Reading-up-time

For reviewing purposes of the problem statements, there is a "reading-up-time" of **10 minutes** prior to the official examination time. During this period it is **not** allowed to start solving the problems. This means explicitly that during the entire "reading-up-time" no writing utensils, e.g. pen, pencil, etc. at all are allowed to be kept on the table. Furthermore the use of carried documents, e.g. books, (electronic) translater, (electronic) dictionaries, etc. is strictly forbidden. When the supervisor refers to the end of the "reading-up-time" and thus the beginning of the official examination time, you are allowed to take your utensils and documents. Please **then**, begin with filling in the **complete** information on the titlepage.

Good Luck!

LAST NAME	
FIRST NAME	
MATRIKEL-No.	
TABLE-No.	

Klausurunterlagen

Ich versichere hiermit, dass ich sämtliche für die Durchführung der Klausur vorgesehenen Unterlagen erhalten, und dass ich meine Arbeit ohne fremde Hilfe und ohne Verwendung unerlaubter Hilfsmittel und sonstiger unlauterer Mittel angefertigt habe. Ich weiß, dass ein Bekanntwerden solcher Umstände auch nachträglich zum Ausschluss von der Prüfung führt. Ich versichere weiter, dass ich sämtliche mir überlassenen Arbeitsunterlagen sowie meine Lösung vollständig zurück gegeben habe. Die Abgabe meiner Arbeit wurde in der Teilnehmerliste von Aufsichtsführenden schriftlich vermerkt.

Duisburg, den __________ ____________________________________
(Unterschrift der/des Studierenden)

Falls Klausurunterlagen vorzeitig abgegeben: _______________Uhr

Bewertungstabelle

Aufgabe 1	
Aufgabe 2	
Aufgabe 3	
Aufgabe 4	
Gesamtpunktzahl	
Angehobene Punktzahl	
%	
Bewertung gem. PO in Ziffern	

____

(Datum und Unterschrift 1. Prüfer, Univ.-Prof. Dr.-Ing. Söffker)

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(Datum und Unterschrift 2. Prüfer, PD Dr.-Ing. Wend)

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(Datum und Unterschrift des für die Prüfung verantwortlichen Prüfers, Söffker)

Fachnote gemäß Prüfungsordnung:

☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐
1,0	1,3	1,7	2,0	2,3	2,7	3,0	3,3	3,7	4,0	5,0
sehr gut		gut			befriedigend			ausreichend		mangelhaft

Bemerkung: ____________________________________

Attention: Give your answers to ALL problems directly below the questions in the exam question sheet.

You are NOT allowed to use a pencil and also NOT red color (red color is used for corrections).

Maximum achievable points:	**100**
Minimum points for the grade 1,0:	**95%**
Minimum points for the grade 4,0:	**50%**

Problem 1 (30 points)

a) (4 points)
A system is fully controllable and asymptotically stable. Can the system be destabilized by control? State reasons.

b) (8 points)
The dynamical behavior of a machine is described by the matrices

$$A = \begin{bmatrix} 0 & a \\ -3 & 4 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

The plant is controlled by a state feedback controller with the matrix $K = \begin{bmatrix} 0 & k_1 \end{bmatrix}$. The parameters are given by $a = -5$ and $k_1 = 2$.

Due to maintenance, the parameters are changed to $a = 4$ and $k_1 = 3$. Furthermore the feedback is changed to positive feedback.

What can be stated about the stability of the closed-loop system (machine with controller) in the two cases? State reasons.

c) (9 points)
Declare the principal strategy for the practical realization of model-based state control. Define the processor-based observer realization using elements of A/D and D/A conversion, the micro controller, and measuring devices by drawing a sketch and denoting the relating devices.

d) (5 points)
A dynamical system is described by

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} u,$$

$$y = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}.$$

State the transfer function of the system and calculate the invariant zeros.

e) (4 points)

The eigenvalues of a system are given by

$$
\begin{aligned}
\lambda_1 &= 0,\\
\lambda_{2,3} &= \pm\ 3\ j,\\
\lambda_{4,5} &= 5\ \pm\ 2\ j,\\
\lambda_6 &= -7,\\
\lambda_7 &= 0\ +\ 0j,\\
\lambda_8 &= 88\ +\ 88j,\\
\lambda_9 &= -88\ -\ 88j,\\
\lambda_{10,11} &= -1 \pm\ 1j,\\
\lambda_{12,13} &= 10^8\ \pm\ 10^{-7}, \text{and}\\
\lambda_{14,15} &= 10^{-8} \pm\ 10^7 j.
\end{aligned}
$$

What is wrong with the statements denoting the eigenvalues? State reasons.

