Reading-up-time

For reviewing purposes of the problem statements, there is a "reading-up-time" of **10 minutes** prior to the official examination time. During this period it is **not** allowed to start solving the problems. This means explicitly that during the entire "reading-up-time" no writing utensils, e.g. pen, pencil, etc. at all are allowed to be kept on the table. Furthermore the use of carried documents, e.g. books, (electronic) translater, (electronic) dictionaries, etc. is strictly forbidden. When the supervisor refers to the end of the "reading-up-time" and thus the beginning of the official examination time, you are allowed to take your utensils and documents. Please **then**, begin with filling in the **complete** information on the titlepage and on page 3.

Good Luck!

LAST NAME	
FIRST NAME	
MATRIKEL-NO.	
TABLE-NO.	

Klausurunterlagen

Ich versichere hiermit, dass ich sämtliche für die Durchführung der Klausur vorgesehenen Unterlagen erhalten, und dass ich meine Arbeit ohne fremde Hilfe und ohne Verwendung unerlaubter Hilfsmittel und sonstiger unlauterer Mittel angefertigt habe. Ich weiß, dass ein Bekanntwerden solcher Umstände auch nachträglich zum Ausschluss von der Prüfung führt. Ich versichere weiter, dass ich sämtliche mir überlassenen Arbeitsunterlagen sowie meine Lösung vollständig zurück gegeben habe. Die Abgabe meiner Arbeit wurde in der Teilnehmerliste von Aufsichtsführenden schriftlich vermerkt.

Duisburg, den ________ ____________________
(Unterschrift der/des Studierenden)

Falls Klausurunterlagen vorzeitig abgegeben: __________Uhr

Bewertungstabelle

Aufgabe 1	
Aufgabe 2	
Gesamtpunktzahl	
Angehobene Punktzahl	
%	
Bewertung gem. PO in Ziffern	

(Datum und Unterschrift 1. Prüfer, Univ.-Prof. Dr.-Ing. Dirk Söffker)

(Datum und Unterschrift 2. Prüfer, Yan Liu, M.Eng.)

(Datum und Unterschrift des für die Prüfung verantwortlichen Prüfers, Söffker)

Fachnote gemäß Prüfungsordnung:

☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐
1,0	1,3	1,7	2,0	2,3	2,7	3,0	3,3	3,7	4,0	5,0
sehr gut		gut			befriedigend			ausreichend		mangelhaft

Bemerkung: ____________________

Attention: Give your answers to ALL problems directly below the questions in the exam question sheet.

You are NOT allowed to use a pencil and also NOT red color (red color is used for corrections).

This exam "System Dynamics" is taken by me as a
☐ mandatory (Pflichtfach)
☐ elective (Wahlfach)
☐ prerequsite (Auflage)
subject (cross ONE option according to your own situation).

Maximum achievable points:	**40**
Minimum points for the grade 1,0:	**95%**
Minimum points for the grade 4,0:	**50%**

Problem 1 (12 points)

a) (3 points)
State the difference between a SISO-system and a MIMO-system.

SISO-system : 1 input , 1 output
"single input single output

MIMO-system: minimum 2 inputs and minimum
2 outputs
multiple inputs multiple outputs

b) (3 points)
State in detail the underlying assumptions of linear, time invariant control technique. Discuss the terms "non linear" and "linear" as well as "time variant" and "time invariant" by mens of two I/O-relations.

Superposition can be used $\Rightarrow$ linear

i.e. not linear : $y(t) = \sin(u(t))$

linear : $y(t) = Ku(t)$

time invariant : $y(t) = Ku(t)$

time variant : $y(t) = K(t)u(t)$

c) (3 points)
The eigenvalue distribution of a system is given in Figure 1.1.

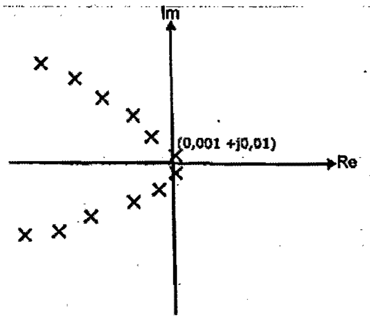


Figure 1.1: Eigenvalue distribution

Is the system asymptocally stable? Justify your statement.

