Reading-up-time

For reviewing purposes of the problem statements, there is a "reading-up-time" of **10 minutes** prior to the official examination time. During this period it is **not** allowed to start solving the problems. This means explicitly that during the entire "reading-up-time" no writing utensils, e.g. pen, pencil, etc. at all are allowed to be kept on the table. Furthermore the use of carried documents, e.g. books, (electronic) translater, (electronic) dictionaries, etc. is strictly forbidden. When the supervisor refers to the end of the "reading-up-time" and thus the beginning of the official examination time, you are allowed to take your utensils and documents. Please **then**, begin with filling in the **complete** information on the titlepage and on page 3.

Good Luck!

LAST NAME	
FIRST NAME	
MATRIKEL-NO.	
TABLE-NO.	

Klausurunterlagen

Ich versichere hiermit, dass ich sämtliche für die Durchführung der Klausur vorgesehenen Unterlagen erhalten, und dass ich meine Arbeit ohne fremde Hilfe und ohne Verwendung unerlaubter Hilfsmittel und sonstiger unlauterer Mittel angefertigt habe. Ich weiß, dass ein Bekanntwerden solcher Umstände auch nachträglich zum Ausschluss von der Prüfung führt. Ich versichere weiter, dass ich sämtliche mir überlassenen Arbeitsunterlagen sowie meine Lösung vollständig zurück gegeben habe. Die Abgabe meiner Arbeit wurde in der Teilnehmerliste von Aufsichtsführenden schriftlich vermerkt.

Duisburg, den __________ ____________________________________

(Unterschrift der/des Studierenden)

Falls Klausurunterlagen vorzeitig abgegeben: _______________Uhr

Bewertungstabelle

Aufgabe 1	
Aufgabe 2	
Aufgabe 3	
Gesamtpunktzahl	
Angepasste Punktzahl	
%	
Bewertung gem. PO in Ziffern	

____

(Datum und Unterschrift 1. Prüfer, Univ.-Prof. Dr.-Ing. Dirk Söffker)

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(Datum und Unterschrift 2. Prüfer, Dr.-Ing. Yan Liu)

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(Datum und Unterschrift des für die Prüfung verantwortlichen Prüfers, Söffker)

Fachnote gemäß Prüfungsordnung:

☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐
1,0	1,3	1,7	2,0	2,3	2,7	3,0	3,3	3,7	4,0	5,0
sehr gut		gut			befriedigend			ausreichend		mangelhaft

Bemerkung: __________________________________

Attention: Give your answers to ALL problems directly below the questions in the exam question sheet.

You are NOT allowed to use a pencil and also NOT red color (red color is used for corrections).

This exam is taken by me as a
☐ mandatory (Pflichtfach)
☐ elective (Wahlfach)
☐ prerequsite (Auflage)
subject (cross ONE option according to your own situation).

Maximum achievable points:	**Points**
Minimum points for the grade 1,0:	**95%**
Minimum points for the grade 4,0:	**50%**

General hints:

1) For the multiple-choice and multiple-choice-similar tasks the following rules are effective:

   i) For correct answers of exam task parts the desired number of points will be given.
   ii) For noncorrect answers of exam task parts the desired number of points will be counted negative.
   iii) No answering will neither lead to positive nor to negative points.
   iv) The points of the task will be summarized. The whole number can not be smaller than zero.

2) If in the exam tasks no information is given for the valid range of numbers for time constants or masses etc. : take for time constants (in sec.), for masses (in kg) positive numbers.

3) If in the exam tasks no information is given for applying negative or positive feedback: use the usual negative feedback.

Problem 1 (30 Points)

1a) (15 Points)
Define the differences between a SISO and a MIMO system with respect to the system description. Which of the following statements is 'True' or 'False'?

