Reading-up-time

For reviewing purposes of the problem statements, there is a "reading-up-time" of **10 minutes** prior to the official examination time. During this period it is **not** allowed to start solving the problems. This means explicitly that during the entire "reading-up-time" no writing utensils, e.g. pen, pencil, etc. at all are allowed to be kept on the table. Furthermore the use of carried documents, e.g. books, (electronic) translater, (electronic) dictionaries, etc. is strictly forbidden. When the supervisor refers to the end of the "reading-up-time" and thus the beginning of the official examination time, you are allowed to take your utensils and documents. Please **then**, begin with filling in the **complete** information on the titlepage and on page 3.

Good Luck!

LAST NAME	
FIRST NAME	
MATRIKEL-NO.	
TABLE-NO.	

Klausurunterlagen

Ich versichere hiermit, dass ich sämtliche für die Durchführung der Klausur vorgesehenen Unterlagen erhalten, und dass ich meine Arbeit ohne fremde Hilfe und ohne Verwendung unerlaubter Hilfsmittel und sonstiger unlauterer Mittel angefertigt habe. Ich weiß, dass ein Bekanntwerden solcher Umstände auch nachträglich zum Ausschluss von der Prüfung führt. Ich versichere weiter, dass ich sämtliche mir überlassenen Arbeitsunterlagen sowie meine Lösung vollständig zurück gegeben habe. Die Abgabe meiner Arbeit wurde in der Teilnehmerliste von Aufsichtsführenden schriftlich vermerkt.

THE ABOVE REQUIRED STATEMENTS AS WELL AS THE SIGNATURE ARE MANDATORY AT THE BEGINNING OF THE EXAM.

Duisburg, ____________ (Date) ____________________ (Student's signature)

Falls Klausurunterlagen vorzeitig abgegeben: ____________Uhr

Bewertungstabelle

Aufgabe 1	
Aufgabe 2	
Aufgabe 3	
Gesamtpunktzahl	
Angepasste Punktzahl	
%	
Bewertung gem. PO in Ziffern	

____

(Datum und Unterschrift 1. Prüfer, Univ.-Prof. Dr.-Ing. Dirk Söffker)

____

(Datum und Unterschrift 2. Prüfer, Dr.-Ing. Yan Liu)

____

(Datum und Unterschrift des für die Prüfung verantwortlichen Prüfers, Söffker)

Fachnote gemäß Prüfungsordnung:

☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐
1,0	1,3	1,7	2,0	2,3	2,7	3,0	3,3	3,7	4,0	5,0
sehr gut		gut			befriedigend			ausreichend		mangelhaft

Bemerkung: ____

Attention: Give your answers to ALL problems directly below the questions in the exam question sheet.

You are NOT allowed to use a pencil and also NOT red color (red color is used for corrections).

This exam is taken by me as a
☐ mandatory (Pflichtfach)
☐ elective (Wahlfach)
☐ prerequsite (Auflage)
subject (cross ONE option according to your own situation).

Maximum achievable points:	**90 Points**
Minimum points for the grade 1,0:	**95%**
Minimum points for the grade 4,0:	**50%**

General hints:

1) For the multiple-choice and multiple-choice-similar tasks the following rules are effective:
   i) For correct answers of exam task parts the desired number of points will be given.
   ii) For noncorrect answers of exam task parts the desired number of points will be counted negative.
   iii) No answering will neither lead to positive nor to negative points.
   iv) The points of the task will be summarized. The whole number can not be smaller than zero.

2) If in the exam tasks no information is given for the valid range of numbers for time constants or masses etc. : take for time constants (in sec.), for masses (in kg) positive numbers.

3) If in the exam tasks no information is given for applying negative or positive feedback: use the usual negative feedback.

Problem 1 (30 Points)

1a) (3 x 5 Points)
Define the differences between a SISO and a MIMO system with respect to the system description. Which of the following statements is 'True' or 'False'?