The system is neither stable nor asymptocally stable because an eigenvalue pair has positive real part.

d) (3 points)
A system in equilibrium condition is excited at $t = 0$ with a harmonical signal with amplitude $A = 2$ and frequency $\omega = 0.5$. The measured output signal of the system is a harmonic signal with amplitude $A = 4$ and frequency $\omega = 0.5$. Classify the stability of the system and justify your statemant.

→ Proportional System:
asymptotic stable

e) (3 points)
The state space of a system is described by

$$\dot{x} = Ax + bu \quad \text{and} \quad y = Cx$$

where

$$A = \begin{bmatrix} 0 & 1 \\ -34 & 0.1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 \end{bmatrix} \text{ and } \quad b = \begin{bmatrix} 0 \\ 1 \end{bmatrix}.$$

Determine the ODE of the system and classify its stability.

$$\begin{bmatrix} \dot{x} \\ \ddot{x} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -34 & 0.1 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

$$Y = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ \dot{x} \end{bmatrix}$$

$$\ddot{x} = -34x + 0.1\dot{x} + u$$

$$\ddot{x} - 0.1\dot{x} + 34x = u$$

$$\frac{1}{34}\ddot{x} - \frac{1}{340}\dot{x} + x = \frac{1}{34}u$$

$$\frac{1}{34}\ddot{y} - \frac{1}{340}\dot{y} + y = \frac{1}{34}u$$

Stability of the System:

$$\ddot{x} - 0.1\dot{x} + 34x = u$$

$$\lambda^2 - 0.1\lambda + 34 \overset{!}{=} 0$$

$$\lambda_{1,2} = 0.05 \pm \sqrt{(0.05)^2 - 34}$$

$$\lambda_{1,2} = 0.05 \pm j\sqrt{\quad}$$

$Re\{\lambda_{1,2}\} > 0$ not stable

alternatively: Stodola-criterion not fulfilled

$\Rightarrow$ not stable

$\sum \square$

Aufgabe 2 (25 Points)
In Fig. 2.1, a block diagram of a system with four transfer elements is given. The input is denoted as u and the output as y.

