Reading-up-time

For reviewing purposes of the problem statements, there is a "reading-up-time" of **10 minutes** prior to the official examination time. During this period it is **not** allowed to start solving the problems. This means explicitly that during the entire "reading-up-time" no writing utensils, e.g. pen, pencil, etc. at all are allowed to be kept on the table. Furthermore the use of carried documents, e.g. books, (electronic) translater, (electronic) dictionaries, etc. is strictly forbidden. When the supervisor refers to the end of the "reading-up-time" and thus the beginning of the official examination time, you are allowed to take your writing utensils. Please **then**, begin with filling in the **complete** information on the titlepage and on page 3.

Good Luck!

LAST NAME	
FIRST NAME	
MATRIKEL-NO.	
TABLE-NO.	

Klausurunterlagen

Ich versichere hiermit, dass ich sämtliche für die Durchführung der Klausur vorgesehenen Unterlagen erhalten, und dass ich meine Arbeit ohne fremde Hilfe und ohne Verwendung unerlaubter Hilfsmittel und sonstiger unlauterer Mittel angefertigt habe. Ich weiß, dass ein Bekanntwerden solcher Umstände auch nachträglich zum Ausschluss von der Prüfung führt. Ich versichere weiter, dass ich sämtliche mir überlassenen Arbeitsunterlagen sowie meine Lösung vollständig zurückgegeben habe. Die Abgabe meiner Arbeit wurde in der Teilnehmerliste von Aufsichtsführenden schriftlich vermerkt.
Durch die Teilnahme versichere ich, dass ich prüfungsfähig bin. Bei Krankheit werde ich die Klausur vorzeitig beenden und unmittelbar eine Ärztin/einen Arzt aufsuchen.

THE ABOVE REQUIRED STATEMENTS AS WELL AS THE SIGNATURE ARE MANDATORY AT THE BEGINNING OF THE EXAM.

Duisburg, ______________ ____
(Date) (Student's signature)

Falls Klausurunterlagen vorzeitig abgegeben: ______________Uhr

Bewertungstabelle

Aufgabe 1	
Aufgabe 2	
Die Bewertung gem. PO in Ziffern ist der xls-Tabelle bzw. dem Papierausdruck zu entnehmen.	

____

(Datum und Unterschrift 1. Prüfer, Univ.-Prof. Dr.-Ing. Dirk Söffker)

____

(Datum und Unterschrift 2. Prüfer, Prof. Dr.-Ing. Mohieddine Jelali, Priv.-Doz.)

____

(Datum und Unterschrift des für die Prüfung verantwortlichen Prüfers, Söffker)

Fachnote gemäß Prüfungsordnung: (alternativ: siehe xls-Tabelle bzw. beigefügter Papierausdruck)

☐	☐	☐	☐	☐	☐	☐	☐	☐	☐	☐
1,0	1,3	1,7	2,0	2,3	2,7	3,0	3,3	3,7	4,0	5,0
sehr gut		gut			befriedigend			ausreichend		mangelhaft

Bemerkung: ______________________________

Attention: Give your answers to ALL problems directly below the questions in the exam question sheet.

You are NOT allowed to use a pencil and also NOT red color (red color is used for corrections).

This exam is taken by me as a
☐ mandatory (Pflichtfach)
☐ elective (Wahlfach)
☐ prerequsite (Auflage)
subject (cross ONE option according to your own situation).

Maximum achievable points:	**60**
Minimum points for the grade 1,0:	**95%**
Minimum points for the grade 4,0:	**50%**

General hints:

1) For the multiple-choice and multiple-choice-similar tasks the following rules are effective:
   i) For tasks with individual evaluation of subtasks, the following applies: Only correct answers are evaluated with the intended number of points.
   ii) The positive points achieved in a subtask are summed up.
   iii) If it is not explicitly described, only one of the given solution options is correct.
   iv) If subtasks contain more than two answer options and only one solution exists: The marking of multiple answer options is interpreted as a non-response due to the not clear declaration of intention. As a result, no points can be given in this case.
2) If in the exam tasks no information is given for the valid range of numbers for time constants or masses etc.: take for time constants (in sec.), for masses (in kg) positive numbers.
3) If in the exam tasks no information is given for applying negative or positive feedback: use the usual negative feedback.

Problem 1 (35 Points)

Mark the correct solution in the following statements.

1a) (2 × 5 × 1 Point, 10 Points)

A1) (1 Point)

Based on $0 = f(x, \dot{x}, u, t),\ y = g(u, x)$

- ◯ a linear SISO-system in the related typical form
- ◯ a nonlinear system in explicit form
- ◯ a nonlinear system in implicit form

is described.

A2) (1 Point)

An open-loop system acts with the purpose to affect the output through the input

- ◯ without feedback.
- ◯ with feedback.
- ◯ not without feedback.

A3) (1 Point)

A signal-flow graph is an equivalent description to a

- ◯ block diagram.
- ◯ state vector.
- ◯ vector field.

A4) (1 Point)

The advanced improvement of the dynamics is one of the central aims

- ◯ in designing control loop with closed effecting sequence.
- ◯ in designing control loop with open effecting sequence.
- ◯ in selecting suitable sensors to measure the disturbance.

A5) (1 Point)

The input variable is $u(t) = a\sin(\omega_0 t)$. From the output side $y(t) = b\sin(\omega_0 t + \pi)$ is measured. With a change on $u(t)$ to $u_2(t) = ab\sin(\omega_0 t)$, $y_2(t) = b^2(\sin(\omega_0 t + b\pi))$ is obtained from the output side. The system is

- ◯ not linear.
- ◯ linear.
- ◯ chaotic.

