Reading-up-time

For reviewing purposes of the problem statements, there is a "reading-up-time" of **10 minutes** prior to the official examination time. During this period it is **not** allowed to start solving the problems. This means explicitly that during the entire "reading-up-time" no writing utensils, e.g. pen, pencil, etc. at all are allowed to be kept on the table. Furthermore the use of carried documents, e.g. books, (electronic) translater, (electronic) dictionaries, etc. is strictly forbidden. When the supervisor refers to the end of the "reading-up-time" and thus the beginning of the official examination time, you are allowed to take your writing utensils. Please **then**, begin with filling in the **complete** information on the titlepage and on page 3.

Good Luck!

NAME	
VORNAME	
MATRIKEL-NR.	
TISCH-NR.	

Klausurunterlagen

Ich versichere hiermit, dass ich sämtliche für die Durchführung der Klausur vorgesehenen Unterlagen erhalten, und dass ich meine Arbeit ohne fremde Hilfe und ohne Verwendung unerlaubter Hilfsmittel und sonstiger unlauterer Mittel angefertigt habe. Ich weiß, dass ein Bekanntwerden solcher Umstände auch nachträglich zum Ausschluss von der Prüfung führt. Ich versichere weiter, dass ich sämtliche mir überlassenen Arbeitsunterlagen sowie meine Lösung vollständig zurück gegeben habe. Die Abgabe meiner Arbeit wurde in der Teilnehmerliste von Aufsichtsführenden schriftlich vermerkt.

THE ABOVE REQUIRED STATEMENTS AS WELL AS THE SIGNATURE ARE MANDATORY AT THE BEGINNING OF THE EXAM.

Duisburg, ______________ ____
(Date) (Student's signature)

Falls Klausurunterlagen vorzeitig abgegeben: ______________Uhr

Bewertungstabelle

Aufgabe 1	
Aufgabe 2	
Aufgabe 3	
Die Bewertung gem. PO in Ziffern ist der xls-Tabelle bzw. dem Papierausdruck zu entnehmen.	

____

(Datum und Unterschrift 1. Prüfer, Univ.-Prof. Dr.-Ing. Dirk Söffker)

____

(Datum und Unterschrift 2. Prüfer, Dr.-Ing. Fateme Bakhshande)

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(Datum und Unterschrift des für die Prüfung verantwortlichen Prüfers, Söffker)

Fachnote gemäß Prüfungsordnung: (alternativ: siehe xls-Tabelle bzw. beigefügter Papierausdruck)

☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐
1,0	1,3	1,7	2,0	2,3	2,7	3,0	3,3	3,7	4,0	5,0
sehr gut		gut			befriedigend			ausreichend		mangelhaft

Bemerkung: __________________________________

Attention: Give your answers to ALL problems directly below the questions in the exam question sheet.

You are NOT allowed to use a pencil and also NOT red color (red color is used for corrections).

This exam is taken by me as a
☐ mandatory (Pflichtfach)
☐ elective (Wahlfach)
☐ prerequsite (Auflage)
subject (cross ONE option according to your own situation).

Maximum achievable points:	**56**
Minimum points for the grade 1,0:	**95%**
Minimum points for the grade 4,0:	**50%**

General hints:

1) For the multiple-choice and multiple-choice-similar tasks the following rules are effective:
   i) For tasks with an individual evaluation of subtasks, the following applies: Only correct answers are evaluated with the intended number of points.
   ii) The points achieved in a subtask will be summed up.
   iii) Unless explicitly stated otherwise, only one of the given solution options is correct.
   iv) If subtasks contain more than two answer options and only one solution exists: The marking of multiple answer options is interpreted as a non-response due to the not clear declaration of intention. As a result, no points can be given in this case.

2) If in the exam tasks no information is given for the valid range of numbers for time constants or masses etc.: take for time constants (in sec.), for masses (in kg) positive numbers.

3) If in the exam tasks no information is given for applying negative or positive feedback: use the usual negative feedback.