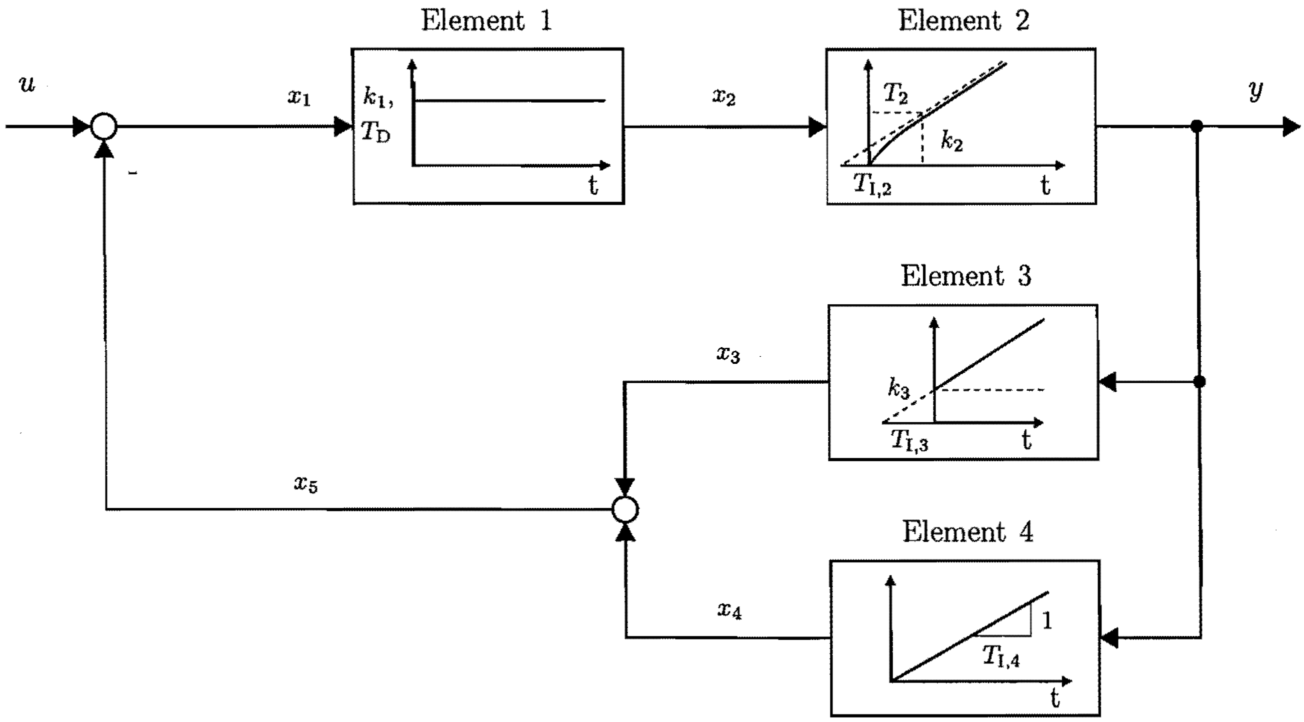


Figure 2.1: Block diagram of the given system

2a) (4 Points)
Classify the transfer behavior of the four elements of the system and determine the four corresponding differential equations in a suitable form for classification under consideration of the given inscriptions.

Element 1: PD $\quad x_2(t) = k_1\left(x_1(t) + T_D\,\dot{x}_1(t)\right)$

Element 2: PIT_1 $\quad T_2\,\dot{y}(t) + y(t) = k_2\left(x_2(t) + \frac{1}{T_{I,2}}\int x_2(t)\,dt\right)$

Element 3: PI $\quad x_3(t) = k_3\left(y(t) + \frac{1}{T_{I,3}}\int y(t)\,dt\right)$

Element 4: I $\quad x_4(t) = \frac{1}{T_{I,4}}\int y(t)\,dt$

2b) (4 Points)
Give the differential equation of the whole system with respect to the input u and the output y as given in task a). The parameters are given as $T_D = T_2 = T_{I,2} = T_{I,3} = T_{I,4} = k_1 = k_2 = k_3 = 1$.

$$x_1 = u - x_5 \qquad x_2 = x_1 + \dot{x}_1$$

$$x_5 = x_3 + x_4 \qquad \dot{y} + y = x_2 + \int x_2$$

$$x_3 = y + \int y$$

$$x_4 = \int y$$

$$\Rightarrow 2\dddot{y} + 5\ddot{y} + 5\dot{y} + 2y = \dddot{u} + 2\ddot{u} + \dot{u}$$

The mathematical model of an electrical system is described by the equations

$$\dot{x}_1(t) + x_1(t) = Kx_2(t),$$
$$\int x_1(t)\ \mathrm{d}t = T_1 y(t) + \int y(t)\ \mathrm{d}t, \text{ and}$$
$$x_2(t) = u(t) + \frac{1}{T_\mathrm{I}} \int u(t)\ \mathrm{d}t.$$

2c) (4 Points)
Utilize the given equations to determine the scalar differential equation of the whole system (with input u and output y) and classify the resulting transfer behavior.

Which kind of transfer behavior of the controller is suitable for realizing stationary accuracy of the whole system (state reasons)?

$\dot{x}_1(t) + x_1(t) = k\,x_2(t)$ (1)

$x_1(t) = T_1 \dot{y}(t) + y(t)$

$\Leftrightarrow \dot{x}_1(t) = T_1 \ddot{y}(t) + \dot{y}(t)$ (2)

aus (1) und (2):

$\Rightarrow T_1 \ddot{y}(t) + (1 + T_1)\dot{y}(t) + y(t) = k\left(u(t) + \frac{1}{T_I}\int u(t)\,dt\right)$

$\Rightarrow PIT_2$

$\Rightarrow$ Controller with P-behavior is sufficient because the plant has an integral part.

2d) (5 Points)
In Fig. 2.2, the eigenvalues of four different systems are illustrated graphically. Furthermore, the step response behaviors of two of the four systems are measured and given in Fig. 2.3. Assign the two measurements to the related systems and state the reasons for your decisions (why or why not for each of the four systems).

