Reading-up-time

For reviewing purposes of the problem statements, there is a “reading-up-time” of **10 minutes** prior to the official examination time. During this period it is **not** allowed to start solving the problems. This means explicitly that during the entire “reading-up-time” no writing utensils, e.g. pen, pencil, etc. at all are allowed to be kept on the table. Furthermore the use of carried documents, e.g. books, (electronic) translater, (electronic) dictionaries, etc. is strictly forbidden. When the supervisor refers to the end of the “reading-up-time” and thus the beginning of the official examination time, you are allowed to take your utensils and documents. Please **then**, begin with filling in the **complete** information on the titlepage and on page 3.

Good Luck!

LAST NAME	
FIRST NAME	
MATRIKEL-NO.	
TABLE-NO.	

Klausurunterlagen

Ich versichere hiermit, dass ich sämtliche für die Durchführung der Klausur vorgesehenen Unterlagen erhalten, und dass ich meine Arbeit ohne fremde Hilfe und ohne Verwendung unerlaubter Hilfsmittel und sonstiger unlauterer Mittel angefertigt habe. Ich weiß, dass ein Bekanntwerden solcher Umstände auch nachträglich zum Ausschluss von der Prüfung führt. Ich versichere weiter, dass ich sämtliche mir überlassenen Arbeitsunterlagen sowie meine Lösung vollständig zurück gegeben habe. Die Abgabe meiner Arbeit wurde in der Teilnehmerliste von Aufsichtsführenden schriftlich vermerkt.

Duisburg, den __________ ____________________________________
(Unterschrift der/des Studierenden)

Falls Klausurunterlagen vorzeitig abgegeben: _______________Uhr

Bewertungstabelle

Aufgabe 1	
Aufgabe 2	
Aufgabe 3	
Gesamtpunktzahl	
Angehobene Punktzahl	
%	
Bewertung gem. PO in Ziffern	

____
(Datum und Unterschrift 1. Prüfer, Univ.-Prof. Dr.-Ing. Dirk Söffker)

____
(Datum und Unterschrift 2. Prüfer, Dr.-Ing. Yan Liu)

____
(Datum und Unterschrift des für die Prüfung verantwortlichen Prüfers, Söffker)

Fachnote gemäß Prüfungsordnung:

☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐
1,0	1,3	1,7	2,0	2,3	2,7	3,0	3,3	3,7	4,0	5,0
sehr gut		gut			befriedigend			ausreichend		mangelhaft

Bemerkung: ____

Attention: Give your answers to ALL problems directly below the questions in the exam question sheet.

You are NOT allowed to use a pencil and also NOT red color (red color is used for corrections).

This exam is taken by me as a
☐ mandatory (Pflichtfach)
☐ elective (Wahlfach)
☐ prerequsite (Auflage)
subject (cross ONE option according to your own situation).

Maximum achievable points:	**60**
Minimum points for the grade 1,0:	**95%**
Minimum points for the grade 4,0:	**50%**

Problem 1 (15 Points)

1a) (2 Points)
Draw the function $m(t)$ of the Laplace transform given by
$M(s) = \frac{6e^{-2s}}{3s} - \frac{3e^{-3s}}{2s} + 6e^{-6s} - \frac{4e^{-8s}}{8s}$.

1b) (2 Points)

In Fig. 1.1, the step response function $h(t)$ is given.