NO.	Task/Question/Judgement	SISO True	SISO False	MIMO True	MIMO False
A1	The system can only be described in time domain.	○	⊗	○	⊗
A2	The system can only be described in frequency domain.	○	⊗	○	⊗
A3	For this type of description (SISO description /MIMO description), using poles is the suitable approach to describe the dynamical full-state behavior.	○	⊗	○	⊗
A4	Root locus design is a suitable control design approach.	⊗	○	○	⊗
A5	The number of inputs and the number of outputs is identical and equal to one.	⊗	○	○	⊗
B1	Poles are real or conjugate complex.	⊗	○	⊗	○
B2	Applying Hurwitz criterion for exact asymptotic stability check is suitable from a principal point of view.	⊗	○	⊗	○
B3	If $r = m$ with Rank $B = m$ and Rank $C = r$, the system is always full controllable.	○	⊗	○	⊗
B4	BIBO-stability check is related to I/O-stability.	⊗	○	⊗	○
B5	A system with the poles $s_{1,2} = 0.0007 \pm j\omega$, $s_3 = -10.000 \pm 0.0007 j\omega$, $s_4 = -1 \pm j\omega$, can be denoted as state stable.	○	⊗	○	⊗
C1	Feedback rules as $u = -ky$ (SISO) and $u = -Kx$ (MIMO) are typical rules for this kind of control.	⊗	○	⊗	○
C2	PID-controller design is a suitable design technique.	⊗	○	○	⊗
C3	State control is a suitable design technique.	⊗	○	⊗	○
C4	Non-controllable eigenvalues are denoted as 'holes', because of the capturing properties of 'black holes with respect to the captured dynamics'.	○	⊗	○	⊗
C5	Non-observable eigenmodes are not measurable using the output variables y.	⊗	○	⊗	○

□

For the following tasks, a system description is given with

$$A = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ a_1 & a_2 & a_3 \end{bmatrix}, \quad B = \begin{bmatrix} b \\ 0 \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} c & 0 & 0 \end{bmatrix}, \quad \text{and} \quad D = \begin{bmatrix} d \end{bmatrix}$$

with $a_{1,2,3} \neq 0$, $b \neq 0$, $c \neq 0$, and $d \neq 0$.

1b) (2 Points)
State the Rosenbrock matrix of the system depending on the parameters $a_{1,2,3}$, b, c, and d.

Solution:

$$P(s) = \begin{bmatrix} s & -1 & 0 & -b \\ 0 & s & -1 & 0 \\ -a_1 & -a_2 & s - a_3 & 0 \\ c & 0 & 0 & d \end{bmatrix}$$

1c) (6 Points)
Calculate the characteristic polynomial of the system matrix A depending on the parameters $a_{1,2,3}$, assuming $d = 0$, $a_1 = -1$, $a_2 = -2$, and $a_3 = -3$. Is the system asymptotic stable?

Solution:

$$\begin{aligned} \det(\lambda I - A) &= \begin{vmatrix} \lambda & -1 & 0 \\ 0 & \lambda & -1 \\ -a_1 & -a_2 & \lambda - a_3 \end{vmatrix} \\ &= \lambda^3 - a_3\lambda^2 - a_2\lambda - a_1 = 0 \\ &= \lambda^3 + 3\lambda^2 + 2\lambda + 1 = 0 \end{aligned}$$

Hurwitz-Criterion:

- All coefficients have same sign.
- Hurwitz determinants:

$$H = \begin{vmatrix} 3 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 3 & 1 \end{vmatrix}$$

$H_1 = 3 > 0$
$H_2 = \begin{vmatrix} 3 & 1 \\ 1 & 2 \end{vmatrix} = 4 > 0$
$H_3 = \begin{vmatrix} 3 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 3 & 1 \end{vmatrix} = 4 > 0$

$\Longrightarrow$ The system is asymptotic stable.

1d) (1 Point)

Assume $A^* = A$ and $B^* = \begin{bmatrix} b & 0 \\ b & -a_1 \\ 0 & 0 \end{bmatrix}$ with $a_1 = a,\ a_2 = a,\ a_3 = -1$. Give the matrices A^* and B^*.

Solution:

$$A^* = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ a & a & -1 \end{bmatrix},\ B^* = \begin{bmatrix} b & 0 \\ b & -a \\ 0 & 0 \end{bmatrix}$$

1e) (2 Points)

Is the eigenvalue $\lambda_1 = -1$ of A^* controllable? State the related condition (use original Hautus criteria based on eigenvectors).