NO.	Task/Question/Judgement	SISO True	SISO False	MIMO True	MIMO False
1	The system can be described in time domain.	⊗	○	⊗	○
2	The system can be described in frequency domain.	⊗	○	⊗	○
3	The number of poles is n with n as the number of highest derivative of the output variable related to the standard description.	⊗	○	○	⊗
4	The number of eigenvalues is n with n as the dimension of the state vector with dim(x)$= n \times 1$.	⊗	○	⊗	○
5	The feedback rules $u = -ky$ (SISO) and $u = -Kx$ (MIMO) are usually denoted as proportional control (P-Controller).	⊗	○	○	⊗
*6	Assuming no pole-zero cancelation: the number of poles and eigenvalues is always equal.	○	○	○	○
7	Eigenvalues are real or conjugate complex.	⊗	○	⊗	○
8	The number of states (dim(x)$= n \times 1$) is equal to the number of outputs.	○	⊗	○	⊗
9	Applying Hurwitz for exact control design (=detailed parameters design) is suitable.	○	⊗	○	⊗
10	The number of inputs and the number of outputs is identical.	⊗	○	○	⊗
11	Observers are adequate techniques to reconstruct inner variables.	⊗	○	⊗	○
12	The system is always full observable.	○	⊗	○	⊗
13	If $r = m$ with Rank $B = m$ and Rank $C = r$, the system is always full controllable.	○	⊗	○	⊗
14	Root locus design is a suitable control design approach.	⊗	○	○	⊗
15	A system with $B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$ and $C = \begin{bmatrix} 1 & -d \\ -3 & 3d \end{bmatrix}$ is a SISO/MIMO system (in the sense of the term).	○	⊗	○	⊗

* Difficult question: Details upon request.

□

For the following tasks, a system description is given with

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ a_1 & a_2 & a_3 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ b \end{bmatrix}, \quad \text{and} \quad C = \begin{bmatrix} 0 & 0 & c \end{bmatrix} \quad \text{with} \quad a_{1,2,3} \neq 0,\ b \neq 0,\ \text{and}\ c \neq 0.$$

It is noted, that no direct transmission between the inputs and the outputs/measurements is given.

1b) (2 Points)
State the Rosenbrock matrix of the system depending on the parameters $a_{1,2,3}$, b, and c.

Solution:

$$P(s) = \begin{bmatrix} s & -1 & 0 & 0 \\ 0 & s & -1 & 0 \\ -a_1 & -a_2 & s - a_3 & -b \\ 0 & 0 & c & 0 \end{bmatrix}$$

1c) (6 Points)
Calculate the characteristic polynomial of the system matrix A depending on the parameters $a_{1,2,3}$. For which values of $a_{1,2,3}$ is the system asymptotic stable?

Solution:

$$\det(\lambda I - A) = \begin{vmatrix} \lambda & -1 & 0 \\ 0 & \lambda & -1 \\ -a_1 & -a_2 & \lambda - a_3 \end{vmatrix}$$
$$= \lambda^3 - a_3\lambda^2 - a_2\lambda - a_1$$

Hurwitz-Criterion:

- $a_{1,2,3} < 0$: All coefficients need to have same sign.
- Hurwitz determinants:

$$\begin{aligned}
&\left|\,-a_3\,\right| < 0 \text{ ok if } a_3 < 0\\
&\begin{vmatrix} -a_3 & -a_1 \\ 1 & -a_2 \end{vmatrix} > 0 \text{ ok if } a_2a_3 + a_1 > 0\\
&\begin{vmatrix} -a_3 & -a_1 & 0 \\ 1 & -a_2 & 0 \\ 0 & -a_3 & -a_1 \end{vmatrix} > 0 \quad -a_1(a_2a_3 + a_1) > 0
\end{aligned} \tag{1.1}$$

$\Rightarrow$ ok if $a_2a_3 + a_1 > 0$

$\Longrightarrow$ For $a_{1,2,3} < 0$ and $a_2a_3 + a_1 > 0$ the system becomes asymptotic stable.

Assume $a_1,\ a_2 = a,\ a_3 = -1,\ B^* = \begin{bmatrix} 0 & 0 \\ 0 & -a_1 \\ b & 0 \end{bmatrix}$.

1d) (1 Point)
Give the actual matrices A, B^*.

Solution:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ a & a & -1 \end{bmatrix},\ B^* = \begin{bmatrix} 0 & 0 \\ 0 & -a \\ b & 0 \end{bmatrix}$$

1e) (4 Points)
Can the eigenvalue $\lambda_1 = -1$ be controlled? State the related condition (use original Hautus).

