = -1$.

j) (5 points)
Calculate the transfer function matrix $G(s)$ of the system using a modified output matrix

$$C = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}.$$

□

$\sum$ □