Problem 1 (27 Points)

1a) (5 × 1 Point, 5 Points)
Mark the correct solution in the following statements.

A1) (1 Point)
The representations

$$f(t) = \frac{A_0}{2} + \sum_{k=0}^{\infty} A_k \cos(k\omega_0 t) + \sum_{k=1}^{\infty} B_k \sin(k\omega_0 t), \qquad (*)$$

$$A_k = \frac{2}{T_0}\int_0^{T_0} f(t)\cos(k\omega_0 t)dt, \qquad k = 0, 1, \ldots \qquad (**)$$

$$B_k = \frac{2}{T_0}\int_0^{T_0} f(t)\sin(k\omega_0 t)dt, \qquad k = 1, 2, \ldots \qquad (**)$$

and

$$f(t) = C_0 + \sum_{k=1}^{\infty} C_k \sin(k\omega_0 t + \phi_k), \qquad (*)$$

$$C_0 = \frac{A_0}{2}, \qquad (**)$$

$$C_k = \sqrt{A_k^2 + B_k^2}, \qquad k = 1, 2, \ldots \qquad (**)$$

$$\phi_k = \arctan\frac{A_k}{B_k}, \qquad k = 1, 2, \ldots \qquad (**)$$

$k = 1 : \omega_0 \quad \rightarrow \quad$ Fundamental wave,

$k > 1 : \omega_i \quad \rightarrow \quad$ Harmonic frequencies of higher order,

describe the approximation of the periodic signal $f(t)$ using a set of specific functions $(*)$ and parameters $(**)$. Here the signal/function $f(t)$ is practically approximated by special harmonic functions $(*)$ with special parameters $(**)$. The two representations (Variant 1: A_0, A_k, B_k, Variant 2: C_0, C_k, ϕ_k) represent

- ○ equal approximations, whereby due to trigonometric connections both variants are equal and Variant 2 allows an easier interpretation (magnitude and phase).
- ○ different approximations with the same accuracy.
- ○ the same approximation, whereby Variant 1 requires more coefficients (A_0,A_k,B_k) and is therefore more accurate.
- ○ different approximations with different accuracy.

A2) (1 Point)

The derivation of Fourier transformed weighting functions ($G(j\omega)$)

$$
\begin{aligned}
&a_n \frac{d^n y}{dt^n} + \cdots + a_1 \frac{dy}{dt} + a_0 y = b_q \frac{d^q u}{dt^q} + \cdots + b_1 \frac{du}{dt} + b_0 u,\\
&\frac{d^i}{dt^i} U e^{j\omega t} = U (j\omega)^i e^{j\omega t}, \qquad \frac{d^i}{dt^i} Y e^{j\omega t} = Y (j\omega)^i e^{j\omega t}, \qquad \text{with } \frac{d^i}{dt^i} e^{j\omega t} = (j\omega)^i e^{j\omega t},\\
&Y(a_n (j\omega)^n + \cdots + a_0) e^{j\omega t} = U(b_q (j\omega)^q + \cdots + b_0) e^{j\omega t},\\
&Y = \underbrace{\frac{b_q (j\omega)^q + \cdots + b_0}{a_n (j\omega)^n + \cdots + a_0}}_{G(j\omega)} U = Y = G(j\omega) U,
\end{aligned}
$$

from a linear Input/Output representation (I/O representation) allows the representation of the I/O relationships in frequency domain. The following applies here:

- ◯ The relations stated in $G(j\omega)$ are only correct in specific points in time based on the model structure assumed to be linear and time-invariant.
- ◯ The relations stated in $G(j\omega)$ are only correct in specific points in time based on the model structure assumed to be linear and time-variant.
- ◯ With $G(j\omega)$, a frequency-independent variable with magnitude and phase is described.
- ◯ With $G(j\omega)$, a frequency-dependent variable with magnitude and phase is described.
- ◯ The relations stated in $G(j\omega)$ describe only transient relationships in general.