Solution:

Yes.
Condition:

$$\lambda I - A = \begin{bmatrix} -1 & -1 & 0 \\ 0 & -1 & -1 \\ -a & -a & 0 \end{bmatrix},$$

$$\begin{bmatrix} \widetilde{x}_{11} \\ \widetilde{x}_{21} \\ \widetilde{x}_{31} \end{bmatrix}^T \begin{bmatrix} -1 & -1 & 0 \\ 0 & -1 & -1 \\ -a & -a & 0 \end{bmatrix} = 0 \Rightarrow \begin{aligned} -\widetilde{x}_{11} - a\widetilde{x}_{31} = 0 \\ -\widetilde{x}_{11} - \widetilde{x}_{21} - a\widetilde{x}_{31} = 0 \\ -\widetilde{x}_{21} = 0 \end{aligned}$$

$$\Rightarrow \begin{aligned} \widetilde{x}_{11} &= a \\ \widetilde{x}_{21} &= 0 \\ \widetilde{x}_{31} &= -1 \end{aligned} \quad \text{or} \quad \begin{aligned} \widetilde{x}_{11} &= -a \\ \widetilde{x}_{21} &= 0 \\ \widetilde{x}_{31} &= 1 \end{aligned}$$

$$\Rightarrow \text{LEV}(\lambda = -1) = [a\ 0\ -1] \Rightarrow [a\ 0\ -1] \begin{bmatrix} 0 & 0 \\ 0 & -a \\ b & 0 \end{bmatrix} = [-b\ 0]$$

$[-b\ 0] \neq 0 \Rightarrow$ For $b \neq 0$, the $\lambda_1 = -1$ related mode is controllable.

1f) (2 Points)

Which conditions for the system A and the elements c_1, c_2 of $C = \begin{bmatrix} c_1 & 0 & 0 \\ 0 & c_2 & 0 \end{bmatrix}$ have to be fullfilled for full observability?

Solution:

$$Q_B = \begin{bmatrix} C \\ CA \\ CA^2 \end{bmatrix} = \begin{bmatrix} c_1 & 0 & 0 \\ 0 & c_2 & 0 \\ 0 & c_1 & 0 \\ 0 & 0 & c_2 \\ 0 & 0 & c_1 \\ ac_2 & ac_2 & -c_2 \end{bmatrix},$$

Rank of Q_B should be equal to 3 to fulfill the full observability condition

$\Rightarrow c_1 \neq 0$ or $c_2 \neq 0 \wedge a \neq 0$ $(c_2 a \neq 0)$

☐

$\sum$ ☐

Problem 2 (37 Points)

Qualify the following statements regarding the analysis of MIMO-systems as well as methods for the design of linear MIMO-systems.

2a) (4 Points)

NO.	Task/Question/Judgement	True	False
1	A system described with eigenvalues $\lambda_{1,2} = -2$, $\lambda_3 = -1 + j$, $\lambda_4 = -1 - j$, $\lambda_5 = \epsilon$ with $\epsilon = 0.001$ is asymptotically stable?	○	⊗
2	A system to be controlled has to be asymptotic stable or stable.	○	⊗
	A system with $A = \begin{bmatrix} 0 & 1 \\ -1 & -a \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ b \end{bmatrix}$ has to be controlled using full state feedback.		
3	In the case $a < 0$, $b \neq 0$ the controlled system can be always stabilized.	⊗	○
4	In the case $a > 0$, $b \neq 0$ the controlled system can never be stabilized.	○	⊗

☐

2b) (4 Points)

From $\det(\lambda_i I - A) = 0$ the left and right eigenvectors $\widetilde{\widetilde{x}}$ and $\widetilde{x}$ can be calculated by $\widetilde{\widetilde{x}}_i^T(\lambda_i I - A) = 0$ and $(\lambda_i I - A)\widetilde{x}_i = 0$ for all λ_i of A. In the detailed case, $\widetilde{x}_i$, $\widetilde{\widetilde{x}}_i$ are calculated as $\widetilde{\widetilde{x}}_1 = \begin{bmatrix} 0 \\ 1 \\ 2 \\ -4 \end{bmatrix}$, $\widetilde{\widetilde{x}}_2 = \begin{bmatrix} 0 \\ -2 \\ -4 \\ 4 \end{bmatrix}$, $\widetilde{x}_1 = \begin{bmatrix} 0 \\ 1 \\ 2 \\ -4 \end{bmatrix}$ and $\widetilde{x}_2 = \begin{bmatrix} 0 \\ -2 \\ 1 \\ 4 \end{bmatrix}$ for a system with $B = \begin{bmatrix} b \\ 0 \\ 2 \\ 1 \end{bmatrix}$, $C = \begin{bmatrix} 0 & -1 & -2 & 0 \end{bmatrix}$.