Solution:

Yes.
Condition:

$$\lambda I - A = \begin{bmatrix} -1 & -1 & 0 \\ 0 & -1 & -1 \\ -a & -a & 0 \end{bmatrix},$$

$$\begin{bmatrix} \widetilde{x}_{11} \\ \widetilde{x}_{21} \\ \widetilde{x}_{31} \end{bmatrix}^T \begin{bmatrix} -1 & -1 & 0 \\ 0 & -1 & -1 \\ -a & -a & 0 \end{bmatrix} = 0 \Rightarrow \begin{array}{l} -\widetilde{x}_{11} - a\widetilde{x}_{31} = 0 \\ -\widetilde{x}_{11} - \widetilde{x}_{21} - a\widetilde{x}_{31} = 0 \\ -\widetilde{x}_{21} = 0 \end{array}$$

$$\Rightarrow \begin{array}{l} \widetilde{x}_{11} = a \\ \widetilde{x}_{21} = 0 \\ \widetilde{x}_{31} = -1 \end{array} \quad \text{or} \quad \begin{array}{l} \widetilde{x}_{11} = -a \\ \widetilde{x}_{21} = 0 \\ \widetilde{x}_{31} = 1 \end{array}$$

$$\Rightarrow \text{LEV}(\lambda = -1) = [a\ 0\ -1] \Rightarrow [a\ 0\ -1] \begin{bmatrix} b & 0 \\ b & -a \\ 0 & 0 \end{bmatrix} = [ab\ 0]$$

$[ab\ 0] \neq 0 \Rightarrow$ For $ab \neq 0$, the $\lambda_1 = -1$ related mode is controllable.

1f) (4 Points)

Which conditions for the system A^* and the elements c_1, c_2 of $C = \begin{bmatrix} c_1 & 0 & 0 \\ 0 & c_2 & 0 \end{bmatrix}$ have to be fulfilled for full observability? Check the observability of the eigenvalues ($\lambda_1 = -1$ and $\lambda_{2,3} = \pm\sqrt{a}$) using the Hautus criteria calculating first the right eigenvectors.

Solution:

Cauculate the right eigenvectors: $(\lambda I - A)\widetilde{x}_i = 0$

$$\lambda I - A = \begin{bmatrix} \lambda & -1 & 0 \\ 0 & \lambda & -1 \\ -a & -a & \lambda+1 \end{bmatrix},$$

for $\lambda_1 = -1$,

$$\begin{bmatrix} -1 & -1 & 0 \\ 0 & -1 & -1 \\ -a & -a & 0 \end{bmatrix} \begin{bmatrix} \widetilde{x}_{11} \\ \widetilde{x}_{12} \\ \widetilde{x}_{13} \end{bmatrix} = 0 \Rightarrow \begin{matrix} -\widetilde{x}_{11} - \widetilde{x}_{12} = 0 \\ -\widetilde{x}_{12} - \widetilde{x}_{13} = 0 \\ -a\widetilde{x}_{11} - a\widetilde{x}_{12} = 0 \end{matrix} \Rightarrow \begin{matrix} \widetilde{x}_{11} = 1 \\ \widetilde{x}_{12} = -1 \\ \widetilde{x}_{13} = 1 \end{matrix}$$

$$C\widetilde{x}_1 = \begin{bmatrix} c_1 & 0 & 0 \\ 0 & c_2 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix} = \begin{bmatrix} c_1 \\ -c_2 \end{bmatrix}$$

$\Rightarrow$ for $c_1 \neq 0$ or $c_2 \neq 0$ is λ_1 observable.

for $\lambda_2 = \sqrt{a}$,

$$\begin{bmatrix} \sqrt{a} & -1 & 0 \\ 0 & \sqrt{a} & -1 \\ -a & -a & \sqrt{a}+1 \end{bmatrix} \begin{bmatrix} \widetilde{x}_{21} \\ \widetilde{x}_{22} \\ \widetilde{x}_{23} \end{bmatrix} = 0 \Rightarrow \begin{matrix} \sqrt{a}\widetilde{x}_{21} - \widetilde{x}_{22} = 0 \\ \sqrt{a}\widetilde{x}_{22} - \widetilde{x}_{23} = 0 \\ -a\widetilde{x}_{21} - a\widetilde{x}_{22} + (\sqrt{a}+1)\widetilde{x}_{23} = 0 \end{matrix} \Rightarrow \begin{matrix} \widetilde{x}_{21} = 1 \\ \widetilde{x}_{22} = \sqrt{a} \\ \widetilde{x}_{23} = a \end{matrix}$$

$$C\widetilde{x}_2 = \begin{bmatrix} c_1 & 0 & 0 \\ 0 & c_2 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ \sqrt{a} \\ a \end{bmatrix} = \begin{bmatrix} c_1 \\ \sqrt{a}c_2 \end{bmatrix}$$