☐

B1) (1 Point)

The state space is an

- ◯ n-dimensional description of the time behavior of a system
- ◯ $n/2$-dimensional description of the state variables and the related velocities
- ◯ $n/2$-fold each combined with 2-dimensional description of the derivation of state variables based on the state variables

described by A with $n = \dim(x)$.

B2) (1 Point)

The dynamics of a linear system is completely described by

- ◯ the matrix A in combination with B, C, and D.
- ◯ the system matrix A.
- ◯ the ratio between output and input.

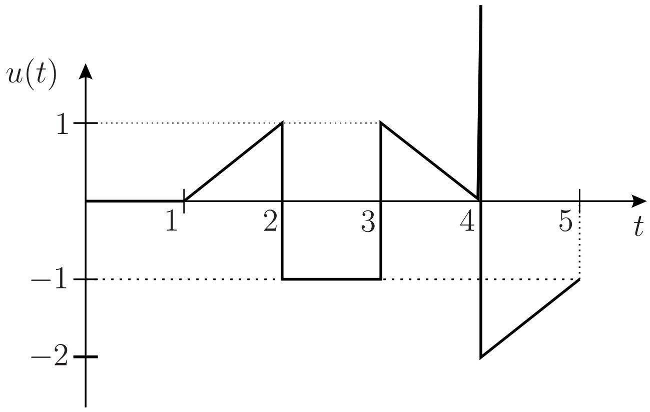
B3) (1 Point)

The dynamics of a PIDT$_1$-system is described by the equation

- ◯ $\frac{1}{{\omega_0}^2}\ddot{y} + \frac{2D}{\omega_0}\dot{y} + y = Ku.$
- ◯ $\frac{T_1}{K}\ddot{y} + \frac{1}{T_\mathrm{D}}\dot{y} = \dot{u} + T_\mathrm{I}u + T_\mathrm{D}\int u\,dt.$
- ◯ $\frac{T_1}{K}\ddot{y} + \frac{1}{K}\dot{y} = \dot{u} + \frac{1}{T_\mathrm{I}}u + T_\mathrm{D}\ddot{u}.$

B4) (1 Point)

The behavior depicted by



can be described by

○ $u(t) = 2(t-2) - 2(t-4)1(t-4) + 2(t-4) - 3(t-3) - (t-3)1(t-3) + \delta(t-4) - 1(t-1) + (t-2)1(t-2).$

○ $u(t) = -(t-2)1(t-2) - 2(t-2) + 2(t-3) - (t-3)1(t-3) + \delta(t-4) - 2(t-4) + (t-1)1(t-1) + 2(t-4)1(t-4).$

○ $u(t) = 1(t-1) - (t-2)1(t-2) + 2(t-2) - 3(t-3) - (t-3)1(t-3) + \delta(t-4) - 2(t-4) + 2(t-4)1(t-4).$

B5) (1 Point)

With respect to $\dot{x} = ax + bu$ the following equation

$$x(t) = \int_0^t e^{a(t-\tau)}\, b\, u(\tau)\, d\tau + e^{at} x_0(t=0)$$

describes

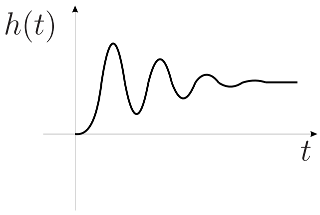
○ the inhomogeneous solution of a 1st order ODE.

○ the homogeneous solution of a 1st order ODE.

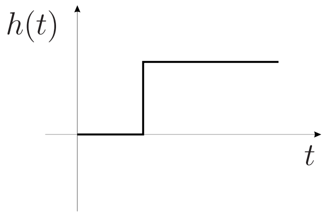
○ the solution of a general 1st order ODE.

1b) (1 Point)

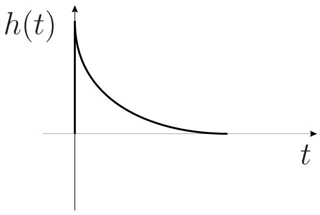
The following step response indicates systems with inertia of higher order ($n \geq 2$).



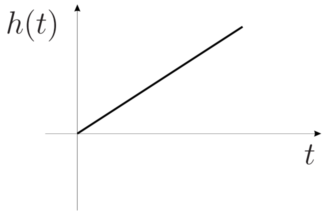
○



○



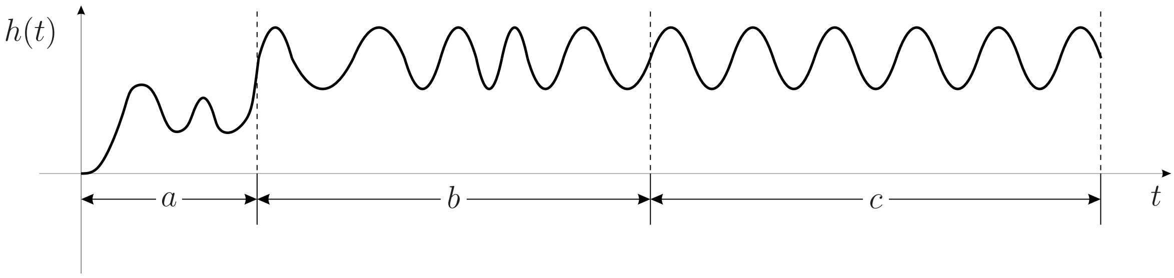
○



○

1c) (1 Point)

The stationary behavior of the system in the following step response is located in



○ the areas a, b, and c.

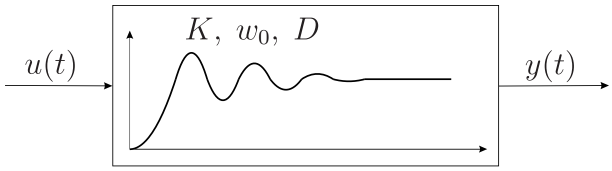
○ the areas b and c.

○ the area c.