What can be concluded from the given facts?

NO.	Task/Question/Judgement	True	False
1	The system has 2 eigenvalues.	○	⊗
2	For $b = 0$ the first mode is not controllable.	⊗	○
3	For $b \neq 0$ the first mode is not controllable.	⊗	○
4	The first mode is unobservable.	○	⊗

☐

2c) (4 Points)

A system with $A = \begin{bmatrix} 0 & 1 \\ -d & -k \end{bmatrix}$, $C = \begin{bmatrix} 0 & c \end{bmatrix}$, $x = \begin{bmatrix} x_1 & x_2 \end{bmatrix}^T$ should be monitored for diagnostic purposes. The damping d can not be modeled, obviously the behavior seems to be undamped, so d has to be assumed as zero.

NO.	Task/Question/Judgement	True	False
1	The system states x_1, x_2 are measured.	○	⊗
2	An observer can be developed to reconstruct x_1, x_2 as $\hat{x}_1$, $\hat{x}_2$.	○	⊗
3	The system description shows that $\dot{x}_1 = x_2$, so x_1 can be calculated as $x_1 = \int x_2 dt$ and therefore no measurement of x_1 is necessary.	○	⊗
4	In the case of negative damping $(d < 0)(=$ excitation) the system is unstable. In this case an observer can be defined for estimation of x_1 and x_2.	⊗	○

| ☐

2d) (7 Points)

For the system with

$$\lambda_{1,2} = -3 \pm j$$
$$\lambda_{3,4} = -1 \pm j$$
$$\lambda_{5,6} = 2 \pm 3j$$
$$\lambda_7 = 4$$
$$\lambda_{8,9} = 0 \pm 2j$$

a controller design has to be realized. It has to be stated that input decoupling zeros are exist with $s_{idz1,2} = 2 \pm 3j$ and $s_{idz3,4} = 0 \pm 2j$, as well as output decoupling zeros with $s_{odz1,2} = -3 \pm j$.

NO.	Task/Question/Judgement	True	False
1	The system is full controllable.	○	⊗
2	The system is not full observable.	⊗	○
3	The system is not stabilizable.	⊗	○
4	The poles are $s_{p1,2} = -1 \pm j$, $s_{p3} = 4$	⊗	○
5	The poles are $s_{p1,2} = 2 \pm 3j$, $s_{p3,4} = 0 \pm 2j$, $s_{p5,6} = -3 \pm j$.	○	⊗
6	A full state observer can not estimate the mode related to $\lambda_{1,2} = 2 \pm 3j$.	○	⊗
7	A full state controller can stabilize the system.	○	⊗

| ☐

2e) (18 Points)

A system is given by

$$A = \begin{bmatrix} 0 & 1 \\ -k & -d \end{bmatrix}, \quad B = \begin{bmatrix} b \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} -c & 0 \end{bmatrix}, \quad \text{and} \quad D = [d].$$

The transfer function matrix $G(s)$ is

- ○ $[s^2 + 2s + 1]$
- ○ $[cbs + d]$
- ○ $[-cbs + d]$
- ⊗ None

☐

Calculate the eigenvalues of A. The result is:

- ⊗ $\lambda_{1,2} = -\frac{d}{2} \pm \sqrt{\frac{d^2}{4} - k}$
- ○ $\lambda_{1,2} = -\frac{d}{2} \pm \sqrt{jk}$
- ○ $\lambda_{1,2} = -\frac{d}{2} \pm j\sqrt{\frac{k}{m}}$
- ○ None

☐

Is the system fully controllable?

- ○ Yes.
- ○ Yes, for $b \neq 0$, arbitrary k.
- ⊗ Yes, for b, $k \neq 0$.
- ○ No.

☐

Is the system fully observable?

- ○ No.
- ○ Yes.
- ○ Yes, for $c = 0$.
- ⊗ Yes, for $c \neq 0$.