A3) (1 Point)

The transfer function can be described in different ways, here

$$
G(s) = k_s \frac{\prod_{i=1}^{q} (T_{0i} s + 1)}{\prod_{i=1}^{n} (T_i s + 1)},
$$

describes

- ◯ the pole-zero (PZ) form.
- ◯ the time-constant (TN) form.
- ◯ a universal description form for the direct (readable) representation of poles, zeros, and time constants.

A4) (1 Point)
The signal $y(s) = G(s)u(s)$ with $G(s) = T_D s + 1$ and $u(s) = 1$ describes with

- ◯ $y(s) = T_D + 1$ the impulse response of a PD-system.
- ◯ $y(s) = T_D s + 1$ the impulse response of a PDT_1-system.
- ◯ $y(s) = T_D + 1$ the step response of a PD-system.
- ◯ $y(s) = T_D + 1$ the step response of a PDT_1-system.

A5) (1 Point)
Consider the transformation of the description of a linear time-invariant SISO-system into a description in frequency domain. This transformation can

- ◯ be achieved dependent on the order of derivatives.
- ◯ be achieved independent of the order of derivatives.
- ◯ not be achieved in general.
- ◯ only be achieved numerically (by machine).

1b) (4 × 1 Point, 4 Points)
Mark the correct solution in the following statements.

B1) (1 Point)
How to interpret the following description of control engineering relationships?

$$K_s = \lim_{t\to\infty} h(t) = \lim_{s\to 0} s \cdot \frac{1}{s} \cdot G(s) = \lim_{s\to 0} G(s), \qquad K_s \neq 0,$$

- ◯ A statement about $G(s)$ can be derived from the static final value when considering a step response via the Laplace boundary theorems.
- ◯ The static gain corresponds to the value of the step response for $t \to \infty$, but can also be derived as a final value from the transfer function for $s \to 0$.
- ◯ The static gain corresponds to the value of the step input for $t \to \infty$, but can also be derived as a limit value from the transfer function by $s \to 0$ multiplied by the Laplace transformed impulse representation.
- ◯ From the limit value of the system to be controlled considered for infinitely long time, a statement about the behavior of $G(s)$ at $s \to 0$ can be derived with a step response using the limit value propositions.

B2) (1 Point)
A polar plot curve along the negative imaginary axis ending at the origin corresponds

- ◯ within the amplitude diagram of the Bode diagram, a continuous increase with increasing frequency.
- ◯ within the amplitude diagram of the Bode diagram, a continuous decrease with increasing frequency.
- ◯ within the phase diagram of the Bode diagram, a continuous increase with increasing frequency.
- ◯ within the phase diagram of the Bode diagram, a continuous decrease with increasing frequency.

B3) (1 Point)

The following pole/zero distributions are given:

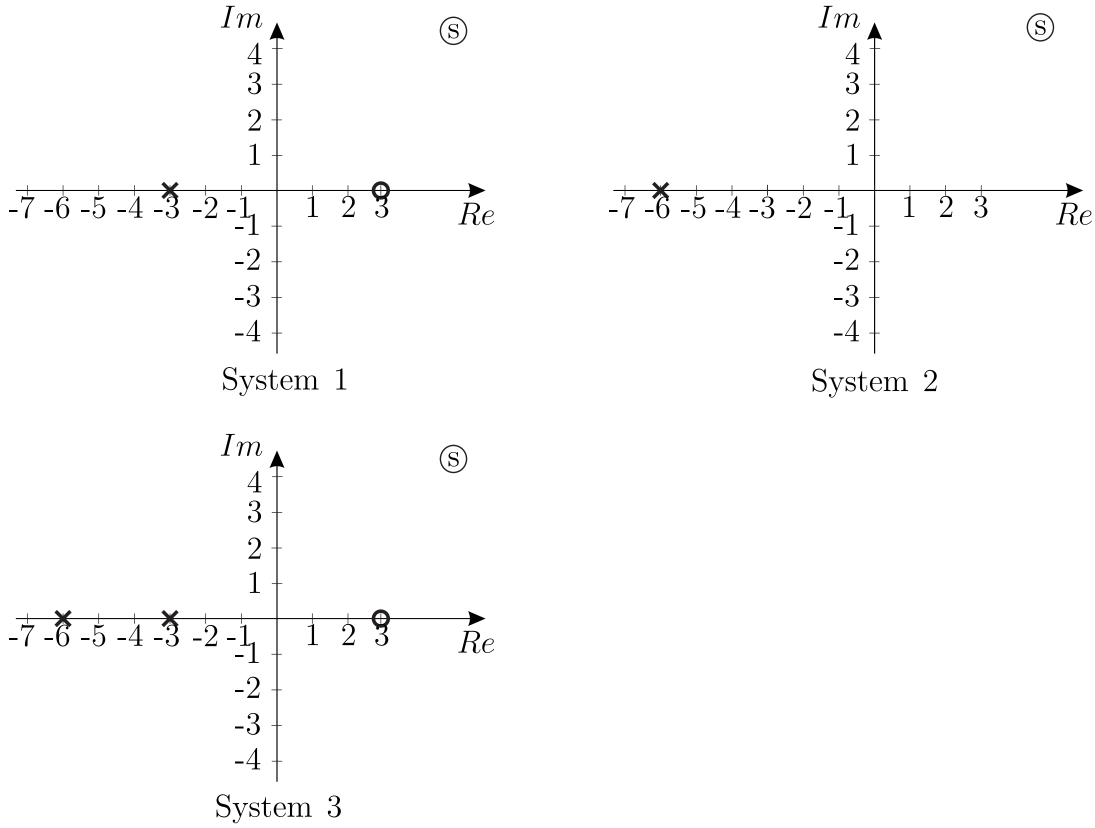

Figure 1.1: Pole/zero distributions

◯ Connecting system 2 and 1 in parallel gives system 3.

◯ Connecting system 2 and 1 in series gives system 3. System 3 is minimum-phase.

◯ Connecting system 2 and 1 in series gives system 3. System 3 is not an all-pass.

◯ Connecting system 2 and 1 in series gives system 3. System system 2 is an all-pass.

◯ Connecting system 2 and 1 in series gives system 3. System 1 is not an all-pass.

B4) (1 Point)

Phase affecting transmission elements are often used to modify the properties of given systems (plants, controllers, controlled systems) accordingly. The system

$$\frac{T_D s + 1}{T s + 1}, \qquad \text{with } T_D < T,$$

will be discussed.

This is a

- ○ phase-reducing transmission element because $\frac{1}{T} < \frac{1}{T_D}$.
- ○ phase-raising transmission element because $\frac{1}{T} < \frac{1}{T_D}$.
- ○ phase-reducing transmission element because the description is in PZ-form.
- ○ phase-reducing transmission element because the description is in TN-form.
- ○ phase-reducing transmission element because $T_D > T$.
- ○ phase-raising transmission element because $T_D > T$.

1c) (2 Points)
Mark the correct solution in the following statements.

C1) (1 Point)
The input/output behavior of a system is defined by the poles $s_{1,2} = 2 \pm j$, $s_3 = -3$, and $s_4 = 0$. Considering the poles of the system the input/output behavior has to be classified as

- ◯ asymptotically stable.
- ◯ boundary stable.
- ◯ unstable.

C2) (1 Point)
An open loop with proportional behavior should be controlled stationary accurate. To achieve this goal

- ◯ a proportional
- ◯ an integral
- ◯ a differential
- ◯ a decelerating
- ◯ an accelerating

part should be integrated into the feedback.

□

1d) (16 Points)
The transfer function of a plant to be controlled is given by

$$G_{\mathrm{S}}(s) = \frac{8(s+2)}{(s^2+4s+53)s},$$

The plant is controlled by a controller with the transfer function

$$G_{\mathrm{R}}(s) = \frac{Ks}{(s-T_1)(s+2)},$$

with $T_1 > 0$ using negative feedback.

1d) i) (3 Points)
Calculate the poles and zeros of plant and controller.