1d) (6 × 1 Point, 6 Points)

1) (1 Point)

The SISO-system described by



is considered. The system can be classified as a

◯ PT_1-system.

◯ PT_2-system.

◯ PDT_3-system.

2) (1 Point)

A P-controller is used to control the system described in task 1d)1). All parameters are assumed as 1. The closed loop system (with negative feedback) is

◯ asymptotically stable.

◯ boundary stable.

◯ unstable.

3) (1 Point)

For the impulse function $\delta(t)$ the following statement is always valid:

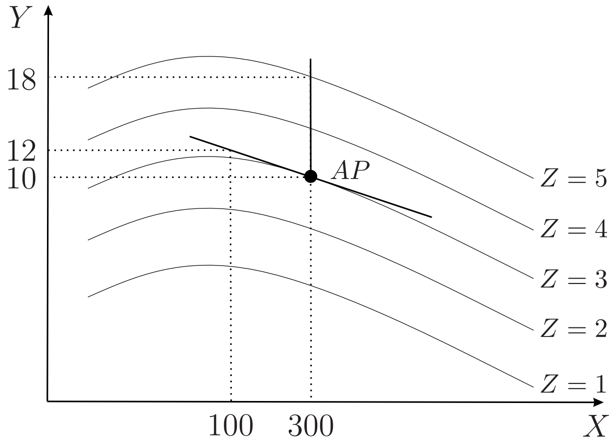
◯ $\delta(t) = \frac{d}{dt}h(t)$.

◯ $\delta(t) = \frac{d}{dt}1(t)$.

◯ $\delta(t) = \frac{d}{dt}g(t)$.

4) (1 Point)

The linearization of the characteristic field around the operating point $AP(X_0, Y_0, Z_0)$



leads to the equation

- ○ $y(x, z) = 0.01x - 4z$.
- ○ $y(x, z) = -0.01x + 4z$.
- ○ $y(x, z) = -0.1x - 4z$.

The origin of the coordinate system (x, y, z) is assumed at the operating point AP.

5) (1 Point)

The I/O-description

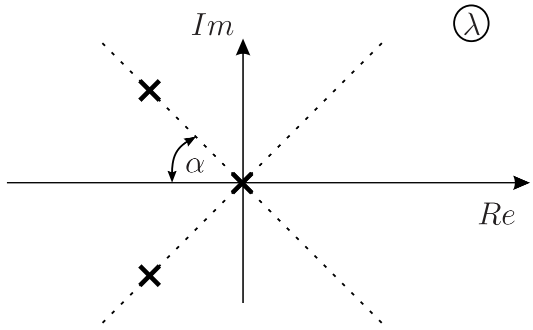
$$y^{(n)} + a_{n-1}y^{(n-1)} + ... + a_0 y = K[u(t) + \frac{1}{T}\int u\ dt];\ a_0 = 1$$

describes

- ○ a SISO-system in general form.
- ○ a MIMO-system with the order n.
- ○ a PI-controller with u as output and y as input.

6) (1 Point)

A system with the eigenvalue distribution



allows variable damping ratios. When the damping ratio increases,

- ◯ the oscillations decay faster.
- ◯ aperiodic boundary case ($D = 1$) cannot appear anymore.
- ◯ stationary/continuous oscillations can occur.

1e) (5 × 1 Point + 4 × 1 Point, 9 Points)

The eigenvalues of four different linear systems without time delay are illustrated in Figure 1.1. Four measured output functions are shown in Figure 1.2. Mark the correct solution in the following statements.