☐

Calculate the feedback gains for state control of the given system using pole placement. The desired eigenvalues of the controlled system should be $\lambda_1 = -4$ and $\lambda_2 = -1$.

- ○ $k_1 = \sqrt{d}$, $k_2 = 2k + d$
- ○ $k_1 = d - 1$, $k_2 = \frac{1}{k}(d^2 - 10)$
- ⊗ $k_1 = \frac{5-d}{b}$, $k_2 = \frac{k-4-d^2+5d}{bk}$
- ○ $k_1 = -5$, $k_2 = \frac{d}{k}$

Solution:

$$|\lambda I - (A - Bk)| \stackrel{!}{=} (\lambda - \lambda_1)(\lambda - \lambda_2)$$

$$(A - Bk) = \begin{bmatrix} 0 & 1 \\ -k & -d \end{bmatrix} - \begin{bmatrix} b \\ 0 \end{bmatrix} \begin{bmatrix} k_1 & k_2 \end{bmatrix} = \begin{bmatrix} -bk_1 & 1 - bk_2 \\ -k & -d \end{bmatrix}$$

$$|\lambda I - (A - Bk)| = \begin{bmatrix} \lambda + bk_1 & bk_2 - 1 \\ k & \lambda + d \end{bmatrix} =$$

$$\lambda^2 + (bk_1 + d)\lambda + dbk_1 - bkk_2 + k$$

$$\stackrel{!}{=} (\lambda + 4)(\lambda + 1) = \lambda^2 + 5\lambda + 4$$

$$\Rightarrow k_1 = \frac{5-d}{b},\ k_2 = \frac{k-4-d^2+5d}{bk}.$$

☐

The observer and the controller design for observer-based control are often performed separately.

NO.	Task/Question/Judgement	True	False
1	According to the separation principle controller and observer design can be decoupled because the resulting eigenvalues are decoupled.	⊗	○
2	The observer and the controller design are not decoupled if different systems are used for design (for disturbance observer-based control).	⊗	○
3	The observer dynamics does not effect the dynamics of the controlled system, so it does not matter how the design is realized.	○	⊗

Solution:

According to the sepatation principle, the dynamical behavior of a system with observer and state feedback is described by

$$\begin{bmatrix}\dot{x}\\ \dot{e}\end{bmatrix} = \begin{bmatrix}A - Bk & Bk\\ 0 & A - LC\end{bmatrix}\begin{bmatrix}x\\ e\end{bmatrix}.$$

Due to the strcture of the matrix of the closed-loop system it can be seen, that the eigenvalues of the controller and those of the observer are independent and thus don't have an influence to each other. So also the design processes are independent of each other, nevertheless is the observer dynamics strongly effecting the overall dynamics because of the resulting eigenvalues.

□

$\sum$ □

Problem 3 (23 Points)

3a) (3 Points)
An electrical system is modeled using the following differential equation

$$m\ddot{x} + d\dot{x} + kx = 4\delta(t),$$

which scalars $m, k > 0$. The state $\dot{x}$ is measured. Taking the vector $z = [z_1,\ z_2]^T = [x,\ \dot{x}]^T$ as the state vector, give the matrices A and C as well as the input vector b of the state space description of this system. For which conditions in terms of d, asymptotic stability can be guaranteed.

Solution:

The equation $m\ddot{x} + d\dot{x} + kx = 4\delta(t)$ can be transformed to

$$\ddot{x} + \frac{d}{m}\dot{x} + \frac{k}{m}x = \frac{4}{m}\delta(t).$$

Substituting the state vector z defined as

$$z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} x \\ \dot{x} \end{bmatrix}$$

into the above equation, one can obtain

$$\dot{z}_1 = z_2,$$

$$\dot{z}_2 = z_2 - \frac{d}{m}z_2 - \frac{k}{m}z_1 + \frac{4}{m}\delta(t).$$

Correspondingly, the state space form of the system can be written as

$$\dot{z} = Az + Bu,$$

$$y = Cz$$

with

$$A = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{d}{m} \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ \frac{4}{m} \end{bmatrix}\delta(t), \text{ and } C = \begin{bmatrix} 0 & 1 \end{bmatrix}.$$

For $d > 0$ the system is asmptotic stable (Stodola).

