

LAST NAME	
FIRST NAME	
MATRIKEL-NO.	

Problem 1

(each with 2 points)

- Draw the basic structure of the classical control loop and denote the signals disturbance, desired (reference) value, and control variable as well as the transfer elements plant and controller.
- Describe the difference between open- and closed-loop control.
- What is a SISO system? Which are the two typical forms used to describe the input/output behavior of a system?
- A system of 3rd order, described by

$$a_3 \frac{d^3 y}{dt^3} + a_2 \frac{d^2 y}{dt^2} + a_1 \frac{dy}{dt} + a_0 y = b_0 u$$

has the input u and the output y . The initial conditions are $y(0) = a$, $\dot{y}(0) = b$ and $\ddot{y}(0) = c$. Find the corresponding Laplace transform.

- The steering behavior of a human driver is approximated by the frequency response $G(j\omega) = K e^{-j\omega T_t} / (1 + T_1 j\omega)$. Describe the character of the parameters T_1 and T_t .

Problem 2

(each with 2 points)

- a) You read in the newspaper: "The **allocation oriented** pension system will be replaced by a **capital oriented** one. [...] The reserves of the pension assurance fund will be reduced to the amount of one month to hold up the pension contributions."
- >>**allocation oriented**<< means, that the income of the pension assurance fund is used to finance the outgo of the fund.
- How would you describe the actual input/output behavior of the fund with respect to the control oriented nomenclature?
- b) What do proportional, integral and derivative transfer elements mean respectively?
- c) In control engineering, feedback is also used to compensate system disturbances. Looking at a proportional plant: what kind of feedback (controller) can be used to cancel the effect of the disturbance by using only small control signals?
- d) Give the basic structure of a PIDT₂-element in the time and in the frequency domain. Thereby denote the input and output signals.
- e) A given system has the transfer function

$$G(s) = K \frac{(s - T_{n1})(s - T_{n2})}{(s - T_{p1})(s - T_{p2})(s - T_{p3})}.$$

Under which conditions is the system stable?

Problem 3

(10 points)

- a) A system with the transfer function

$$G(s) = \frac{10(s + 0.00001)(s - 10)}{s(s - 10000)(s^2 + 0.01s + 0.0001)}$$

is given. Characterize the transfer behavior of the system by drawing the corresponding BODE-diagram including the poles and zeros of the system. Pay attention to the axes labels.

- b) The system given in problem 3a) is now followed by a time delay system with $T_t = 0.1$ s. First point out the principle transfer behavior of the time delay system by drawing its BODE-diagram. Then, draw in the changes caused by the time delay system in the already made figure from problem 3a).
- c) You are working on the synthesis of a closed loop control. Up to now you only know the behavior of the plant because of its measured Nyquist plot. For control you chose a PI-controller with the known controller parameters K and T_I . By means of which (for example graphic-) method can the stability of the whole closed loop system be checked and also the corresponding controller coefficients be determined approximatively?
- d) You are working on the synthesis of a closed loop control. Up to now you only have a mathematical model (differential equation) of the plant, derived by theoretical modelling. For control you choose a PID T_2 -controller with the controller parameters K , T_I , T_D , T_1 and T_2 . By means of which method can the stability of the whole closed loop system be checked and also the corresponding controller coefficients be quantified?
- e) You are working on the synthesis of a closed loop control. Up to now you only know the (measured or mathematically calculated) transfer function of the plant. For control you choose an PDT T_1 -controller with the controller parameters K , T_D , T_1 . In which way can you picture the stability of the whole closed loop system in dependence of a single coefficient? In which way can you thereby take the damping of single eigenvalues into account?

Problem 4

(15 points)

- a) The modeling of the different elements in a technical system leads to the following transfer functions

$$G_1(s) = \frac{K_1}{1 + T_1 s},$$

$$G_2(s) = K_p,$$

$$G_3(s) = \frac{K_I}{s(1 + T_1 s)}, \text{ and}$$

$$G_4(s) = \frac{K_2}{1 + T_2 s + T_3^2 s^2}.$$

The elements are connected in the following manner:

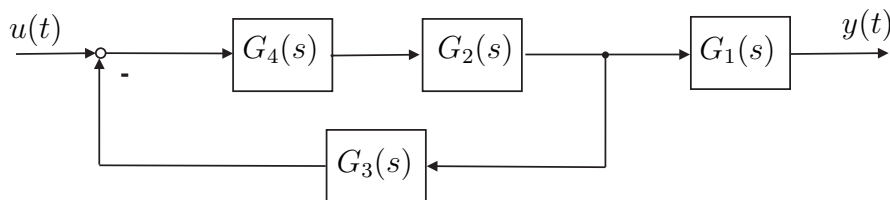


Figure 4.1: Technical system

Please derive the transfer function $G(s)$ of the system from $u(t)$ to $y(t)$.