1d) ii) (2 Points)

State, under which conditions i) the plant and ii) the controller show asymptotically stable behavior. State reasons for each answer.

1d) iii) (2 Points)
Determine the denominator polynomial of the closed loop disturbance transfer behavior.

In the following, the transfer function of the disturbance behavior is

$$G_{\mathrm{Z}}(s) = \frac{(s^2 + 4s + 24)(s - T_1)}{s^3 + (6 - T_1)s^2 + (24 - 6T_1)s + 8K - 24T_1}.$$

1d) iv) (3 Points)
State the conditions under which the closed loop shows asymptotically stable behavior.

System modification and measurement result in the following representation for the open loop.

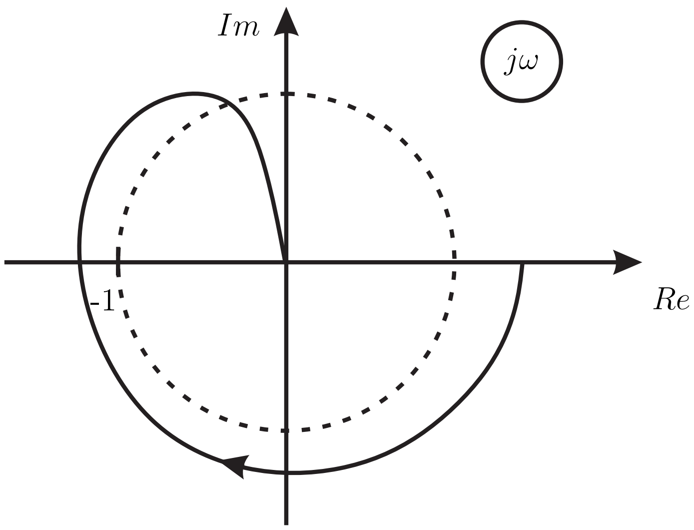

Figure 1.2: Polar plot of the open loop

1d) v) (2 Points)
Is the open loop asymptotically stable? State reasons. Justify which behavior (stable, unstable) you expect for the closed control loop.

1d) vi) (2 Points)

In Figure 1.2, clearly and deferentially draw and name
the amplitude crossing frequency,
the phase crossing frequency,
the phase reserve, and
the amplitude reserve, as well as the line segments relevant to the amplitude reserve.

The transfer function of the open loop is

$$\tilde{G}_0(s) = \frac{\tilde{K}s(s-0,5)(s+3)}{(s+2j)(s-2j)(s+3)(s+4)}.$$

1d) vi) (3 Points)
Can the closed loop show behavior with $D \geq 1$? State reasons based on a root locus sketch. Is the closed loop asymptotically stable?

$\sum$

Problem 2 (17 Points)

2a) (10 Points)
A system with PIT_3-behavior (T_1, T_2, T_3, K, T_I) is controlled (negative feedback) by a controller with PT_1-behavior $(T_1^\star, K^\star)$. For the parameters the relations $T_1 > T_2 > T_3 > T_1^\star > T_I > 0$ are valid.

2a) ii) (4 Points)
Show graphically the principle pole/zero distribution of the open loop.

2a) ii) (6 Points) Mark the correct solution in the following statements.

A1) (1 Point)
The plant is

- ◯ asymptotically stable.
- ◯ unstable.
- ◯ boundary stable.

A2) (1 Point)
The open loop is

- ◯ asymptotically stable.
- ◯ unstable.
- ◯ boundary stable.

A3) (1 Point)
The closed loop is unstable for

- ◯ large control gains.
- ◯ small control gains.
- ◯ $K^\star = 0$.

A4) (1 Point)
Based on a suitable controller parameterization it is

- ◯ possible
- ◯ not possible
- ◯ independent of the plant always and in general possible
- ◯ independent of the plant always and in general not possible

to achieve asymptotically stable behavior of the closed loop.