3b) (6 Points)

A controlled electromechanical system is described in the following state-space form

$$\begin{bmatrix}\dot{x}_1\\\dot{x}_2\\\dot{x}_3\\\dot{x}_4\end{bmatrix} = \underbrace{\begin{bmatrix}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\\ -3+k_1 & 0 & a & b\\ 0 & -4+k_2 & 0 & 0\end{bmatrix}}_{A_{cs}} \begin{bmatrix}x_1\\x_2\\x_3\\x_4\end{bmatrix} + \begin{bmatrix}0\\0\\b_1\\b_2\end{bmatrix} u, \quad y = \begin{bmatrix}m_1 & m_2 & 0 & 0\end{bmatrix}, \text{ and } b_1 = 0,\ b_2 = 1.$$

For which values of the parameters k_1, k_2, a, and b is the system asymptotic stable? Define as an intermediate result the characteristic polynomial of the Hurwitz matrix H.

Solution:

The characteristic polynomial can be calculated as

$$\det(\lambda I - A_{cs}) = \det\begin{bmatrix}\lambda & 0 & -1 & 0\\ 0 & \lambda & 0 & -1\\ 3-k_1 & 0 & \lambda - a & -b\\ 0 & 4-k_2 & 0 & \lambda\end{bmatrix}$$

$$= \lambda^4 - a\lambda^3 + (7-k_1-k_2)\lambda^2 + (k_2-4)a\lambda + k_1k_2 - 4k_1 - 3k_2 + 12.$$

Let $a_0 = k_1k_2 - 3k_2 - 4k_1 + 12$, $a_1 = (k_2-4)a$, $a_2 = 7 - k_1 - k_2$, $a_3 = -a$ and $a_4 = 1$, the Hurwitz matrix can be calculated as

$$H = \begin{bmatrix}a_3 & a_4 & 0 & 0\\ a_1 & a_2 & a_3 & a_4\\ 0 & a_0 & a_1 & a_2\\ 0 & 0 & 0 & a_0\end{bmatrix} = \begin{bmatrix}-a & 1 & 0 & 0\\ (k_2-4)a & 7-k_1-k_2 & -a & 1\\ 0 & k_1k_2-3k_2-4k_1+12 & (k_2-4)a & 7-k_1-k_2\\ 0 & 0 & 0 & k_1k_2-3k_2-4k_1+12\end{bmatrix}$$

According to the Hurwitz criterion, the system is asymptotically stable, if

$$k_1k_2 - 3k_2 - 4k_1 + 12 > 0, \quad (k_2-4)a > 0, \quad 7-k_1-k_2 > 0, \quad -a > 0,$$

$$\Rightarrow a < 0 \wedge k_1 < 3 \wedge k_2 < 4$$

and

$$H_1 = \det[-a] > 0$$

$$H_2 = \det\begin{bmatrix}-a & 1\\ (k_2-4)a & 7-k_1-k_2\end{bmatrix} > 0$$

$$H_3 = \det\begin{bmatrix}-a & 1 & 0\\ (k_2-4)a & 7-k_1-k_2 & -a\\ 0 & k_1k_2-3k_2-4k_1+12 & (k_2-4)a\end{bmatrix} > 0 \Longrightarrow H_3 = 0$$

Because H_3 is always equal to zero, the system is always not asymptotically stable.

3c) (5 Points)
Define the control feedback gain matrix using k_1, k_2 implemented in the system A_{cs} as well as the related input matrix $B = \begin{bmatrix} 0 & 0 & b_1 & b_2 \end{bmatrix}^T$.

Solution:
Assume $K = \begin{bmatrix} k_1 & k_2 & k_3 & k_4 \end{bmatrix}$

$$BK = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ b_1k_1 & b_1k_2 & b_1k_3 & b_1k_4 \\ b_2k_1 & b_2k_2 & b_2k_3 & b_2k_4 \end{bmatrix}$$

Due to

$$A_{cs} = A - BK = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -3+k_1 & 0 & a & b \\ 0 & -4+k_2 & 0 & 0 \end{bmatrix},$$

$$BK \stackrel{!}{=} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ -k_1 & 0 & 0 & 0 \\ 0 & -k_2 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow \begin{cases} b_1k_1 & \stackrel{!}{=} -k_1 \Longrightarrow b_1 \stackrel{!}{=} -1 \\ b_1k_2 & \stackrel{!}{=} 0 \Longrightarrow b_1 \stackrel{!}{=} 0 \\ b_2k_1 & \stackrel{!}{=} 0 \Longrightarrow b_2 \stackrel{!}{=} 0 \\ b_2k_2 & \stackrel{!}{=} -k_2 \Longrightarrow b_2 \stackrel{!}{=} -1 \\ k_3 & \stackrel{!}{=} 0 \\ k_4 & \stackrel{!}{=} 0 \end{cases}$$

$\Longrightarrow$ No suitable feedback gain matrix exists.