- b) For the system given in problem 4a) the following parameters are given:

$$T_1 = T_1 = T_2 = 0.5s$$

$$T_3 = 1s$$

$$K_p = K_2 = 2$$

$$K_I = K_1 = 1$$

Classify the transfer system. Is the system stable?

- c) The system given with problem 4a) is simplified with

$$G_1(s) = 1,$$

$$G_2(s) = K_p,$$

$$G_3(s) = \frac{K_I}{s}, \text{ and}$$

$$G_4(s) = 1$$

with $K_p = -4$, $K_I = 3$.

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Calculate the poles and zeros of the system. Is the system stable?

For feedback a PI-controller with the transfer function $K(s) = K_R \frac{1+T_R s}{T_R s}$ is chosen. Draw the block diagram of the complete closed loop system. Derive the transfer function of the complete closed loop system. Assume $T_R = 1$, calculate the range for K_R in which the closed loop system is stable.

- d) Draw the frequency diagram for the systems given with problem 4c) (the simplified, uncontrolled system and the system controlled by the PI-controller with $K_R = -3.001$).
- e) For a technical system with negative feedback the following open loop Nyquist plot was measured. It is known, that the system has a pole at the origin. On the basis of which method can the stability range for the closed loop system be calculated in dependence on the system amplification K ? Is the closed loop system stable or not? Explain your answer.

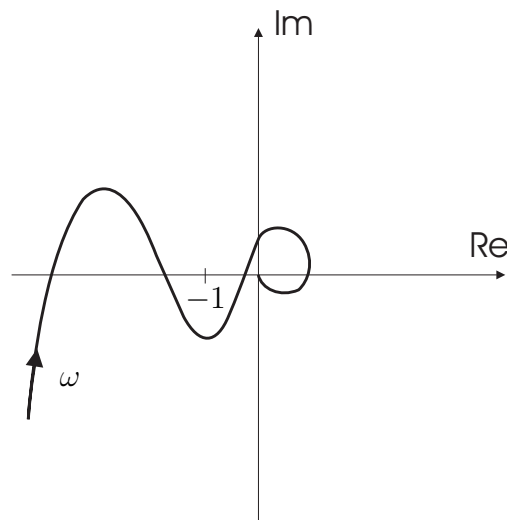
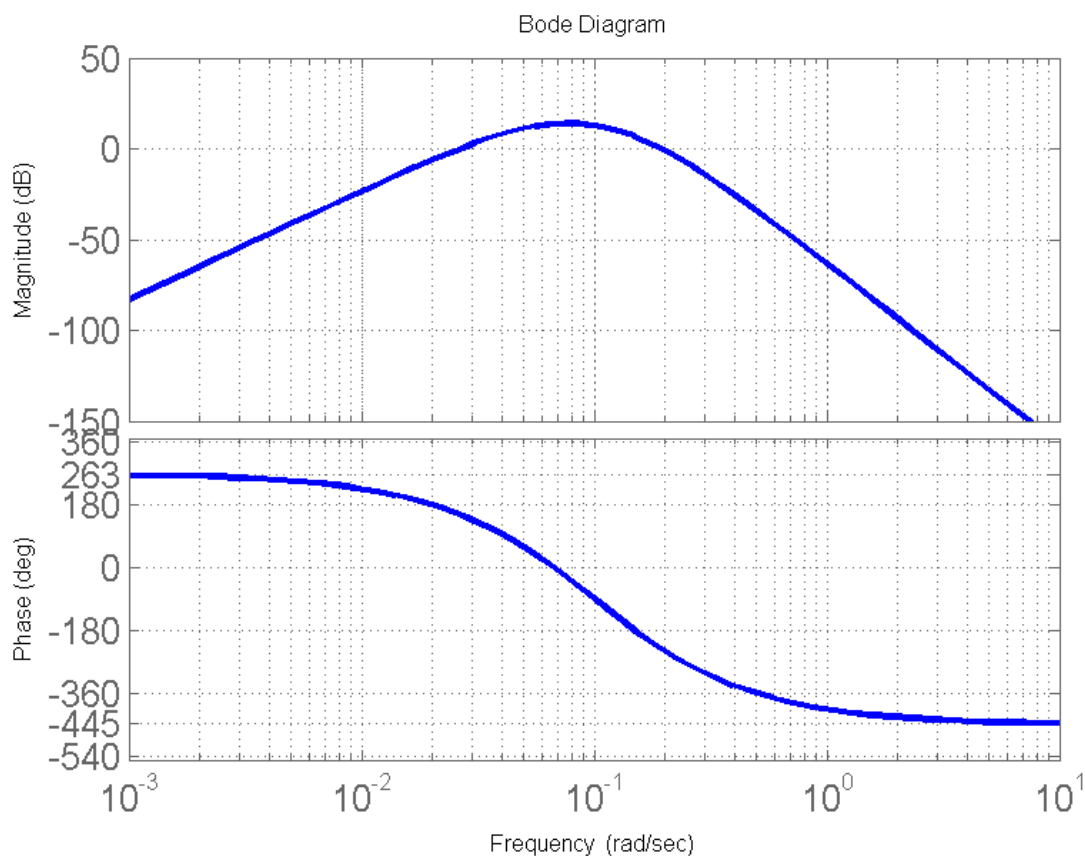