$\Rightarrow$ for $c_1 \neq 0$ or $ac_2 \neq 0$ is λ_2 observable.

for $\lambda_3 = -\sqrt{a}$,

$$\begin{bmatrix} -\sqrt{a} & -1 & 0 \\ 0 & -\sqrt{a} & -1 \\ -a & -a & -\sqrt{a}+1 \end{bmatrix} \begin{bmatrix} \widetilde{x}_{31} \\ \widetilde{x}_{32} \\ \widetilde{x}_{33} \end{bmatrix} = 0 \Rightarrow \begin{matrix} -\sqrt{a}\widetilde{x}_{31} - \widetilde{x}_{32} = 0 \\ -\sqrt{a}\widetilde{x}_{32} - \widetilde{x}_{33} = 0 \\ -a\widetilde{x}_{31} - a\widetilde{x}_{32} + (1-\sqrt{a})\widetilde{x}_{33} = 0 \end{matrix} \Rightarrow \begin{matrix} \widetilde{x}_{31} = 1 \\ \widetilde{x}_{32} = -\sqrt{a} \\ \widetilde{x}_{33} = a \end{matrix}$$

$$C\widetilde{x}_3 = \begin{bmatrix} c_1 & 0 & 0 \\ 0 & c_2 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ -\sqrt{a} \\ a \end{bmatrix} = \begin{bmatrix} c_1 \\ -\sqrt{a}c_2 \end{bmatrix}$$

$\Rightarrow$ for $c_1 \neq 0$ or $ac_2 \neq 0$ is λ_3 observable.

$\Longrightarrow$ for $c_1 \neq 0$ or $ac_2 \neq 0$, the system is fully observable.

Problem 2 (35 Points)

Qualify the following statements regarding the analysis of MIMO-systems as well as methods for the design of linear MIMO-systems.

2a) (4 Points)

NO.	Task/Question/Judgement	True	False
1	A system described with eigenvalues $\lambda_{1,2} = 2,\ \lambda_3 = 1 + j,$ $\lambda_4 = 1 - j,\ \lambda_5 = \epsilon$ with $\epsilon = -0.001$ is asymptotically stable.	$\bigcirc$	$\otimes$
2	A system to be observed ($\hat{=}$ using state observer) has to be asymptotic stable or stable.	$\bigcirc$	$\otimes$
	A system with $A = \begin{bmatrix} 0 & 1 \\ -1 & -a \end{bmatrix}$, $C = \begin{bmatrix} 0 \\ c \end{bmatrix}^T$ has to be observed using full state measurements,		
3	In the case $a < 0,\ c \neq 0$ the observed system can be always detected.	$\otimes$	$\bigcirc$
4	In the case $a > 0,\ c \neq 0$ the observed system can never be detected.	$\bigcirc$	$\otimes$

2b) (4 Points)

From $\det(\lambda_i I - A) = 0$ the left and right eigenvectors $\widetilde{\widetilde{x}}$ and $\widetilde{x}$ can be calculated by $\widetilde{\widetilde{x}}_i^T(\lambda_i I - A) = 0$ and $(\lambda_i I - A)\widetilde{x}_i = 0$ for all λ_i of A. In the detailed case, $\widetilde{x}_i$, $\widetilde{}_i$ are calculated as $\widetilde{\widetilde{x}}_1 = \begin{bmatrix} 6 \\ 1 \\ 0 \\ 2 \end{bmatrix}$, $\widetilde{\widetilde{x}}_2 = \begin{bmatrix} 0 \\ 3 \\ -1 \\ 5 \end{bmatrix}$, $\widetilde{x}_1 = \begin{bmatrix} 2 \\ -5 \\ 1 \\ 3 \end{bmatrix}$, and $\widetilde{x}_2 = \begin{bmatrix} 6 \\ -2 \\ 3 \\ 1 \end{bmatrix}$ for a system with $B = \begin{bmatrix} -1 \\ 0 \\ b \\ 3 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 & -3 & 1 \end{bmatrix}$.

What can be concluded from the given facts?