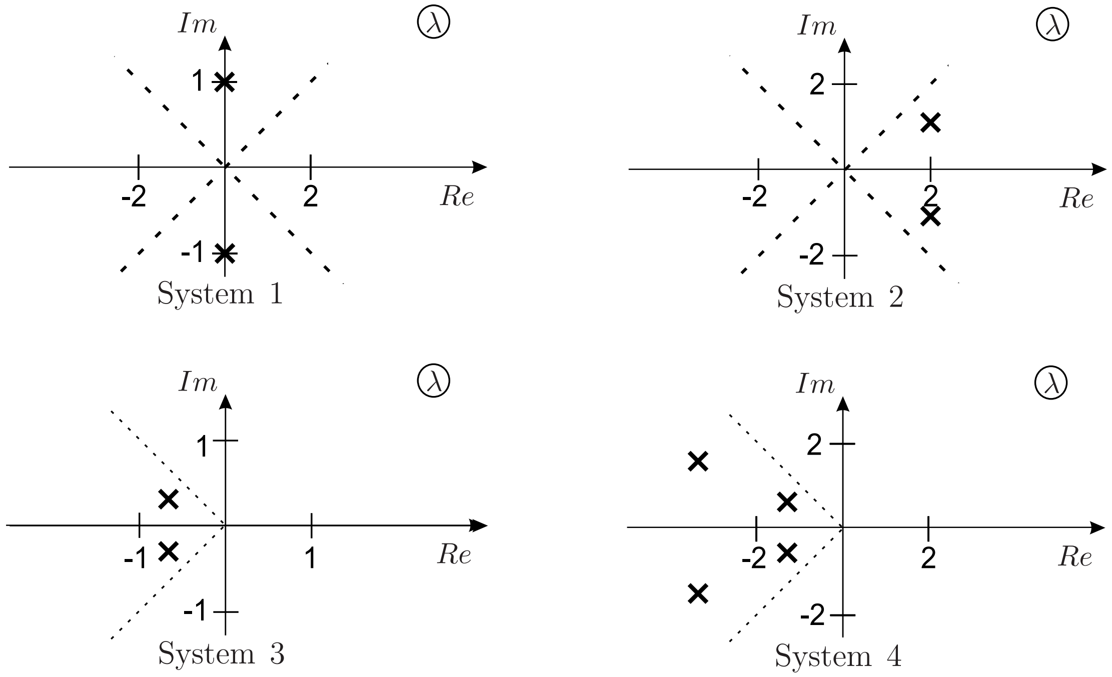


Figure 1.1: Eigenvalue distribution of four different systems

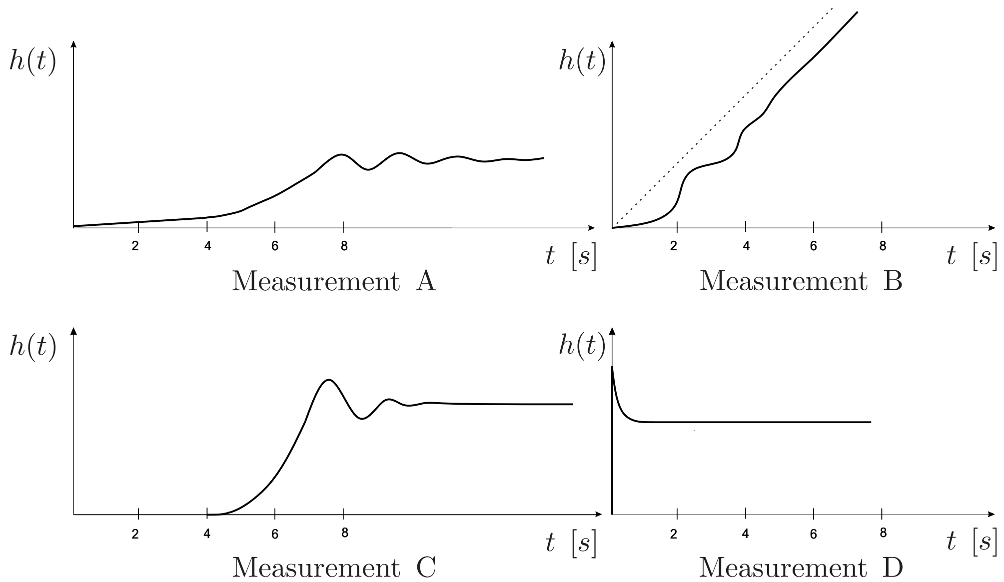


Figure 1.2: Output functions

A1) (1 Point)

Measurement B shows a system

- ◯ able to vibrate/oscillate.
- ◯ with time delay.
- ◯ not able to vibrate/oscillate.

A2) (1 Point)

Measurement D indicates a system behavior, which shows

- ◯ no inertia.
- ◯ inertia.
- ◯ vibration/oscillation.

A3) (1 Point)

Measurement C shows the following behavior:

- ◯ a system able to vibrate/oscillate without time delay.
- ◯ a system able to vibrate/oscillate with time delay.
- ◯ a system able to vibrate/oscillate without damping.

A4) (1 Point)

Measurement D shows the step response of a

- ◯ PD-system.
- ◯ PDT_1-system.
- ◯ DT_1-system.

A5) (1 Point)

Measurement B could

- ◯ correspond to the behavior of a PDT_1T_t-system with $T_t > 0$.
- ◯ not be classified.
- ◯ correspond to the behavior of a IT_2-system.

B1) (1 Point)

System 3 can be described as an example by the equation

- ◯ $\ddot{y} + y = u$.
- ◯ $\ddot{y} + 1.6\dot{y} + y = u$.
- ◯ $\ddot{y} + 3\dot{y} + y = u$.

B2) (1 Point)

System 4 can be described

- ◯ with a technically meaningful equation.
- ◯ with no technically meaningful equation.
- ◯ with an equation though, such systems cannot be found in technology yet.

B3) (1 Point)

System 3 shows damping ratio

- ◯ of $D = \frac{\sqrt{2}}{2}$ at least.
- ◯ of $D = \frac{\sqrt{2}}{2}$ maximally.
- ◯ by which continuous oscillations appear very easily, which is also very dangerous.

B4) (1 Point)

Systems 4 shows 2 asymptotically stable conjugate complex pole pairs. The two pole pairs represent the following characteristic:

- ◯ identical eigenfrequency.
- ◯ identical damping ratio.
- ◯ 4 different eigenmotions.

1f) (8 Points)

The control loop $u \rightarrow y$ has to be considered. The related dynamic behavior of the components is illustrated in Figure 1.3.

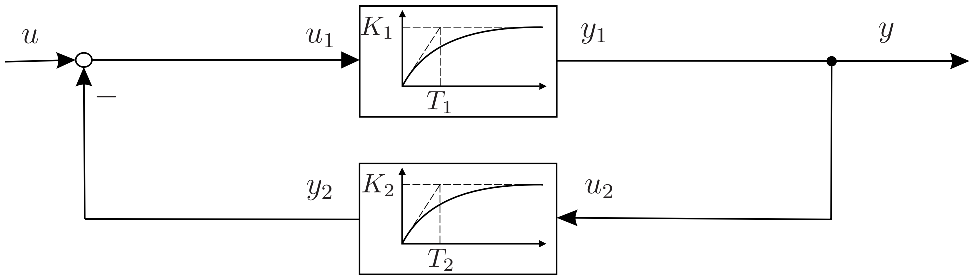


Figure 1.3: Control loop

1f) i) (4 Points)
State reasons (with the help of an eigenvalue calculation), if stable behavior of the control loop can be obtained. Under which conditions does aperiodic boundary case exist?

1f) ii) (2 Points)

Calculate the static gain K_S of the controlled system. For which controller setting K_2 is the static gain of the controlled system minimized? State reasons for your statement by calculation.

1f) iii) (2 Points)
How should the controller parameters be chosen to achieve the slowest possible disturbance accommodation?

$\sum$

Problem 2 (25 Points)
The block diagram of a system consisting of four transfer elements with w, u as inputs and y as output is given (refer to Figure 2.1).