Σ

Problem 2 (25 points)
A system to be controlled is given in state space description by

$$A = \begin{bmatrix} 2 & 3 & a \\ 0 & -4 & 1 \\ 0 & 0 & b \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 3 \\ 4 & 0 \\ 0 & c \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}, \quad \text{and} \quad D = 0.$$

a) (3 points)
Calculate the eigenvalues of the system. Is the system asymptotically stable? State reasons.

For the next tasks, the parameters a and b are given as

$$a = 0 \quad \text{and} \quad b = -1.$$

b) (5 points)
Give the controllability matrix Q_{S} and define the range of c in which the system is fully controllable.

Use $c = 1$ for all following tasks.

c) (9 points)
The system should be controlled by full state feedback (negative feedback). The feedback gain matrix is supposed as

$$K = \begin{bmatrix} k_1 & 0 & k_2 \\ 0 & k_3 & 0 \end{bmatrix}.$$

Determine the parameters k_1, k_2, and k_3 such that the closed-loop system has the eigenvalues

$$\lambda_{1/2} = -2 \quad \text{and} \quad \lambda_3 = -1.$$

In order to realize the control task (without measurements of all states), an identity observer has to be designed.

d) (3 points)
Give the observability matrix Q_{B}. Why is the system fully observable?

e) (5 points)
The dynamics of the observer should have the eigenvalues

$$\lambda_1 = -1, \quad \lambda_1 = -2, \quad \text{and} \quad \lambda_3 = -3.$$

Define the related parameters l_1, l_2, and l_3 of the observer gain matrix

$$L = \begin{bmatrix} l_1 \\ l_2 \\ l_3 \end{bmatrix}.$$

□

$\sum$ □

Problem 3 (20 points)
A control plant is given with the state equations

$$\begin{aligned} \dot{x}_1(t) + ax_1(t) &= u(t), \\ \dot{x}_2(t) &= bu(t), \\ \dot{x}_3(t) + cx_3(t) &= x_1(t) + x_2(t), \text{ and} \\ y &= x_3(t) \end{aligned}$$

in time domain.

a) (2 points)
Develop the state space model with the state vector $x = [x_1(t) \quad x_2(t) \quad x_3(t)]^T$ and give the matrices A, B, C, and D.

b) (1 point)
For a modified system the matrix A is given by

$$A = \begin{bmatrix} -3 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & -2 \end{bmatrix}.$$

Is the system asymptotically stable?

For the following subtasks c) - g) an unstable system with the state space representation

$$\dot{x}(t) = \begin{bmatrix} -2 & 4 & -1 \\ 0 & 1 & 5 \\ 0 & 0 & -3 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} u(t)$$

is given.
The system should be stablized with the help of a state feedback controller $u = -Kx$, where

$$K = [k_1 \quad k_2 \quad k_3] \,.$$

c) (4 points)
State the eigenvalues and the corresponding eigenvectors of the uncontrolled system.

d) (1 point)
Why is the uncontrolled system unstable? State reasons.