NO.	Task/Question/Judgement	True	False
1	The system has 2 eigenvalues.	$\bigcirc$	$\otimes$
2	For $b = 0$ the first mode is not controllable.	$\otimes$	$\bigcirc$
3	For $b \neq 0$ the first mode is not controllable.	$\otimes$	$\bigcirc$
4	The first mode is unobservable.	$\bigcirc$	$\otimes$

2c) (4 Points)

A system with $A = \begin{bmatrix} 0 & 1 \\ -d & -k \end{bmatrix}$, $C = \begin{bmatrix} 0 & c \end{bmatrix}$, $x = \begin{bmatrix} x_2 & x_1 \end{bmatrix}^T$ should be monitored for diagnostic purposes. The damping d can not be modeled, obviously the behavior seems to be perfectly damped, so d is assumed as 10.

NO.	Task/Question/Judgement	True	False
1	The system states x_2, x_1 are measured.	○	⊗
2	An observer has to be developed, because x_2, x_1 are not measured completely.	○	⊗
3	The system description shows that $\dot{x}_2 = x_1$, so x_2 can be calculated as $x_1 = \int x_2 dt$ and therefore no measurement for the absolute value of x_2 is necessary.	○	⊗
4	In the case of negative damping $(d < 0)(=$ excitation) the system is unstable. In this case of an unstable system an observer can be defined for estimation of x_2 and x_1.	⊗	○

| □

2d) (7 Points)

For the system with

$$\lambda_{1,2} = -3 \pm j$$
$$\lambda_{3,4} = -1 \pm j$$
$$\lambda_{5,6} = 2 \pm 3j$$
$$\lambda_7 = 4$$
$$\lambda_{8,9} = 0 \pm 2j$$

a controller design has to be realized. It has to be stated that ounput decoupling zeros exist with $s_{odz1,2} = 2 \pm 3j$ and $s_{odz3,4} = 0 \pm 2j$, as well as input decoupling zeros with $s_{idz1,2} = -3 \pm j$.

NO.	Task/Question/Judgement	True	False
1	The system is fully controllable.	○	⊗
2	The system is not fully observable.	⊗	○
3	The system is stabilizable.	⊗	○
4	The poles are $s_{p1,2} = -1 \pm j$, $s_{p3} = 4$	⊗	○
5	The eigenvalues are $\widetilde{\lambda}_{1,2} = -3 \pm j$, $\widetilde{\lambda}_{3,4} = -1 \pm j$, $\widetilde{\lambda}_{5,6} = 2 \pm 3j$, $\widetilde{\lambda}_7 = 4$ and $\widetilde{\lambda}_{8,9} = 0 \pm 2j$.	⊗	○
6	A full state observer can not estimate the mode related to $\widetilde{\lambda}_{8,9} = 0 \pm 2j$.	⊗	○
7	An observer-based full state controller can not stabilize the system.	⊗	○

| □

2e) (16 Points)

A system is given by

$$A = \begin{bmatrix} 0 & 1 \\ -k & -d \end{bmatrix}, \quad B = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}, \quad C = \begin{bmatrix} c & 0 \end{bmatrix}, \quad \text{and} \quad D = [0].$$

The transfer function matrix $G(s)$ is

○ $[s^2 + 2s + 1]$ ○ $[cbs + d]$

○ $[-cbs + d]$ ⊗ None

☐

Calculate the eigenvalues of A. The result is:

⊗ $\lambda_{1,2} = -\frac{d}{2} \pm \sqrt{\frac{d^2}{4} - k}$ ○ $\lambda_{1,2} = -\frac{d}{2} \pm \sqrt{jk}$

○ $\lambda_{1,2} = -\frac{d}{2} \pm j\sqrt{\frac{k}{m}}$ ○ None

☐

Is the system fully controllable?

○ Yes. ○ Yes, for $b_1 \neq 0$ or $b_2 \neq 0$, arbitrary k and d.

○ No. ⊗ Yes, for $b_1 k \neq 0$ or $b_2 \neq 0$ or $b_2 \neq (-\frac{d}{2} \pm \sqrt{\frac{d^2}{4} - k})b_1$.

☐

Is the system fully observable?

○ No. ○ Yes.

○ Yes, for $c = 0$. ⊗ Yes, for $c \neq 0$.

☐

Calculate the feedback gains for state control of the given system using pole placement. The desired eigenvalues of the controlled system should be $\lambda_1 = 4$ and $\lambda_2 = -1$.