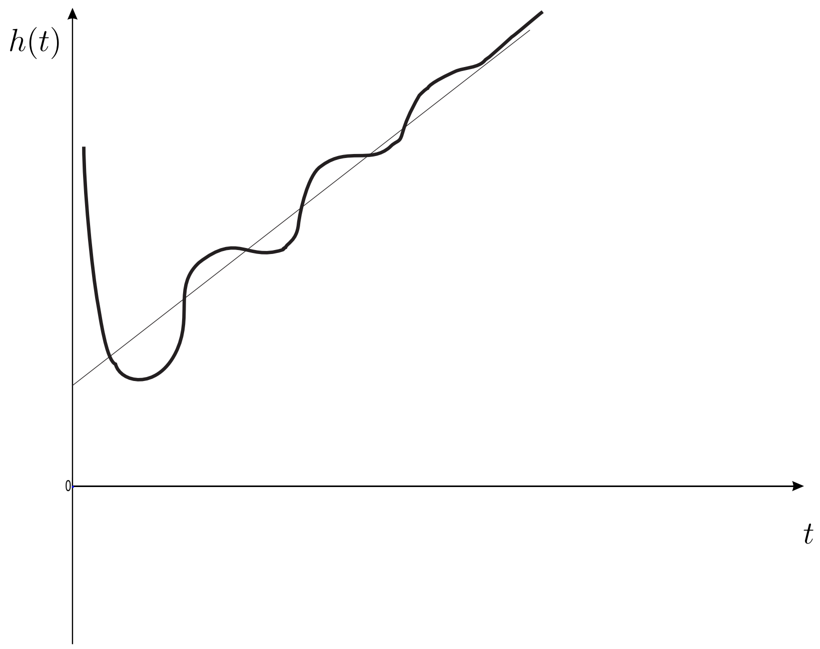


Figure 1.1: Step response function

Which transfer function describes the behavior given in Fig. 1.1?

Check the correct answer(s). Several solutions may be possible.

○ $G(s) = \mathrm{P}$

○ $G(s) = \mathrm{PIDT}_1$

○ $G(s) = \mathrm{PDT}_1$

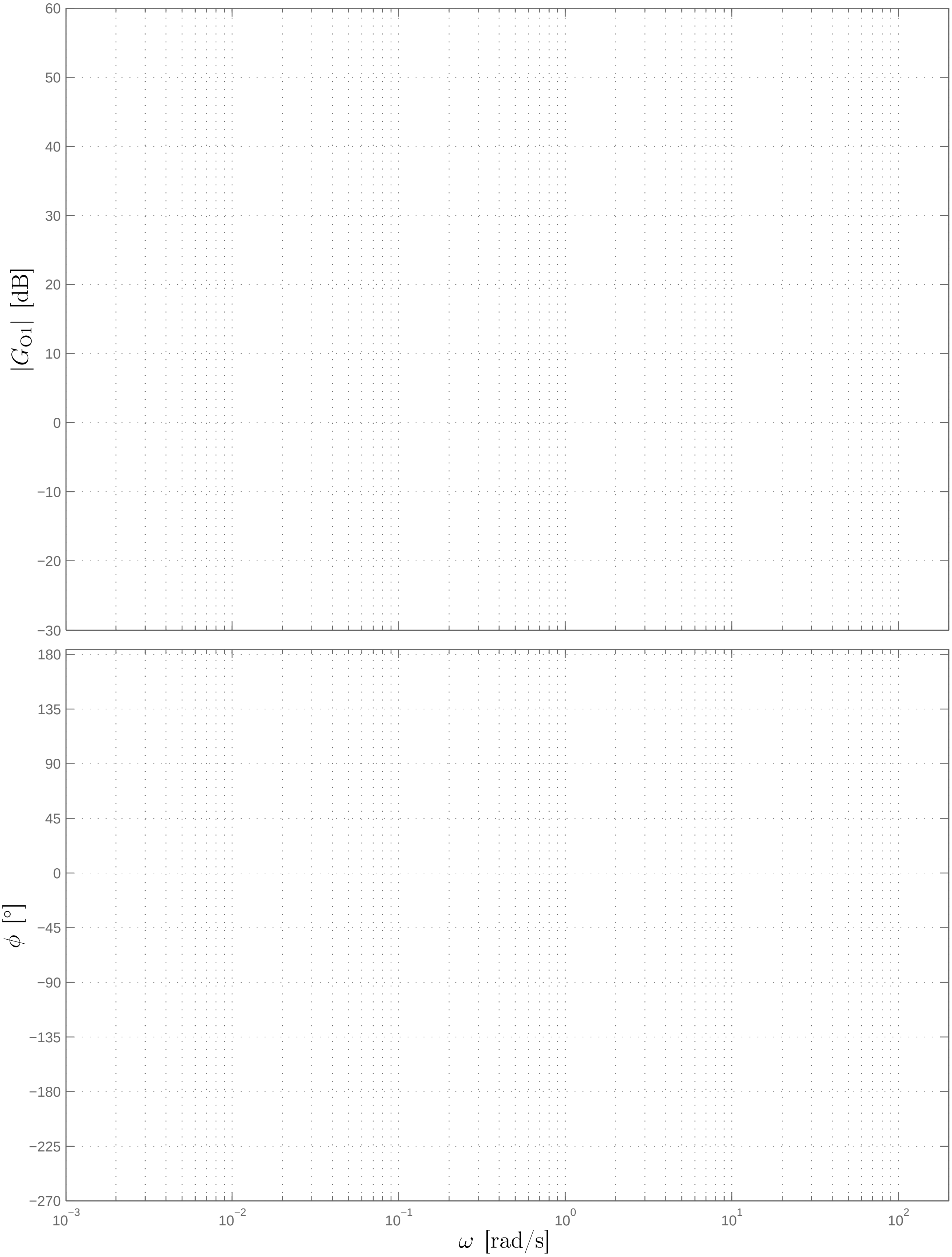
○ $G(s) = \mathrm{PIDT}_2$

☐

1c) (4 Points)