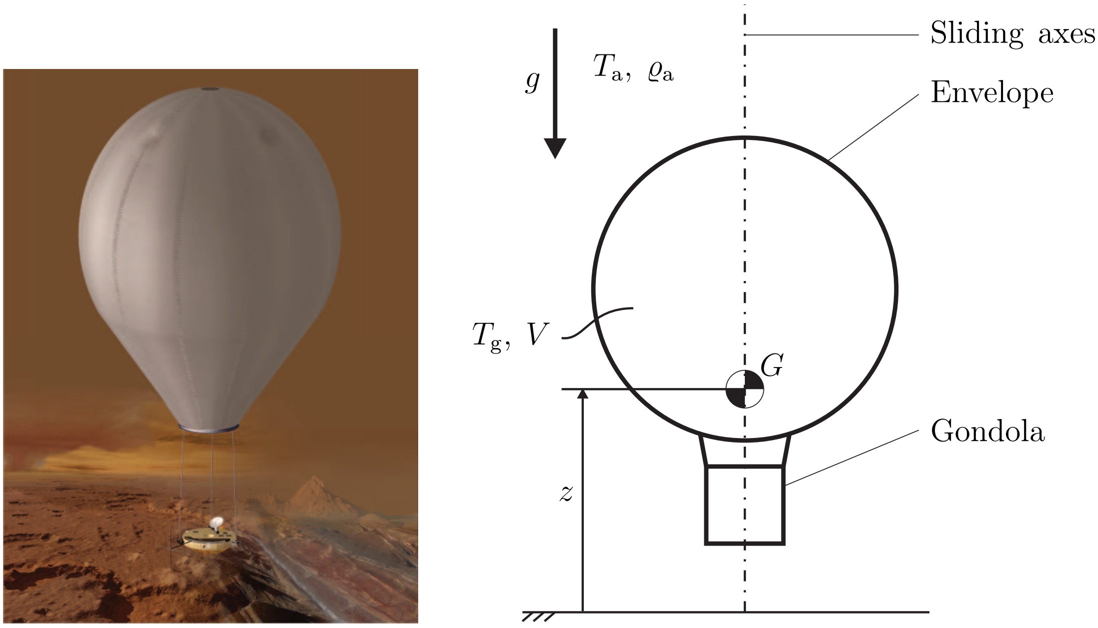
e) (4 points)
Is the given system fully controllable? If not, which eigenvalues are not controllable?

f) (4 points)
Determine the characteristic polynomial of the controlled system. Determine the gain matrix K so that the poles are placed at $s_{1,2} = -4 \pm j$ and $s_3 = -3$.

g) (4 points)
For cost reasons only one state of the system can be measured. In order to perform state feedback control, which state should be measured? State reasons.
(Hint: Check the three possibilities $C_1 = [1 \quad 0 \quad 0]$, $C_2 = [0 \quad 1 \quad 0]$, and $C_3 = [0 \quad 0 \quad 1]$).

$\sum$ □

Problem 4 (25 points)
There has been intensive research using hot-air balloons for a proposed longterm planetary exploration of the moon Titan. Due to Titan's atmosphere, which is up to four times thicker than on Earth, the application of balloons for longterm studies of Titan's crust seems to be suitable. Balloon spacecrafts are highly energy efficient as they are traveling with the wind, however there are other major problems to solve as for example the altitude control with respect to optimal energy efficiency.
In Fig. 4.1(a) a conceptual design of a possible exploration aircraft is shown. The corresponding simplified multi body model is shown in Fig. 4.1(b). The balloon consists of two bodies - the gondola and the envelope - with G representing the center of gravity of both.



(a) Conceptual design (JPL, NASA)

(b) Multi body model

Figure 4.1: Balloon model

The differential equations describing the system above can be obtained by Newton's law and the heat balance equation of the lifting gas as

$$(m_{\mathrm{tot}} + C_{\mathrm{m}}\varrho_{\mathrm{a}}V)\,\ddot{z} = g\,(\varrho_{\mathrm{a}}V - m_{\mathrm{tot}}) - \frac{1}{2}\varrho_{\mathrm{a}}AC_{\mathrm{D}}|\dot{z}|\dot{z},$$

$$m_{\mathrm{g}}c_{\mathrm{pg}}\dot{T}_{\mathrm{g}} = \dot{q}_{\mathrm{g}} - \left(\frac{g m_{\mathrm{g}} T_{\mathrm{g}}}{T_{\mathrm{a}}}\right)\dot{z} + \dot{q}_{\mathrm{i}},$$

with m_{tot} denoting the total mass of the balloon, z its position measured from ground, g the gravitational constant, ϱ_{a} the density of the ambient air, C_{m} the virtual mass coefficient, V the enclosed Volume of the envelope, A the projected area perpendicular to the moving direction, C_{D} the drag coefficient, m_{g} the mass of the gas filling, c_{pg} the specific heat of the gas filling, T_{g} the temperature of the gas filling, T_{a} the ambient temperature, and $\dot{q}_{\mathrm{i}}$ the rate of the heat input.