○ $k_1 = \frac{1}{b_1}$, $k_2 = \frac{-4+d}{b_2}$

○ $k_1 = 1$, $k_2 = \frac{-3-b_1-d}{b_2}$

⊗ $k_1 = \frac{-4b_2-k(3b_1+b_2+b_1d)}{kb_1^2+db_1b_2+b_2^2}$, $k_2 = \frac{-(3b_2-4b_1+3b_1d+b_2d-b_1k+b_1d^2)}{kb_1^2+db_1b_2+b_2^2}$

○ $k_1 = -1$, $k_2 = \frac{-3+b_1-d}{b_2}$

Solution:

$$|\lambda I - (A - BK)| \overset{!}{=} (\lambda - \lambda_1)(\lambda - \lambda_2)$$

$$(A - BK) = \begin{bmatrix} 0 & 1 \\ -k & -d \end{bmatrix} - \begin{bmatrix} b_1 \\ b_2 \end{bmatrix} \begin{bmatrix} k_1 & k_2 \end{bmatrix} = \begin{bmatrix} -b_1k_1 & 1 - b_1k_2 \\ -k - b_2k_1 & -d - b_2k_2 \end{bmatrix}$$

$$|\lambda I - (A - BK)| = \begin{bmatrix} \lambda + b_1k_1 & b_1k_2 - 1 \\ k + b_2k_1 & \lambda + d + b_2k_2 \end{bmatrix} =$$

$$\lambda^2 + (b_1k_1 + d + b_2k_2)\lambda - (k + b_2k_1)(b_1k_2 - 1) - b_1k_1(d + b_2k_2)$$

$$\overset{!}{=} (\lambda - 4)(\lambda + 1) = \lambda^2 - 3\lambda - 4$$

$$\Rightarrow k_1 = \frac{-4b_2-k(3b_1+b_2+b_1d)}{kb_1^2+db_1b_2+b_2^2},\ k_2 = \frac{-(3b_2-4b_1+3b_1d+b_2d-b_1k+b_1d^2)}{kb_1^2+db_1b_2+b_2^2}.$$

□

Hautus (Mode-related) analysis is considered.

NO.	Task/Question/Judgement	True	False
1	Hautus analysis allows a detailed calculation of variable and parameter dependences between the system A and related output defined by C.	⊗	○
2	Hautus approach is a perfect approach combining stability analysis with observer design.	○	⊗
3	Hautus analysis does not allow mode-wise consideration of observability and controllability.	○	⊗

□

$\sum$ □

Problem 3 (25 Points)

3a) (3 Points)
An electrical system is modeled using the following differential equation

$$m\ddot{x} + d\dot{x} + kx = 4 \cdot 1(t),$$

which scalars $m, d > 0$ and $u(t) = 1(t)$. The variable x is measured. Taking the vector $z = [z_1,\ z_2]^T = [x,\ \dot{x}]^T$ as the state vector, give the matrices A and C as well as the input vector b of the state space description of this system. For which conditions in terms of k the system behavior becomes unstable?

Solution:

The equation $m\ddot{x} + d\dot{x} + kx = 4 \cdot 1(t)$ can be transformed to

$$\ddot{x} + \frac{d}{m}\dot{x} + \frac{k}{m}x = \frac{4}{m} \cdot 1(t).$$

Substituting the state vector z defined as

$$z = \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = \begin{bmatrix} x \\ \dot{x} \end{bmatrix}$$

into the above equation, one can obtain

$$\dot{z}_1 = z_2,$$

$$\dot{z}_2 = z_2 - \frac{d}{m}z_2 - \frac{k}{m}z_1 + \frac{4}{m} \cdot 1(t).$$

Correspondingly, the state space form of the system can be written as

$$\dot{z} = Az + Bu,$$

$$y = Cz$$

with

$$A = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{d}{m} \end{bmatrix}, \quad b = \begin{bmatrix} 0 \\ \frac{4}{m} \end{bmatrix}, \text{ and } C = \begin{bmatrix} 1 & 0 \end{bmatrix}.$$