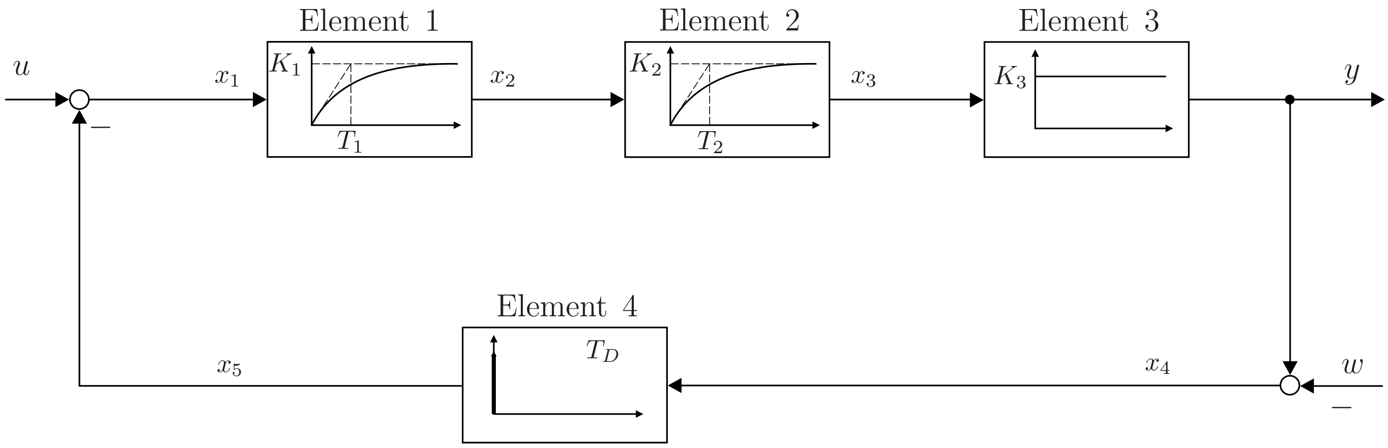


Figure 2.1: Block diagram of the system

2a) (4 Points)
Classify the transfer behaviors (type of single transfer element behavior) of the elements 1 to 4 and give the corresponding differential equations using the given notation of inputs and outputs in a form suitable for classification.

2b) (3 Points)
Determine the differential equation of the entire system illustrated in Figure 2.1 with the parameters $T_1 = T_2 = T_\mathrm{D} = K_1 = K_2 = K_3 = 1$. Here, the input is defined as w and the output as y.
Classify the transfer behavior of the entire system.

The mathematical model of a hydraulic propulsion (refer to Figure 2.2) is piecewise described by the equation

$$T_1\dot{y} + y = K[u + \frac{1}{T_{I1}}\int u\,dt],$$

where the symbols denote

y : position of cylinder and
u : valve voltage.

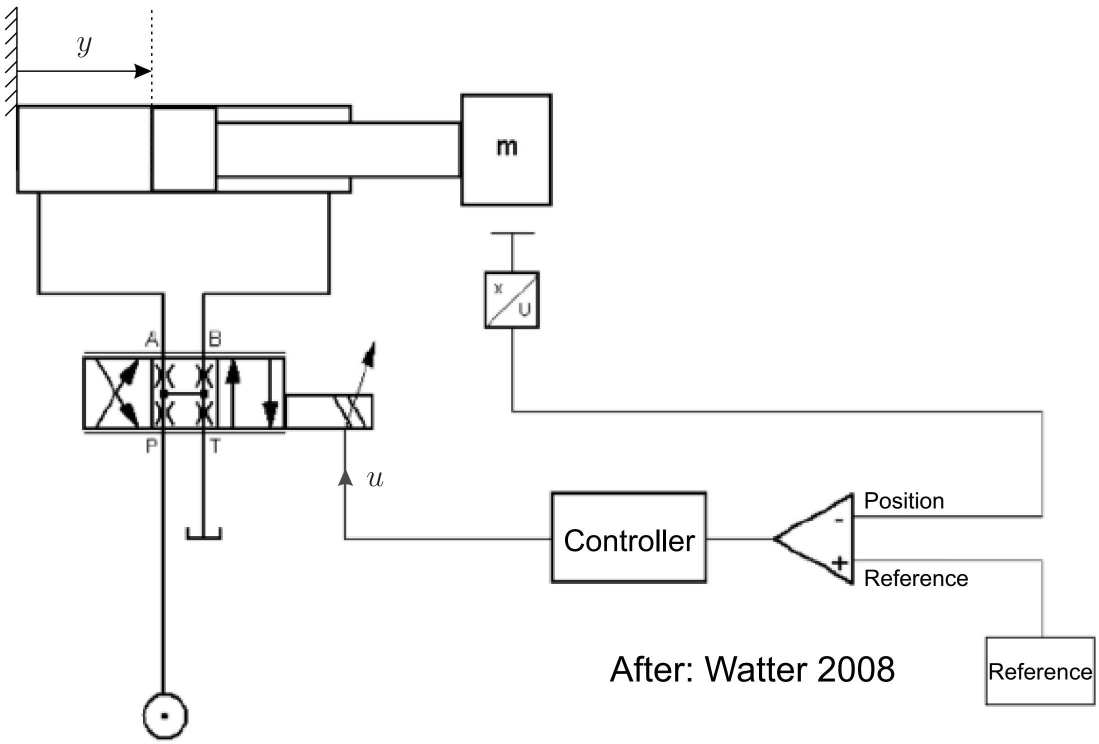


Figure 2.2: Model of a hydraulic system

2c) (2 Points)
Define the state space model of the system using $x = [y \quad \int u]^T$ as unconventional state vector, u as the input, and y as the output.