State the input/output behavior of a PIT_2-system as a transfer function $G(s)$. Define the poles and zeros of this system in a general way. Draw the corresponding Bode diagram with the given system parameters $k = 10$, $w_0 = 2$, $D = 0$, and $T_\text{I} = 0.5$ in the diagram in the sequel. Draw first the precise, quantitatively approximated Bode diagram and then additionally the exact behavior (Hint: lg(10)=1).

Bode diagram



1d) (3 Points)

The system A, B, C with $A = \begin{bmatrix} 0 & 1 \\ -1 & -1 \end{bmatrix}$, $B = \begin{bmatrix} 0 & b \end{bmatrix}^T$, $C = \begin{bmatrix} c & 0 \end{bmatrix}$, and $c = 1$, is given. Which transfer function describes the system.

Check the correct answer(s). Several solutions may be possible.

○ $G(s) = \frac{b}{s^2 + s + 1}$

○ $G(s) = \frac{bc}{s^2 + s + 1}$

○ $G(s) = \frac{b}{c(s^2 + s - 1)}$

○ $G(s) = \frac{c}{b(s^2 + s + 1)}$

☐

1e) (4 Points)
The following approximated Bode diagram (Fig. 1.2) describes the behavior of a dynamical system.

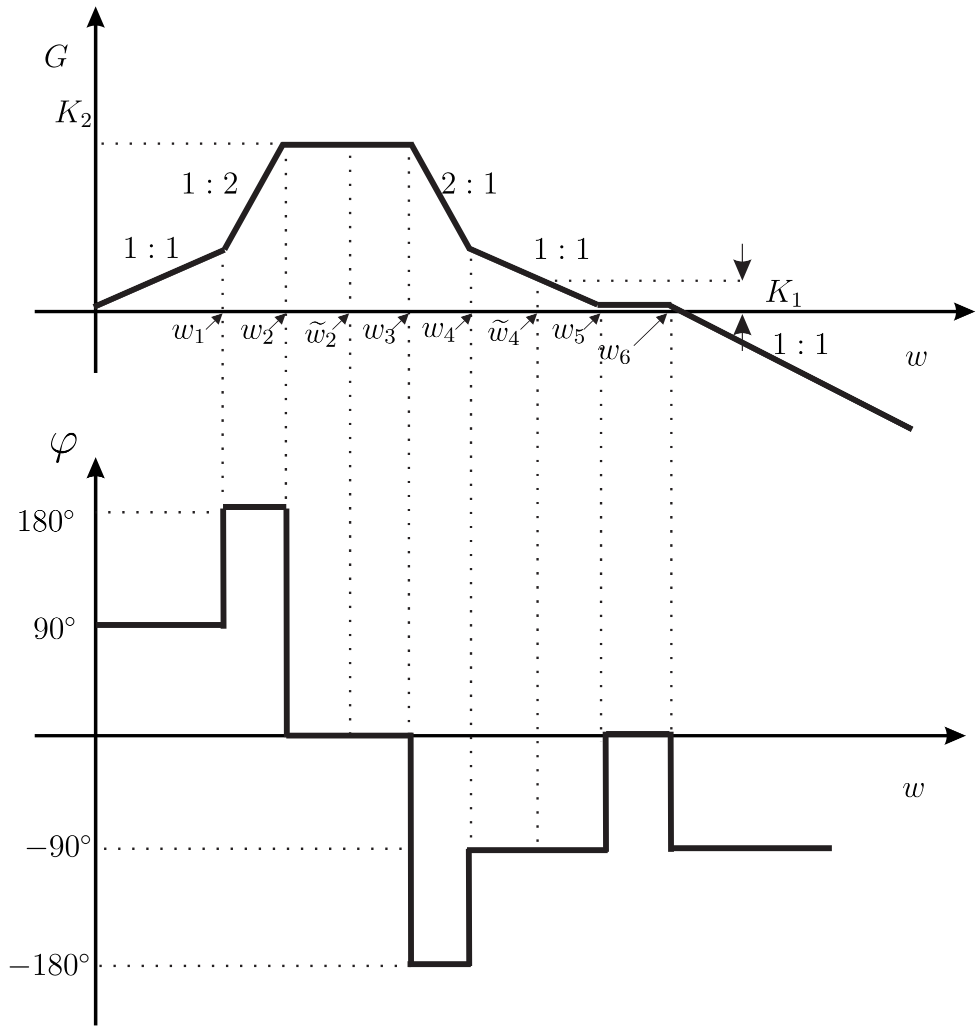


Figure 1.2: Approximated Bode diagram

For the functional capability of the system, it is necessary that the phase delay at the output is as minimal as possible, specifically in the area $\varphi \in [-35°; 35°]$. In which frequency range, the system has to be operated?