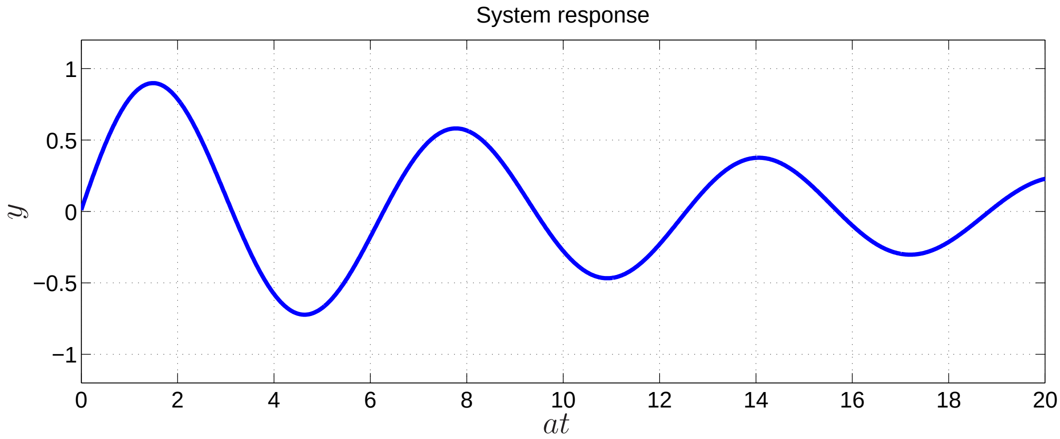
For $k < 0$ the system becomes unstable (Stodola).

3b) (5 Points)

From a given system it is known that the eigenvalues are $\lambda_{1,2} = -1 \pm j$, $\lambda_{3,4} = -2 \pm 2j$, $\lambda_{5,6} = -6 \pm 6j$. The system's response is measured using one measurement channel y. The following experiments are performed:

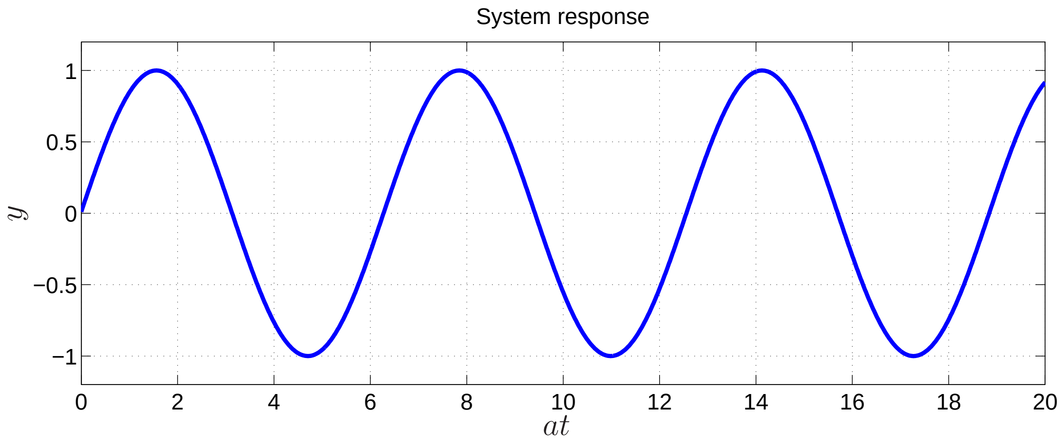
i) Free motions with suitable initial conditions

The following measurements (all with the same scaling) are taken from the sensor:



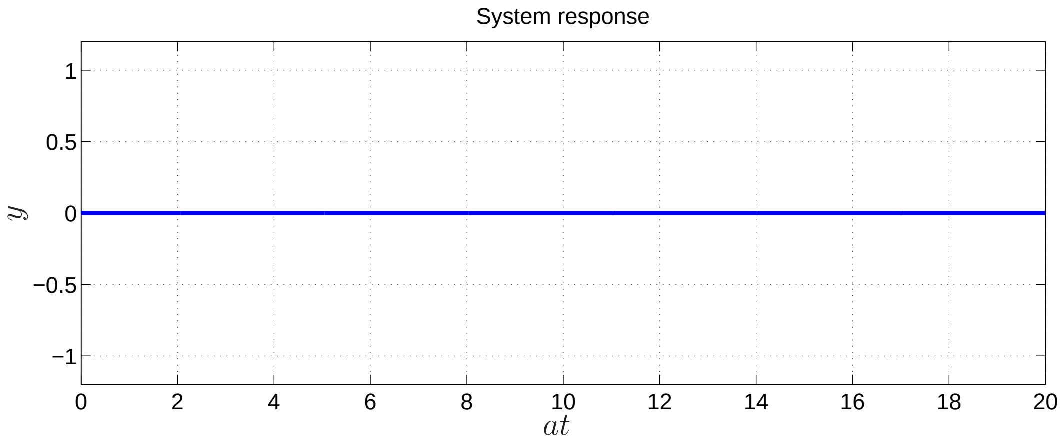
ii) Quasistationary excitation with $u(t) = sin(\omega t)$

The following output is taken from the sensor:



iii) Quasistationary excitation with $u(t) = sin(2\omega t) + sin(6\omega t)$

The following output is taken from the sensor:



Analyze the system properties:

-What can be concluded with respect to the observability?
-What can be concluded about the properties of the eigenvalues $\lambda_{1,2} = -1 \pm j$?
-What can be concluded about the controllability of $\lambda_{3,4,5,6}$?