Figure 4.2: Nyquist plot

Problem 5

(15 points)

- a) The behavior of a plant is given by the IT₁-Element $G(s) = \frac{K_I}{s(1+T_1s)}$. For the control of the plant the PI-element with $G_C(s) = \frac{K_P(1+T_n s)}{T_n s}$ is used. Derive the conditions for the stability of the closed loop system in dependence on the controller coefficients ($T_1, T_n > 0$). Hint: Use the Hurwitz criterion.
- b) For the measured BODE-diagram (see below) of an open loop system the gain margin for the corresponding closed loop system has to be determined. Draw the Nyquist plot of the open loop system. Both figures: Draw in the gain margin! What can you say about the stability of the closed loop system using the gain margin criterion?

**Figure 5.1:** Bode-diagram for problem 5b)

- c) The transfer function of a plant is given by

$$G_P(s) = \frac{K}{(s + a - ib)(s + a + ib)(s + 4)}$$

The system is controlled by a PIDT₁-feedback with:

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$$G_C(s) = \frac{K_p(T_n s + T_n s^2 T_D + 1 + T_D s + T_v s^2 T_n)}{T_n s(1 + T_D s)}.$$

It is imperative: $T_n = 1$, $T_v = 2$, $T_D = 4$, $a = 2$, $b = 2$, $K = 2$. Derive the transfer function of the open control loop. Calculate the poles and the zeros. Is the open control loop stable?

- d) Draw the root locus of the system given in problem 5c).
- e) Draw in the stability limit K_{crit} and describe a way to calculate K_{crit} .

Problem 6

(40 points)

The control loop given in figure 6.1 has to be analyzed:

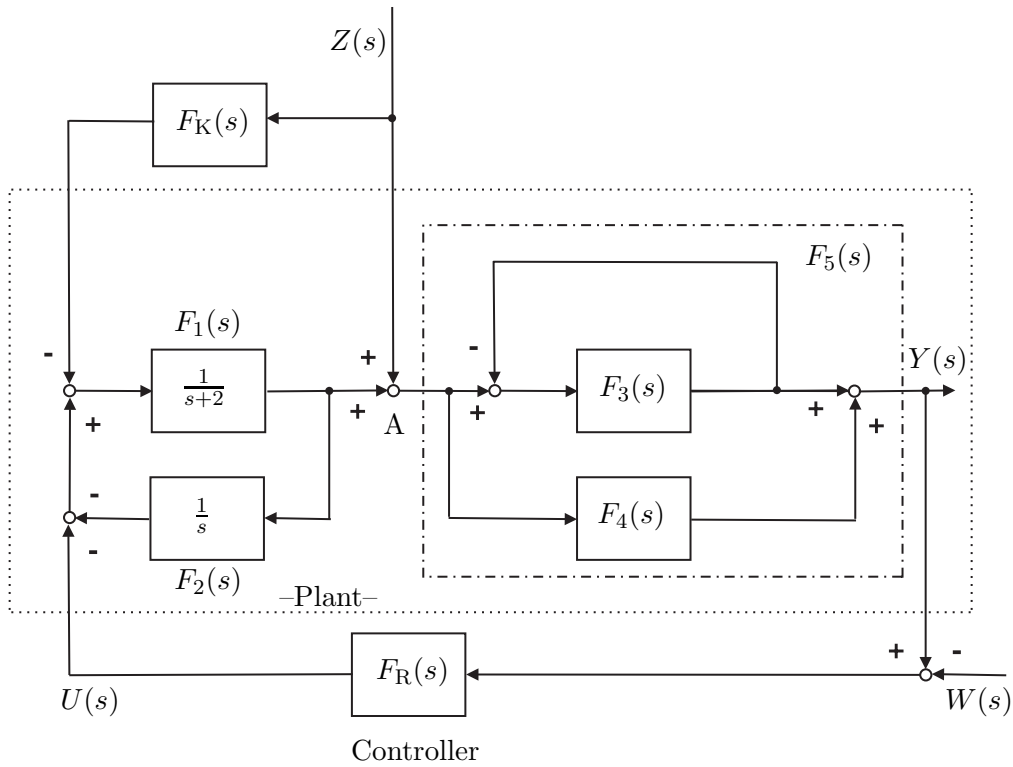


Figure 6.1: Block diagram

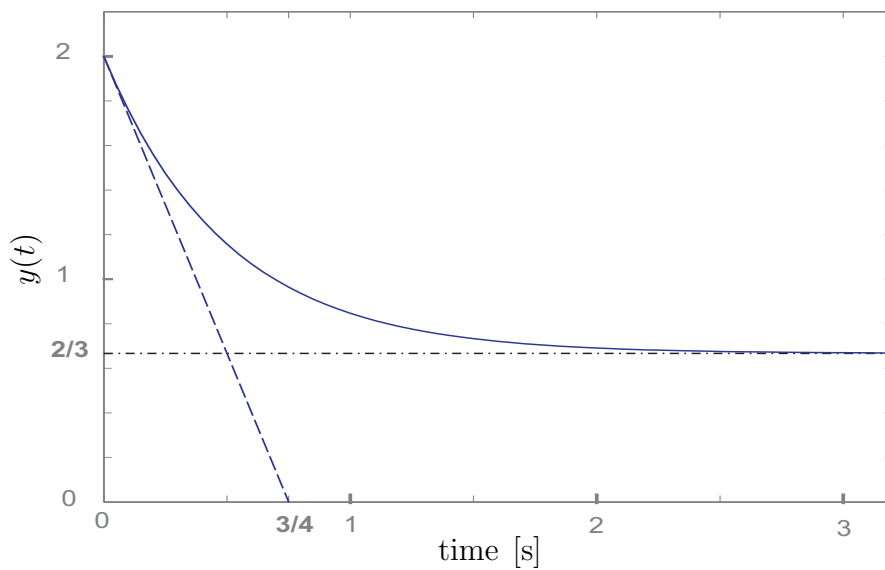


Figure 6.2: Impulse response

The subsystem described by the transfer function $F_5(s)$ is a PIT₁-System with

$$F_5(s) = K \frac{1 + \frac{1}{T_I s}}{1 + T_1 s} .$$

The measured impulse response of this subsystem is given with figure 6.2.

- a) Determine the transfer function $F_5(s)$: calculate the needed parameters K and T_I for given $T_1 = 0.5\text{s}$ with the help of figure 6.2. Hint: use the finite-value or initial-value theorem of the Laplace transformation!
- b) Determine the transfer functions $F_3(s)$ and $F_4(s)$. Each of the hereby described blocks contain an integral-part.

For the following calculations use the parameters $K = 1$, $T_I = 1$, $T_1 < T_I$ for the transfer function $F_5(s)$.

- c) Determine the transfer function $F_s = Y(s)/U(s)$ of the complete plant! Sketch the Nyquist plot of the corresponding frequency response $F(j\omega)$.
- d) For the control of the plant a PI-controller with

$$F_R = K_R \left(1 + \frac{1}{s}\right)$$

is used. Sketch the Nyquist plot of the opened loop system. Is the closed loop system stable or not? Why?

- e) In the following, the disturbance reaction of the plant has to be analyzed. The measurable disturbance $Z(s)$ acts on the plant at point A. By a disturbance feed-forward through $F_K(s)$, the influence of $Z(s)$ on the plant output $Y(s)$ can be compensated.

Is such a disturbance feedforward necessary, considering only steady state system behavior and $K_R < \infty$? If yes, determine the transfer function $F_K(s)$.

Hint: First determine the disturbance-transfer-function. Then use again the limit theorem of the Laplacetransformation and analyze the case $w(t) \equiv 0$, $z(t) = 1(t)$ and $0 < K_R < \infty$.

Maximum achievable points:	100
Minimum points for the grade 1,0:	95
Minimum points for the grade 4,0:	50