The linearized system equations yield the following set structurally

$$a_1\ddot{z} = a_2\dot{z} \quad \text{and}$$
$$b_1\dot{T}_{\mathrm{g}} = -b_2\left(\dot{z} + T_{\mathrm{g}}\right) + \dot{q}_{\mathrm{i}}.$$

If not stated differently use the equations above to complete the following tasks.

a) (1 point)
What is the physical difference of using either the state vector $x_1 = \begin{bmatrix} z & \dot{z} & T_{\mathrm{g}} \end{bmatrix}^T$ or the state vector $x_2 = \begin{bmatrix} \dot{z} & T_{\mathrm{g}} \end{bmatrix}^T$ to describe the balloon system?

b) (4 points)
Give the state space representation of the vertical dynamics of the balloon system using the state vector $x_1 = \begin{bmatrix} z & \dot{z} & T_{\mathrm{g}} \end{bmatrix}^T$ by stating the $A, B, C,$ and D matrices with the rate of heat $\dot{q}_{\mathrm{i}}$ as the input. The position z of the balloon and the temperature T_{g} of the filling gas are measured.

Additionally, define the type of the system (SISO, SIMO, MISO, or MIMO).

At a certain ambient condition and several assumptions on the caloric equation, which can be assumed for specific locations on Titan, the state space representation can be approximated by

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & -2 & 0 \\ c & -1 & -4 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ 0 \\ 2 \end{bmatrix}, \qquad C = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix}, \quad \text{and} \qquad D = 0.$$

If not stated differently use the matrices above to complete the following tasks.

c) (1 point)
For which parameters of c is the system completely controllable? State your reason refering to the rank of the controllability matrix Q_{S}.

d) (2 points)
For which parameters of c is the system completely observable? State your reason refering to the rank of the observability matrix Q_{B}.

e) (2 points)
For which values of the parameter c is the system asymptotically stable?

The system is transferred to the reduced state space representation with the state vector x_2 as

$$A = \begin{bmatrix} -2 & d \\ -1 & -4 \end{bmatrix}, \qquad B = \begin{bmatrix} 0 \\ 2 \end{bmatrix}, \qquad C = \begin{bmatrix} 0 & 1 \end{bmatrix}, \quad \text{and} \qquad D = 0.$$

If not stated differently use the matrices above with the parameter $d = 0$ to complete the following tasks.

f) (8 points)
Determine the transfer function matrix $G(s)$ of the state space representation. Determine the eigenvalues, the poles, the invariant zeros, the decoupling zeros, and the transmission zeros of the system.

In case of decoupling zeros, state the type (input/output decoupling zero) additionally.

g) (5 points)
For the design of an optimal controller for an altitude control of the balloon, the matrices

$$Q = \begin{bmatrix} 20 & 10 \\ 10 & 20 \end{bmatrix}, \qquad P = \begin{bmatrix} p_1 & p_2 \\ p_3 & p_4 \end{bmatrix}, \quad \text{and} \qquad R = 4$$

are given. Calculate $K^\star$ of a linear quadratic optimal controller (assume $p_2 = p_3 = 1$, $p_1, p_4 > 0$).

The following transfer function matrix $G(s)$ of the closed loop of the balloon system is obtained by using a slightly modified input and output matrix to

$$G(s) = \begin{bmatrix} \dfrac{s+2}{s^2+4ds+4d^2} \\ \\ \dfrac{s}{s^2+2ds+4s+8d} \end{bmatrix}.$$

h) (2 points)
Determine the values of the parameter d for which the closed loop is input/output stable.

$\sum$ □