3d) (4 Points)
Check whether an observer feedback gain matrix L exists, so that the real parts of the eigenvalues of the observer system $\dot{x} = (A - LC)x$ with

$$A = \begin{bmatrix} 1 & 1 & -1 \\ 8 & 0 & 8 \\ -1 & 1 & 1 \end{bmatrix} \text{ and } C = \begin{bmatrix} c_1 & c_2 & 1 \end{bmatrix}, c_{1,2} = 0$$

are less than 2 (i.e., $\text{Re}(\lambda_i) < 2$ for $i = 1, \dots n$, where λ_i is the i-th eigenvalue of the observer system and $n = 3$ as the system order).

Solution:

If $(A,\ C)$ is fully observable the observer system $\dot{x} = (A - LC)x$ is fully observable, and it is possible to allocate eigenvalues of the observer arbitrarily to obtain $\text{Re}(\lambda_i) < 2$.
Use the Kalman criterion to check the observability with

$$Q_b = \begin{bmatrix} C \\ CA \\ CA^2 \end{bmatrix} = \begin{bmatrix} c_1 & c_2 & 1 \\ c_1 + 8c_2 - 1 & c_1 + 1 & 8c_2 - c_1 + 1 \\ 10c_1 + 6 & 16c_2 & 6c_1 + 10 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ -1 & 1 & 1 \\ 6 & 0 & 10 \end{bmatrix}.$$

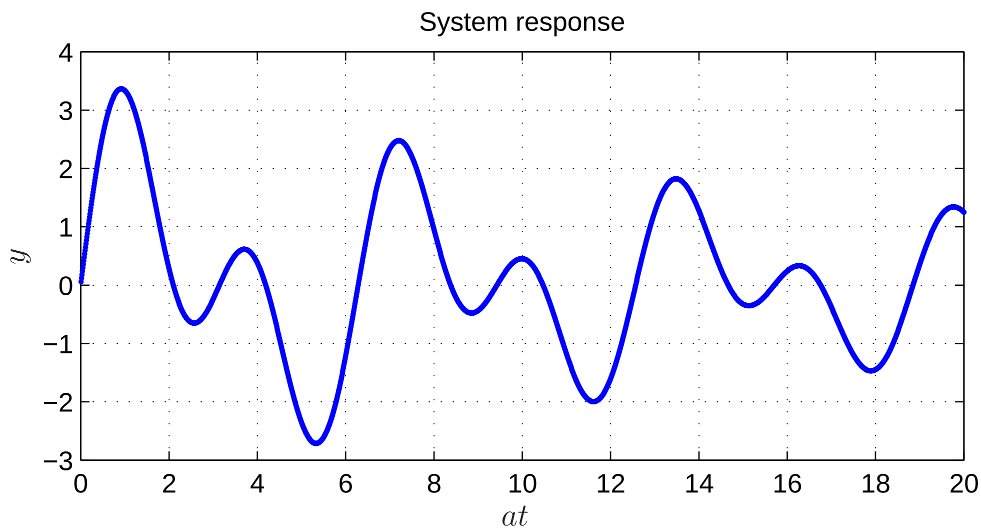
$\text{rank}(Q_b) = 3 = n$ appears.