Solution:

-The eigenvalue pairs $\lambda_{3,4}$ and $\lambda_{5,6}$ are not observable.
or only the eigenvalue pair $\lambda_{1,2}$ are observable.(results from i)

-The eigenvalues $\lambda_{1,2}$ are also poles (can be concluded from ii).
They are both controllable and observable(can be concluded from i and ii).

-Controllability of the eigenvalue pairs $\lambda_{3,4}$ and $\lambda_{5,6}$ can not be concluded from the measurements.

3c)

Given is the standard description of a linear MIMO system. The system described by

$$A = \begin{bmatrix} 0 & 0 & 0.5 \\ 2 & -1 & 1 \\ -2 & -3 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} b_1 \\ 0 \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \text{ and } D = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

should be controlled using the state feedback matrix $K = \begin{bmatrix} k_1 & k_2 & k_3 \end{bmatrix}$.

3c1) (3 Points)
Is it necessary for practical purposes to build an observer for the given system to realize full state feedback control? State reason.

Solution:

Yes.
Because of Rank $C = 2 \neq n$, not all states are measured.

3c2) (3 Points)
For which b_1 ($b_1 \neq 0$) the system is fully controllable?

Solution:

$$Q_s = \begin{bmatrix} B & AB & A^2B \end{bmatrix} = \begin{bmatrix} b_1 & 0 & -b_1 \\ 0 & 2b_1 & -4b_1 \\ 0 & -2b_1 & -6b_1 \end{bmatrix}$$

Rank $Q_B \overset{!}{=} n = 3$

$\Rightarrow$ det $Q_B \neq 0$

Full controllability for
$b_1 \neq 0$

3c3) (1 Points)
For task 3c2), is it possible to design the controller eigenvalues arbitrarily?

Solution:

Yes, if the conclusion for full controllability is fulfilled.

3c4) (5 Points)
A modified system is given with $A = \begin{bmatrix} 0 & 1 \\ a_1 & a_2 \end{bmatrix}$ and $B = \begin{bmatrix} b & 0 \end{bmatrix}^T$ with $b \neq 0$. Calculate k_1 and k_2 so that the eigenvalues of the closed-loop system are $\lambda_1 = -3$ and $\lambda_2 = 2$.

Solution:

Characteristic equation of the closed-loop system: $\det(\lambda I - A + BK)$=0

$$\lambda I - A + BK = \begin{bmatrix} \lambda + bk_1 & -1 + bk_2 \\ -a_1 & \lambda - a_2 \end{bmatrix}$$

$$\Longrightarrow \lambda^2 + (bk_1 - a_2)\lambda - a_1 + a_1 bk_2 - a_2 bk_1 = 0$$

$$\lambda_1 = -3 \Rightarrow -(3b + a_2 b)k_1 + a_1 bk_2 + 3a_2 - a_1 + 9 = 0$$

$$\lambda_2 = 2 \Rightarrow (2b - a_2 b)k_1 + a_1 bk_2 - 2a_2 - a_1 + 4 = 0$$

or

$$|\lambda I - (A - BK)| \stackrel{!}{=} (\lambda + 3)(\lambda - 2) = \lambda^2 + \lambda - 6$$
$$bk_1 - a_2 \stackrel{!}{=} 1 \text{ and } -a_1 + a_1 bk_2 - a_2 bk_1 \stackrel{!}{=} -6$$

$$\Longrightarrow k_1 = (a_2 + 1)/b \qquad k_2 = (a_2^2 + a_2 + a_1 - 6)/(a_1 b)$$

3c5) (5 Points)

A modified system is given with $A = \begin{bmatrix} 0 & 1 \\ a & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$. Is the system stabilizable (use Hautus criterion with $\tilde{\tilde{x}}_i = [\tilde{\tilde{x}}_{i1} \ 1]$ as the left eigenvector of A)? If yes, State conditions. If No, state reason.

Solution:

No. Because $B = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$

$\Rightarrow$ the system is not controllable.

$\Rightarrow$ the system is not stabilizable.

$\sum$