2d) (1 Point)
The position of the cylinder in the system is controlled with a PI-controller. Give the corresponding equation of the controller. Use the given notations from the system.

2e) (4 Points)
Determine the differential equation of the transfer behavior with respect to the desired value with negative feedback. Classify the transfer behavior with respect to the desired value of the controlled system. The second order derivative of the input is filtered out with a filter (set to zero).

2f) (4 Points)

For the following task the parameters are given: $K_1 = T_1 = T_{\text{I1}} = T_{\text{I2}} = 1$ and $\lambda_1 = -1$.
Can the system have conjugate complex eigenvalues? State the corresponding conditional equation. Under which conditions does the system show the damping $D = 0.5$?

2g) (5 Points)

A system is described by

$$A = \begin{bmatrix} 6 & 0 & 8 \\ a & b & -4 \\ 10 & 0 & -5 \end{bmatrix}.$$

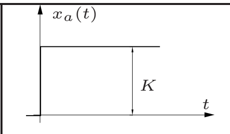
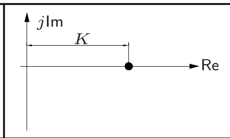
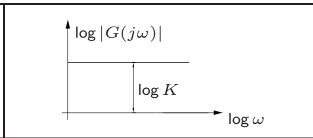
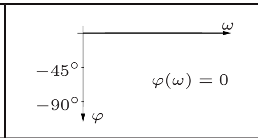
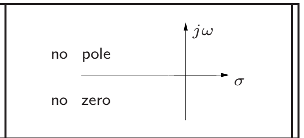
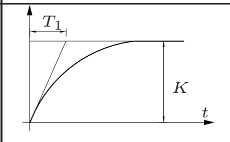
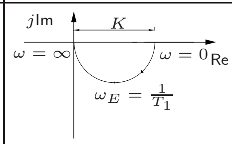
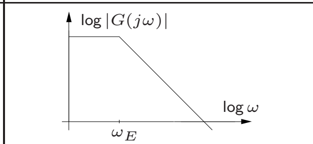
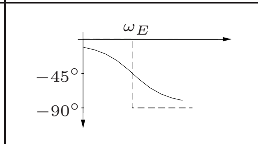
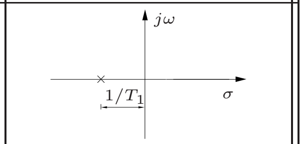
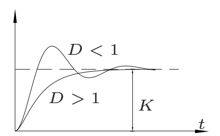
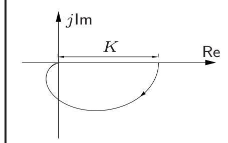
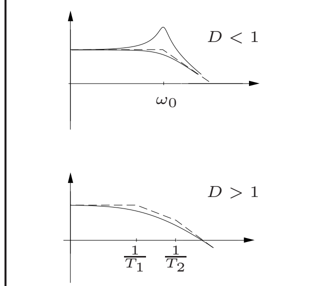
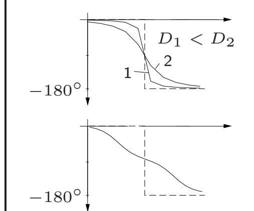
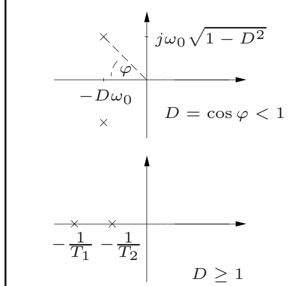
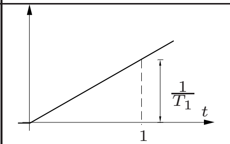
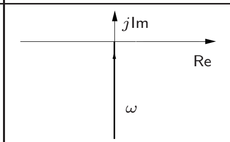
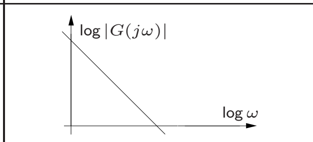
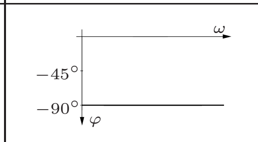
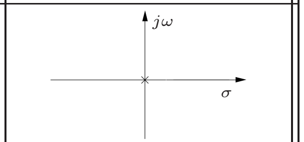
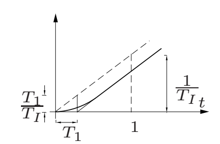
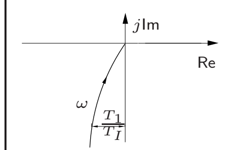
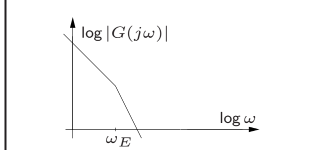
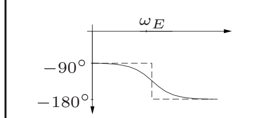
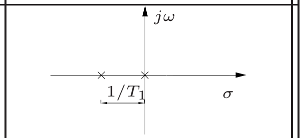
One eigenvalue is $\lambda_1 = b$ and the corresponding eigenvector is $V_1 = [0 \quad 1 \quad 0]^T$. Calculate the eigenvalues λ_2 and λ_3 as well as the missing elements v_{22}, v_{23}, v_{31}, and v_{32} of the corresponding eigenvectors $V_2 = [1 \quad v_{22} \quad v_{23}]^T$ and $V_3 = [v_{31} \quad v_{32} \quad 1]^T$. Show mathematically if $V_4 = [-\frac{1}{2} \quad 2 \quad 1]^T$ can be eigenvector of the system. (*Hint:* $\sqrt{441} = 21$.)