A5) (1 Point)
Based on a suitable controller parameterization it is

- ◯ possible
- ◯ not possible
- ◯ independent of the plant always and in general possible
- ◯ independent of the plant always and in general not possible

to achieve boundary stable behavior of the closed loop.

A6) (1 Point)
The closed loop has

- ◯ three
- ◯ four
- ◯ five

poles.

2b) (7 Points)
A minimal-phase system is given with the amplitude behavior shown in Figure 2.1. The system is controlled by a P-controller ($K_{\mathrm{P}} = 1$).

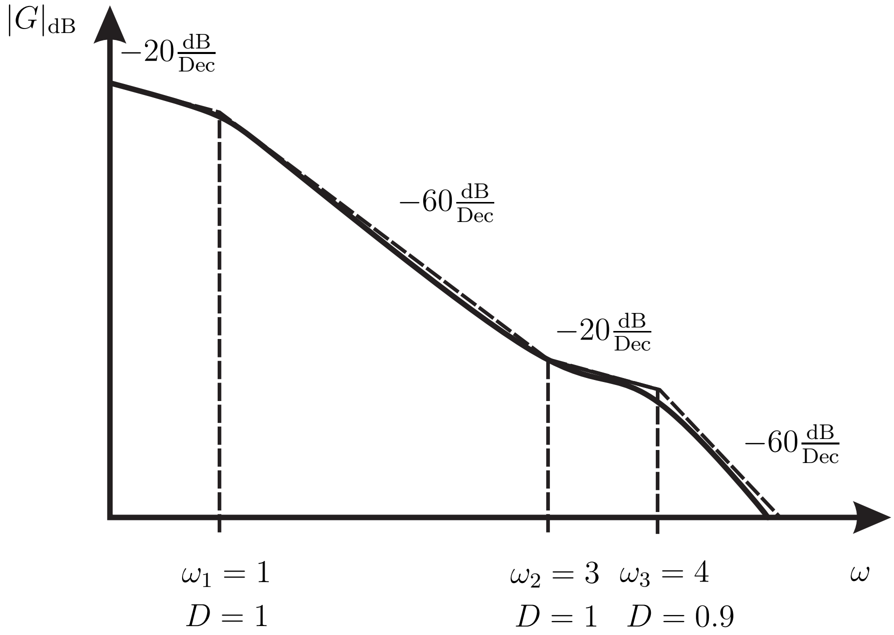

Figure 2.1: Amplitude behavior

i) (2 Points)
Determine the poles and zeros of the open loop.

ii) (5 Points)
Draw the phase shift behavior qualitatively (real and approximated behavior) and draw the polar plot of the open loop.

$\sum$

Problem 3 (12 Points)

3a) (8 Points)
Determine the transfer function $G(s) = \frac{Y(s)}{U(s)}$ for the transfer systems given in the figures (block diagrams).

3a) i) (2 Points)

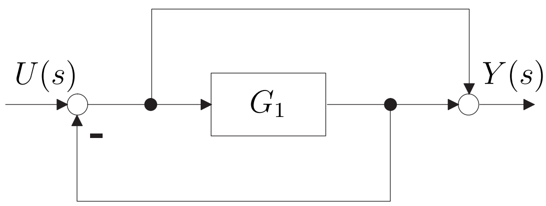

Figure 3.1: Block diagram of the control loop

3a) ii) (2 Points)

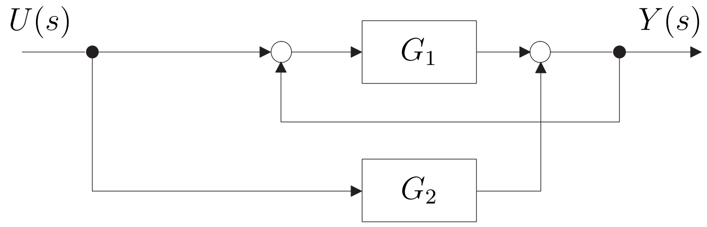

Figure 3.2: Block diagram of the control loop

3a) iii) (4 Points)