Therefore, the system is fully observable so it is possible to allocate eigenvalues of the observer system to obtain $\text{Re}(\lambda_i) < 2$; here

3e) (5 Points)
From a given system it is known that the eigenvalues are $\lambda_{1,2} = -1 \pm j$, $\lambda_{3,4} = -2 \pm 2j$, $\lambda_{5,6} = -6 \pm 6j$. The systems response is measured using one measurement channel y. The following experiments are performed:

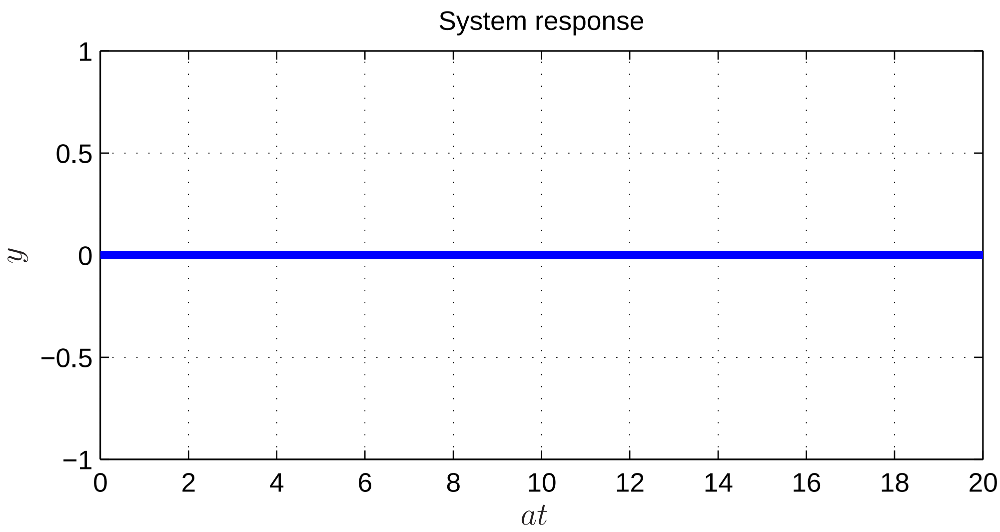
i) Free motions with suitable initial conditions:

The following measurements (all with the same scaling) are taken from the sensor:



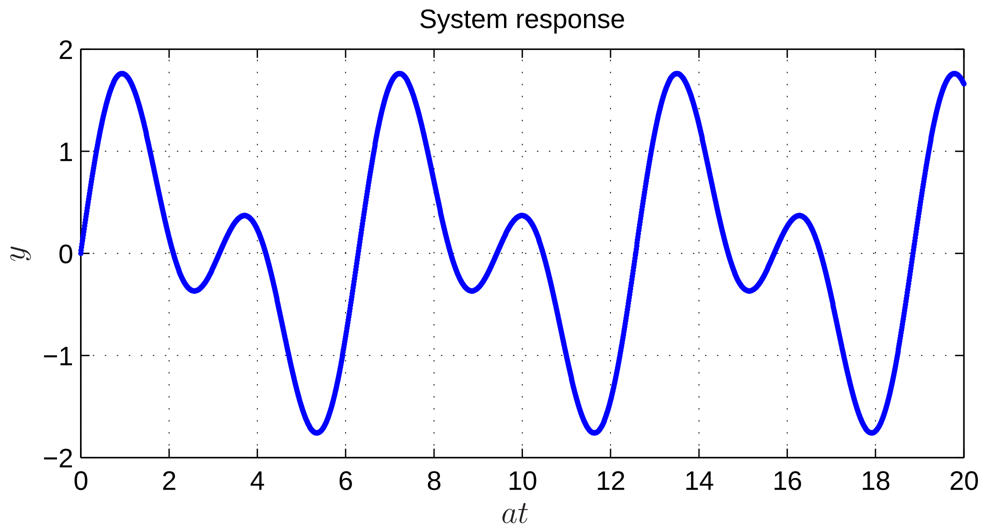
ii) Quasistationary excitation with $u(t) = sin(6\omega t)$.

The following output is taken from the sensor:



iii) Quasistationary excitation with $u(t) = sin(2\omega t) + sin(\omega t)$.

The following output is taken from the sensor:



Analyze the system properties with respect to the

-relation between eigenvalues and poles,
-observability aspects, and
-controllability aspects.

Solution:

The eigenvalues $\lambda_{1,2,3,4}$ are also poles (can be concluded from iii)).
The eigenvalue pair $\lambda_{5,6}$ is not observable (results from i)). Controllability of the eigenvalue pair $\lambda_{5,6}$ can not be concluded from the measurements. All poles are controllable and observable.

$\sum \square$