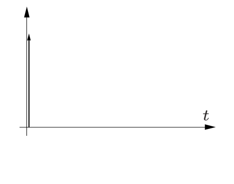
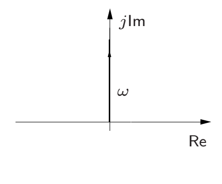
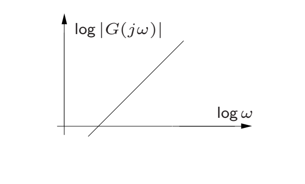
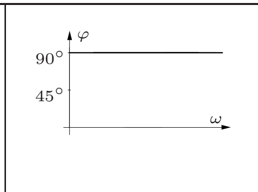
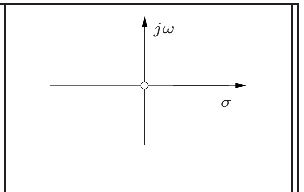
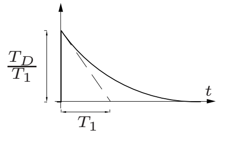
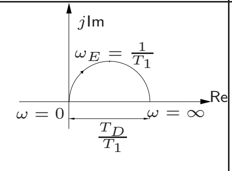
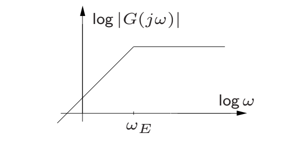
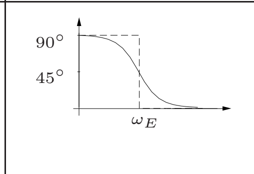
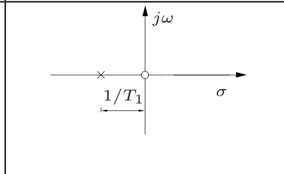
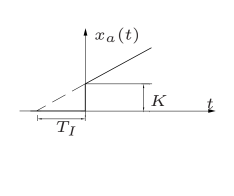
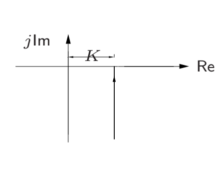
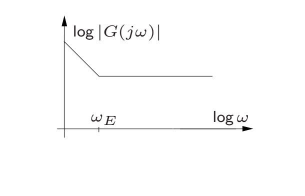
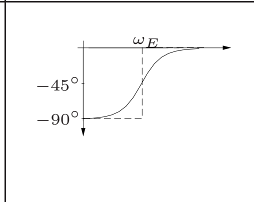
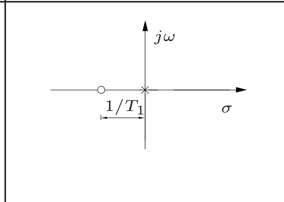
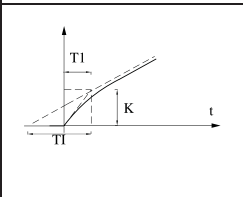
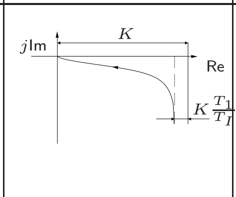
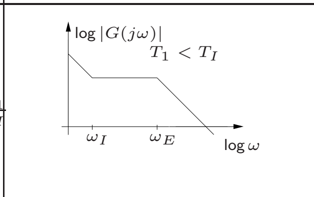
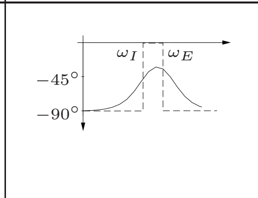
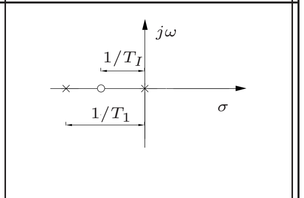
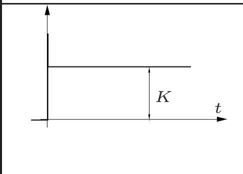
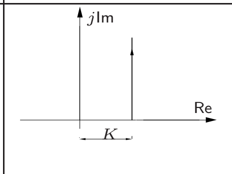
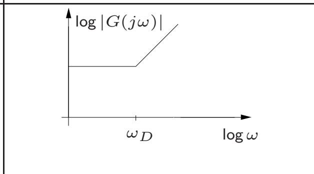
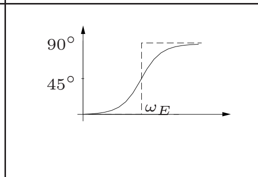
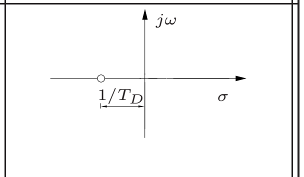
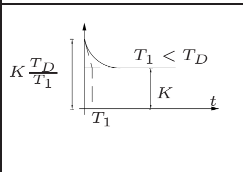
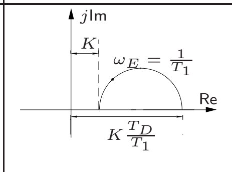
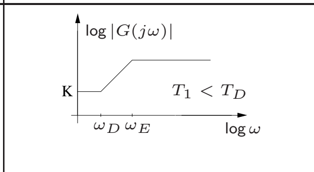
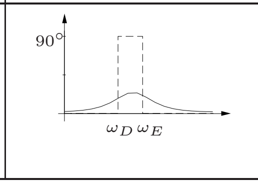
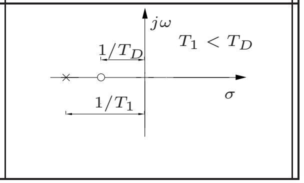
2h) (2 Points)
The system matrix of a control system is described by