Check the correct answer(s). Several solutions may be possible.

○ $\omega_{operate} \in [0, \omega_2]$

○ $\omega_{operate} \in [\omega_3, \omega_5]$

○ $\omega_{operate} \in [\omega_2, \omega_3]$

○ $\omega_{operate} \in [\omega_5, \omega_6]$

$\square$

$\sum \square$

Problem 2 (20 Points)
The control loop shown in Figure 2.1 consists of a controller $G_\mathrm{R}(s)$, a plant $G_\mathrm{S}(s)$, and a transfer element $G_\mathrm{M}(s)$, which describes the dynamic behavior of a measuring device.

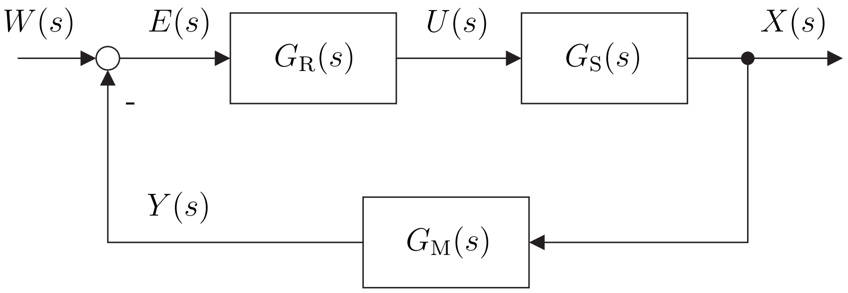


Figure 2.1: Block diagram

The transfer behavior of the plant $G_\mathrm{S}(s)$ is described by the differential equation

$$10\ddot{x}(t) + 7\dot{x}(t) + 3x(t) = u(t) + \int u(t)dt \text{ mit } \dot{x}(0) = x(0) = 0.$$

The transfer behavior of the controller $G_\mathrm{R}(s)$ is described by the differential equation

$$u(t) = 3e(t) + 2\dot{e}(t).$$

The transfer behavior of the transfer element $G_\mathrm{M}(s)$ is described by the differential equation

$$2y(t) + 3\dot{y}(t) = x(t) \text{ mit } y(0) = 0.$$

2a) (3 Points)
Determine the transfer functions of the plant $G_\mathrm{S}(s) = \frac{X(s)}{U(s)}$, the controller $G_\mathrm{R}(s) = \frac{U(s)}{E(s)}$, and the transfer element $G_\mathrm{M}(s) = \frac{Y(s)}{X(s)}$.

2b) (2 Points)

Determine the transfer function of the open loop $G_{\mathrm{O}}(s) = \frac{X(s)}{E(s)}$.

Is the open loop $G_{\mathrm{O}}(s)$ asymptotically stable? State reason. Check the correct answer(s). There are several possible solutions.

- ◯ Asymptotic stable, because $s_{pi} < 0$.
- ◯ Not asymptotic stable, because $s_{pi} > 0$.
- ◯ Not asymptotic stable, because $s_{pi} \leq 0$.
- ◯ Asymptotic stable, because $s_{pi} \leq 0$.

In the sequel, the following open loop transfer function has to be used.

$$G_\mathrm{O}(s) = \frac{(3s-1)(2s+3)}{s(10s^2+7s+1)}$$

2c) (5 Points)
Calculate the phase and amplitude values for $\omega \to 0$ and $\omega \to +\infty$. Draw the approximated Bode diagram as well as the polar plot. Denote the characteristic frequencies and the approximated gradients of the transfer function.

2d) (4 Points)
The stability behavior of the closed loop system resulting from the open loop system $G_{\mathrm{O}}(s)$ with

$$G_{\mathrm{O}}(s) = \frac{2s + 3}{10s^2 + 7s + 1}$$

with negative feedback has to be determined using the root-locus approach. Draw the approximated root-locus plot.