Measurement 1 -> System 1
(low damping)
Measurement 2 -> System 3
(high damping)

System 2: not stable -> can not be measured
System 4: damping is too large for strong oscillation
(maximal one oscillation)

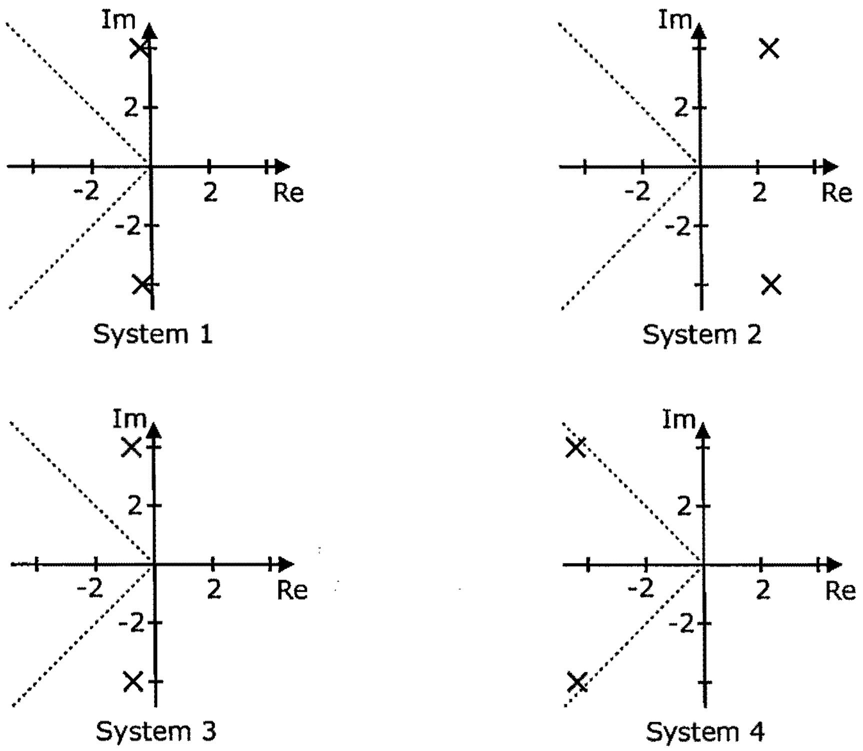


Figure 2.2: Eigenvalue distribution of four different systems

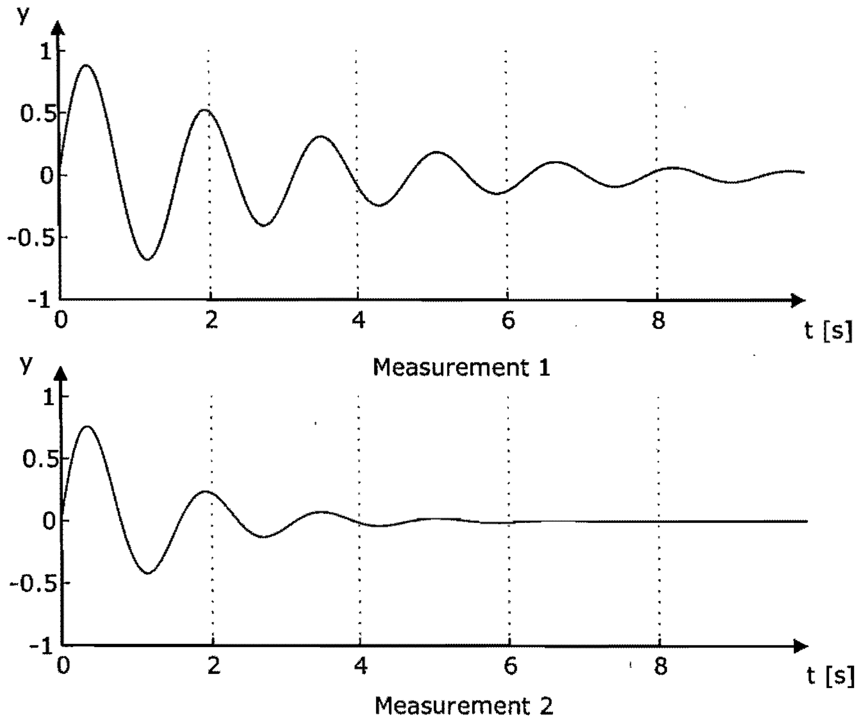
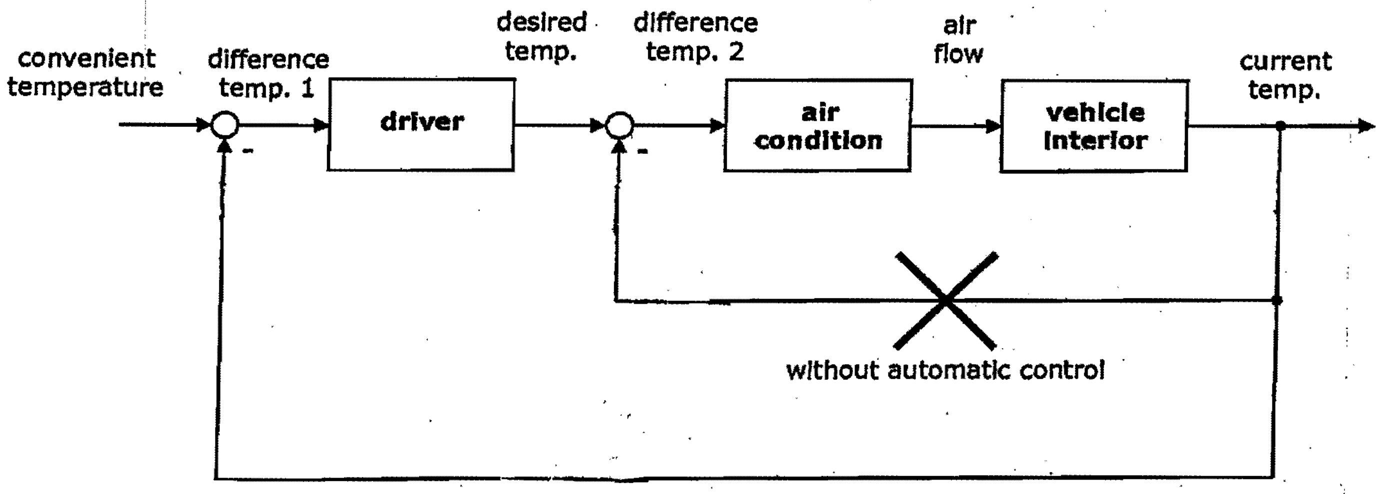