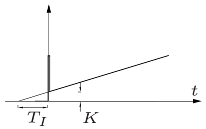
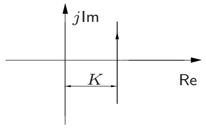
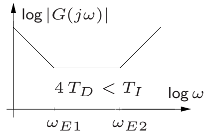
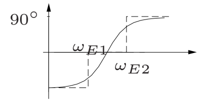
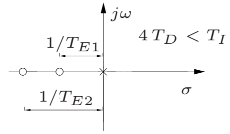
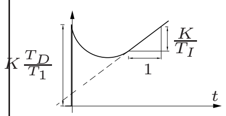
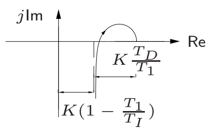
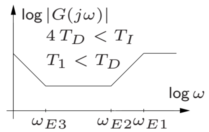
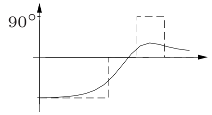
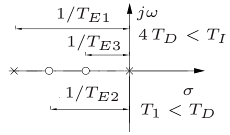
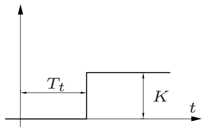
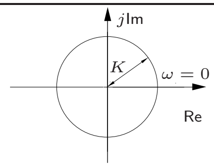
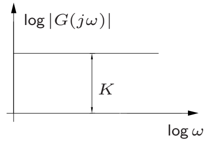
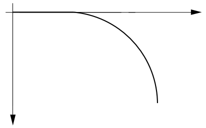
$$A = \begin{bmatrix} -3a & 2 \\ -4a^2 & -9a \end{bmatrix}.$$

Calculate the characteristic polynomial $p(\lambda)$ of the system depending on the parameter a. For which parameters a can the system be asymptotically stable?

Σ

System	Time domain	Step response	Polar plot	Bode–diagram (Amplitude behavior)	(Phase behavior)	s–Plane × Pole ○ Zero
P	$x_a(t) = K\,x_e(t)$ $G(s) = K$					
PT_1	$T_1\,\dot{x}_a(t) + x_a(t) = K\,x_e(t)$ $G(s) = K\frac{1}{1+T_1\,s}$					
PT_2	$\frac{1}{\omega_0^2}\ddot{x}_a(t) + \frac{2\,D}{\omega_0}\dot{x}_a(t) + x_a(t) = K\,x_e(t)$ $G(s) = K\frac{1}{\frac{1}{\omega_0^2}s^2 + \frac{2\,D}{\omega_0}s + 1}$ $D < 1$: Complex conjugated roots of the charact. equation $\lambda_{1,2} = -\omega_0(D \pm j\sqrt{1-D^2})$ $D \geq 1$: Real roots of the charact. equation $\lambda_{1,2} = -\omega_0(D \pm j\sqrt{1-D^2}) = -1/T_{1,2}$					
I	$x_a(t) = \frac{1}{T_I}\int x_e \mathrm{d}t$ $G(s) = \frac{1}{T_I\,s}$					
IT_1	$T_1\,\dot{x}_a(t) + x_a(t) = \frac{1}{T_I}\int x_e(t)\mathrm{d}t$ $G(s) = \frac{1}{T_I\,s(1+T_1\,s)}$					

System	Time domain	Step response	Polar plot	Bode–diagram (Amplitude behavior)	Bode–diagram (Phase behavior)	s–Plane × Pole ○ Zero
D	$x_a(t) = T_D \frac{dx_e}{dt}$ $G(s) = T_D\, s$					
DT_1	$T_1\dot{x}_a(t) + x_a(t) = T_D \frac{dx_e}{dt}$ $G(s) = T_D \frac{s}{1 + T_1\, s}$					
PI	$x_a(t) =$ $K\left[x_e(t) + \frac{1}{T_I}\int x_e(t)dt\right]$ $G(s) = K\left[1 + \frac{1}{T_I\, s}\right]$					
PIT_1	$T_1\,\dot{x}_a(t) + x_a(t) =$ $K\left[x_e(t) + \frac{1}{T_I}\int x_e dt\right]$ $G(s) = K\frac{1 + \frac{1}{T_I\, s}}{1 + T_1\, s}$					
PD	$x_a(t) = K\left[x_e(t) + T_D\frac{dx_e}{dt}\right]$ $G(s) = K\,[1 + T_d\, s]$					
PDT_1	$T_1\dot{x}_a(t) + x_a(t) =$ $K\left[x_e(t) + T_D\frac{dx_e}{dt}\right]$ $G(s) = K\frac{1 + T_D\, s}{1 + T_1\, s}$					

<table>
<tr><th rowspan="2">System</th><th rowspan="2">Time domain</th><th rowspan="2">Step response</th><th rowspan="2">Polar plot</th><th colspan="2">Bode–diagram</th><th>s–Plane</th></tr>
<tr><th>(Amplitude behavior)</th><th>(Phase behavior)</th><th>× Pole ◯ Zero</th></tr>
<tr><td>PID</td><td>$x_a(t) =$
$K\left[x_e(t) + \frac{1}{T_I}\int x_e \mathrm{d}t + T_D\frac{\mathrm{d}x_e}{\mathrm{d}t}\right]$
$G(s) = K\left[1 + T_D\, s + \frac{1}{T_I\, s}\right]$</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>$PIDT_1$</td><td>$T_1\dot{x}_a(t) + x_a(t) =$
$K\left[x_e(t) + \frac{1}{T_I}\int x_e \mathrm{d}t + T_D\frac{\mathrm{d}x_e}{\mathrm{d}t}\right]$
$G(s) = K\frac{1 + T_D\, s + \frac{1}{T_I\, s}}{1 + T_1\, s}$</td><td></td><td></td><td></td><td></td><td></td></tr>
<tr><td>$(P)T_t$</td><td>$x_a(t) = K\, x_e(t - T_t)$
$G(s) = K\, e^{-s\, T_t}$
$K = 1$ for pureT_t-element</td><td></td><td></td><td></td><td></td><td>Pole at $-\infty$
Zero at $+\infty$</td></tr>
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