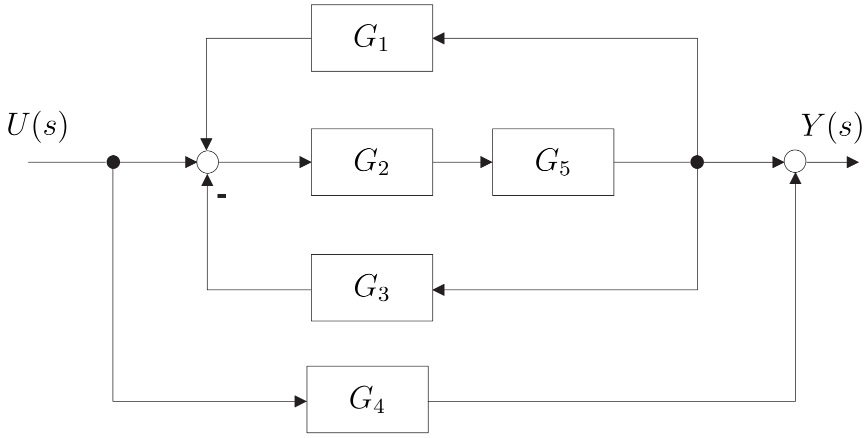

Figure 3.3: Block diagram of the control loop

3b) (9 Points)
Figure 3.4 shows the pole-zero distribution of a plant $G_\mathrm{S}(s)$ controlled by an ideal P-controller $G_\mathrm{R}(s)$ using negative feedback. The plant has the gain $K_\mathrm{S} = 2$.

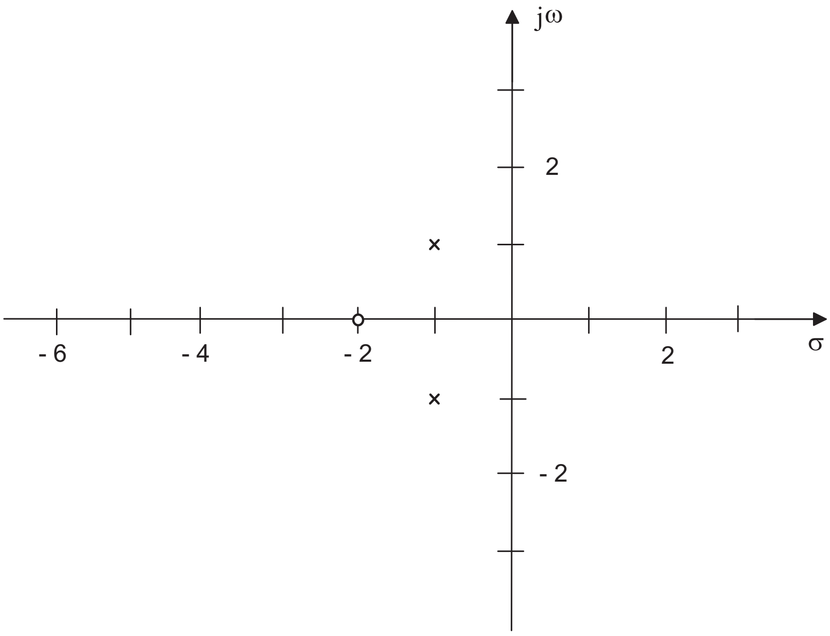

Figure 3.4: Pole-zero distribution of the plant

3b) i) (2 Points)
What are the transfer functions of the plant $G_S(s)$ and the controller $G_R(s)$?

3b) ii) (2 Points)
What is the denominator polynomial of the closed loop?

3b) iii) (3 Points)
Sketch the closed loop root locus curve ($K_R > 0$). Calculate the bifurcation point(s) of the root locus. (Note: The bifurcation points are determined using the formula given in the appendix).

3b) iv) (2 Points)

Determine the value of the gain K_{R} of the P-controller for which the eigenmotion of the closed loop has the damping $D = 0.9$. (Note: For $D = 0.9$, $s_1 = -2 + j$ holds).

$\sum$