Figure 2.3: Measurement of the dynamic behaviors of two different systems

2e) (3 Points)
In the following, the block diagram describing a vehicle's automatic air condition system (ac) and a driver's behavior (adjusting the ac) has to be given.

The driver senses the temperature inside the vehicle. The difference between the temperature convenient for the driver and the real temperature within the vehicle causes that the driver changes the desired value of the automatic ac to a new one. Depending on the difference between the desired and the real temperature inside the vehicle, the automatic ac heats or cools the air flowing inside the vehicle by affecting humidity.

Mark additionally the part of the block diagram, which does not exist if an air conditioner does not support automatic control regarding the temperature inside the car.


convenient temperature
difference temp. 1
driver
desired temp.
difference temp. 2
air condition
air flow
vehicle interior
current temp.
without automatic control

2f) (5 Points)
The system matrix of a dynamic system is given by

$$A = \begin{bmatrix} -2 & 0 & k_1 \\ 0 & -1 & 2 \\ k_2 & 0 & 0 \end{bmatrix}.$$

Determine the characteristic polynomial $p(\lambda)$ of the system. For which values of the parameter k_1 and k_2 is the system asymptotically stable? Use the Hurwitz criterion for your calculation.

2|| $\det(\lambda E - A) \overset{!}{=} 0$

$$\Leftrightarrow \begin{vmatrix} \lambda+2 & 0 & -k_1 \\ 0 & \lambda+1 & -2 \\ -k_2 & 0 & \lambda \end{vmatrix} \overset{!}{=} 0$$

$$\Leftrightarrow \underbrace{\lambda^3 + 3\lambda^2 + \lambda(2-k_1k_2) - k_1k_2}_{p(\lambda)} \overset{!}{=} 0$$

$a_i > 0$ für $i = 0, \dots, 3$

$\Rightarrow \underline{\underline{k_1k_2 < 0}}$

$$\Rightarrow H = \begin{vmatrix} 3 & -K_1K_2 & 0 \\ 1 & 2-K_1K_2 & 0 \\ 0 & 3 & -K_1K_2 \end{vmatrix}$$

$H_j > 0$ für $j = 1, \dots, 3$

$H_1 = |\ 3\ | > 0$

$H_2 = \begin{vmatrix} 3 & -K_1K_2 \\ 1 & 2-K_1K_2 \end{vmatrix} > 0 \Rightarrow \underline{\underline{k_1k_2 < 3}}$

$H_3 = (-K_1K_2) \cdot H_2 > 0 \Rightarrow K_1K_2 < 0$ and $K_1K_2 < 3$

$\Rightarrow$ asymptotic stable for $k_1k_2 < 0$