2e (2 Points)
Which qualitative advice (small, ..., very large, spesific value) can you give for the choice of the resulting gain, when the system behavior has to obtain a maximum degree of stability r (with $r = |Re(\lambda_i)|$ for the stable λ_i with minimal real part)? Please, state your reason based on a segmentation of the root-locus plot in 2d), or draw the position of the corresponding poles.

2f) (4 Points)
Two systems are arranged according to Figure 2.2. The dynamic behavior of system 1 is described by

$$T\dot{u}_1 + u_1 = K_2 y,$$

and the dynamic behavior of system 2 is described by

$$u_2 = (K_1 - K_2)y + T_D K_2 \dot{y}.$$

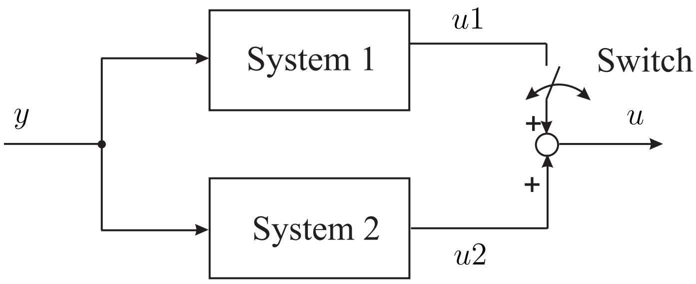


Figure 2.2: Block diagram of the system

Classify the transfer behaviors of the two components.
Give the resulting transfer behavior of the entire system if the switch is closed.
Give the resulting transfer behavior of the entire system if the switch is closed as input/output relation in time domain.($u(t = 0) = y(t = 0) = \dot{y}(t = 0) = 0$).

$\square$

$\sum \square$

Problem 3 (25 Points)
For an electric linear motor a position control has to be designed.
Dependend on the input voltage $u(t)$ the velocity $v(t) = \dot{p}(t)$ can approximately be described by

$$T_2 \iint v(t)\ dt\ dt + T_1 \int v(t)\ dt + v(t) = k_1 \int u(t)\ dt + k_2 \iint u(t)\ dt\ dt.$$

The position control to be realized by a controller is described by

$$u(t) = k_{\mathrm{R}}\ p(t).$$

This controller is connected to the linear motor by a negative feedback.
Hint: Use first a suitable form to derive the goal.

3a) (6 points)
Classify the transfer behaviors $G_{\mathrm{S}}(s) = \frac{v(s)}{u(s)}$ and $G_{\mathrm{R}}(s) = \frac{u(s)}{p(s)}$ of the plant and the controller respectively. State the transfer function $G_{\mathrm{O}}(s)$ of the open loop system with the related classification.

3b) (3 points)
State the reference transfer behavior $G_{\mathrm{W}}(s)$ and the disturbance transfer behavior $G_{\mathrm{D}}(s)$ of the entire system (linear motor and position control with negative feedback connection) and classify the behaviors if possible.

3c) (9 points)
Three systems are controlled with a P-controller in negative feedback configuration. For each open-loop system the transfer functions are determined as given below.

System 1:
Zeros: $s_{01} = -4 + i$; $s_{02} = -4 - i$; $s_{03} = -5$
Poles: $s_1 = -2 + i$; $s_2 = -2 - i$; $s_3 = -2$ $s_4 = -1$

System 2:
Zeros: $s_{01} = -3 + i$; $s_{02} = -3 - i$
Poles: $s_1 = -1$; $s_2 = -0,5$; $s_3 = -1 + i$; $s_4 = -1 - i$

System 3:
Zeros: $s_{01} = -1$; $s_{02} = -2$
Poles: $s_1 = 1 + i$; $s_2 = 1 - i$ $s_3 = 1$

Using qualitative root locus plots, state reasons, which of these systems is stable. If necessary, apply suitable distinction of cases.

3d) (7 points)
A system $G_{\mathrm{S}}(s)$ with

$$G_{\mathrm{S}}(s) = \frac{4 + \frac{1}{s}}{s^2 + 3s + 2}$$

has to be controlled by a feedback controller with $G_{\mathrm{R}} = \frac{4}{2s+1}\ e^{sT_t}$. For the resulting open-loop system the transfer function and the related zeros and poles have to be determined. Additionally the qualitative Bode diagram (plot the approximated behavior first) and the polar plot have to be determined. Draw the phase margin and the amplitude margin of the closed loop system.

Determine approximatively the maximal delay time T_t to achieve the stability bound.
Hint: For the phase margin of a dead time element $\varphi(\omega)$ the relation

$$\varphi(\omega) = -\omega T_t$$

holds.

$\sum$ ☐